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S.E. (Mech/Auto./S/W) (I Sem.) EXAMINATION, 2019

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours

Maximum Marks : 50

- N.B. :—** (i) Neat diagrams must be drawn wherever necessary.
(ii) Figures to the right indicate full marks.
(iii) Use of electronic pocket calculator is allowed.
(iv) Assume suitable data, if necessary.

1. (a) Solve any two of the following differential equations : [8]

(i) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = e^{-3x} \cos 4x + 6e^{2x}$

(ii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 16y = x^2 + 2^{\log x} + 4 \cosh(\log x)$

(iii) $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$, (by using method of variation of parameters)

(b) Solve the integral equation : [4]

$$\int_0^{\infty} f(x) \cos \lambda x \, dx = e^{-2\lambda}, \lambda > 0$$

P.T.O.

Or

2. (a) A 8 lb weight is placed at one end of a spring suspended from the ceiling. The weight is raised to 5 inches above the equilibrium position and left free. Assuming the spring constant 12 lb/ft, find the equation of motion, the displacement function, amplitude and period. [4]

- (b) Solve any *one* of the following : [4]

(i) $L[te^{2t} \cos 3t]$

(ii) $L^{-1}\left[\frac{2s+7}{s^2+4s+29}\right]$.

- (c) Solve the differential equation by Laplace transform method : [4]

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = te^t$$

where $y(0) = 0$, $y'(0) = 3$.

3. (a) The first four moments of a distribution about the value 2.5 are 1, 10, 20 and 25. Obtain first four central moments. Also calculate coefficient of skewness (β_1) and coefficient of kurtosis (β_2). [4]

- (b) A dice is thrown five times. If getting an odd number is a success, then what is the probability of getting : [4]

(i) four successes

(ii) at least four successes.

- (c) Find the directional derivative of $\phi = xy^2 + yz^2 + zx^2$ at (1, 1, 1) along the vector $\vec{i} + 2\vec{j} + 2\vec{k}$. [4]

Or

4. (a) Obtain the regression line of y on x for the following data : [4]

x	y
1	2
2	5
3	3
4	8
5	7

- (b) Prove the following (any one) : [4]

(i) $\nabla \cdot \left(\frac{\vec{a} \times \vec{r}}{r} \right) = 0$

(ii) $\nabla^2 (r^9 \log r) = (90 \log r + 19)r^7$.

- (c) Show that the vector field : [4]

$$\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$$

is irrotational. Also find the scalar potential ϕ such that $\vec{F} = \nabla\phi$.

5. (a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = zi + xj + yk$ and C is the arc of the curve $\vec{r} = \cos ti + \sin tj + tk$ from $t = 0$ to $t = 2\pi$. [5]

- (b) Using Gauss divergence theorem, evaluate $\iiint_V \nabla \cdot \vec{F} dV$ where $\vec{F} = 2x^2yi - y^2j + 4xz^2k$ over the region bounded by the cylinder $y^2 + z^2 = 9$ and the plane $z = 2$ in the first octant. [4]

- (c) Using Stoke's theorem evaluate $\iint_S \nabla \times \vec{F} \cdot \hat{n} dS$ where $\vec{F} = (x + y) \mathbf{i} + (y + z) \mathbf{j} - xk$ and S is the surface of the plane $2x + y + z = 2$ which is in the first octant. [4]

Or

6. (a) Using Green's theorem, evaluate $\int_C e^{-x} (\sin y dx + \cos y dy)$ where 'C' is the rectangle with vertices $(0, 0)$, $(\pi, 0)$, $(\pi, \frac{\pi}{2})$, $(0, \frac{\pi}{2})$. [4]

- (b) Using Gauss divergene theorem, evaluate

$$\iiint_S [(x^2 - yz) dydz + (y^2 - xz) dx dz + (z^2 - xy) dx dy]$$

taken over rectangular parallelopiped $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$. [4]

- (c) Using stoke's theorem evaluate $\iint_S \nabla \times \vec{F} \cdot \hat{n} dS$. Where $\vec{F} = yi + zj + xk$ over the surface $x^2 + y^2 = 1 - z$, $z > 0$. [5]

7. (a) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions : [7]

(i) $u(0, t) = 0$

(ii) $u(4, t) = 0$

(iii) $\frac{\partial u}{\partial t} = 0$ when $t = 0$

(iv) $u(x, 0) = 25$.

(b) Solve $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions : [6]

(i) $u(0, t) = 0$

(ii) $u(2, t) = 0$

(iii) $u(x, 0) = x, 0 < x < 2$

Or

8. (a) Solve $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$, given that : [6]

(i) $V(0, y) = 0$

(ii) $V(C, y) = 0$

(iii) $V \rightarrow 0$ as $y \rightarrow \infty$

(iii) $V = V_0$ when $y = 0$.

(b) Use fourier transform to solve the equation [7]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}, 0 \leq x < \infty, t > 0$$

subject to conditions :

(i) $u(0, t) = 0, t > 0$

(ii) $u(x, 0) = \begin{cases} 6 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$

(iii) $u(x, t)$ is bounded.