Total No. of Questions : 8]

SEAT No. :

P4402

[Total No. of Pages : 3

[5458]-108

F.E.

ENGINEERING MATHEMATICS - I

(2015 Pattern) (Credit System)

Time : 2 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Solve Q.1 or Q.2, Q.3 or Q.4 , Q.5 or Q.6, Q.7 or Q.8.*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Figures to the right indicate full marks.*
4) *Use of electronic pocket calculator is allowed.*
5) *Assume suitable data, if necessary.*

Q1) a) Examine for consistency of system of equations **[4]**

$x + y - 3z = -1$

$4x - 2y + 6z = 8$

$15x - 3y + 9z = 21$

if consistent solve it.

b) Find eigen values of the matrix. **[4]**

$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

Also find eigen vector corresponding to smallest eigen value.

c) Two opposite vertices of a square are represented by complex numbers $9 + 12i$ and $-5 + 10i$. Find the complex number representing the other two vertices of the square. **[4]**

OR

Q2) a) Examine for Linear dependence or independence of vectors $x_1 = (3, 1, -4)$, $x_2 = (2, 2, -3)$, $x_3 = (0, -4, 1)$. If dependent find the relation between them. **[4]**

b) Solve $x^4 + x^3 + x^2 + x + 1 = 0$, by using DeMoivre's theorem. **[4]**

c) If $\sinh(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that $\sinh^4\theta = \cos^4\phi$. **[4]**

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Q3) a) Solve any one : **[4]**

i) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(n+1)^n}{n!}$.

ii) Test the convergence of the series $\frac{1}{1+2^{-1}} + \frac{2}{1+2^{-2}} + \frac{3}{1+2^{-3}} + \ldots\ldots$

b) Prove that $\log(1 + x + x^2 + x^3 + x^4) = x + x^2/2 + x^3/3 + x^4/4 - \frac{4}{5}x^5 + \ldots\ldots$ **[4]**

c) Find n^{th} derivative of $y = \frac{1}{(x-1)^2(x-2)}$. **[4]**

OR

Q4) a) Solve any one : **[4]**

i) Find a & b, if $\lim_{x\to 0} \frac{x(-a\cos x + 1) + b\sin x}{x^3} = \frac{1}{3}$.

ii) Prove that $\lim_{x\to\infty} \left(\frac{ax+1}{ax-1}\right)^x = e^{2/a}$.

b) Expand $2x^3 + 7x^2 + x - 6$ in powers of $(x - 3)$. **[4]**

c) If $y = a\cos(m\log x) + b\sin(m\log x)$, show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+m^2)y_n = 0$. **[4]**

Q5) Solve any two :

a) If $u = \log(x^3 + y^3 - x^2y - xy^2)$, prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = \frac{-4}{(x+y)^2}$. **[6]**

b) If $x = e^u \tan v$, $y = e^u \sec v$, find the value of $\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right] \cdot \left[x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right]$. **[7]**

c) If $v = f(e^{x-y}, e^{y-z}, e^{z-x})$ then show that $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = 0$. **[6]**

OR

Q6) Solve any two :

a) Find $\frac{du}{dx}$ if $u = x.\log(xy)$ and $x^3 + y^3 + 3xy = 0$. **[6]**

b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$$ **[7]**

c) If $x^2 = au + bv$, $y^2 = au - bv$ prove that $(u_x)_y \cdot (x_u)_v = (v_y)_x \cdot (y_v)_u$ where a, b are constants. **[6]**

Q7) a) If $ux = yz$, $vy = zx$, $wz = xy$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **[4]**

b) Examine for functional dependence $u=y+z$, $v=x+2z^2$, $w=x-4yz-zy^2$. **[4]**

c) Find the extreme values of $f(x, y) = 3x^2 - y^2 + x^3$. **[5]**

OR

Q8) a) If $u = x + y^2$, $v = y + z^2$, $w = z + x^2$ find $\left(\frac{\partial x}{\partial u}\right)_{v,w}$ by using Jacobians. **[4]**

b) The area of a triangle ABC, is calculated from the formula $\Delta = \frac{1}{2}\, bc \sin A$. Errors of 1%, 2% & 3% respectively are made in measuring b, c, A. If the correct values of A is 45°. Find the % error in the calculated values of Δ. **[4]**

c) Find stationary values of $a^3x^2 + b^3y^2 + c^3z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$. **[5]**

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