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[5559]-220

S.E. (Chemical) (I Sem.) EXAMINATION, 2019

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours

Maximum Marks : 50

- N.B. :—** (i) Neat diagrams must be drawn wherever necessary.
(ii) Use of logarithmic tables, slide rule, electronic pocket calculator is allowed.
(iii) Assume suitable data, if necessary.
(iv) Solve Q. Nos. **1 or 2**, Q. Nos. **3 or 4**, Q. Nos. **5 or 6**, Q. Nos. **7 or 8**.

- 1. (a)** Solve any *two* of the following : [8]
(i) $(D^2 + 3D + 2)y = \sin(e^x)$
(ii) $(D^2 + 2D + 1)y = xe^{-x} \cos x$
(iii) $(D^4 + 4)y = \sec 2x$ by variation parameter.
(b) Find Fourier sine transform of e^{-mx} (M20) and hence show that : [4]

$$\int_0^{\infty} \frac{\lambda \sin \lambda x}{\lambda^2 + m^2} d\lambda = \frac{\pi}{2} e^{-mx}, \quad m > 0, x > 0.$$

P.T.O.

Or

2. (a) A body of weight $w = 1$ N is suspended from a spring stretches it 4 cm. If the weight is pulled down 8 cm below its equilibrium position and then released. Find the position and velocity as a function of time. [4]
- (b) What is the function $f(x)$ where Fourier cosine transform is $\frac{\sin a\lambda}{\lambda}$? [4]
- (c) Find Fourier cosine transform of the function : [4]
- $$f(x) = \cos x, \quad 0 < x < a$$
- $$= 0, \quad x > a.$$

3. (a) Attempt any one : [4]

(i) Find Laplace Transform of $e^{-3t} \int_0^t t \cdot \sinh 2t \, dt$.

(ii) Find Inverse Laplace Transform of

$$\frac{4s - 7}{(3s + 5)(2s - 3)}.$$

- (b) Find Laplace Transform of $(t^2 + 3t + 2) U(t - 1) + \frac{\sin t}{t} \delta(t - 3)$. [4]
- (c) Find the directional derivatives of $\phi = xy^2 + yz^3$, at $(1, -1, 1)$ in the direction normal to the surface $x^2 + y^2 + z^2 = 9$ at $(1, 2, 2)$. [4]

Or

4. (a) Attempt any one : [4]

(i) $\nabla^2 \left[\nabla \cdot \left(\frac{\vec{r}}{r^2} \right) \right] = \frac{2}{r^4}$

(ii) $\nabla^2 [r^n \log r] = [n(n + 1) \log r + 2n + 1] r^{n-2}$

- (b) If vector field $\vec{F} = (x + 4y + az)\vec{i} + (bx - 6y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational. Find a, b, c and determine scalar function ϕ such that $\vec{F} = \nabla\phi$. [4]
- (c) Solve by Laplace Transform method $\frac{dy}{dt} + 3y(t) + 2\int_0^t y(t) dt = t$, given $y(0) = 0$. [4]
5. (a) Find the work done by the force $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ along the curve $x = t, y = t^2, z = t^3$ from $t = 0$ to $t = 1$. [5]
- (b) Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ for vector field $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ over the curved surface paraboloid $z = 4 - x^2 - y^2$ above the XOY plane. [4]
- (c) Evaluate $\iint_S (\nabla \times \vec{F}) \cdot \hat{n} dS$ where $\vec{F} = (x^3 - y^3)\vec{i} - xyz\vec{j} + y\vec{k}$ and S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$. [4]
- Or
6. (a) Using Green's theorem evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = y^3\vec{i} - x^3\vec{j}$ where C is circle $x^2 + y^2 = a^2, z = 0$. [5]
- (b) Use Gauss divergence theorem to evaluate $\iint_S \vec{F} \cdot \hat{n} ds$ for vector field $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ over the surface of the sphere $x^2 + y^2 + z^2 = 1$. [4]
- (c) Evaluate $\iint_S (\nabla \times \vec{F}) \cdot \hat{n} ds$ where $\vec{F} = (x - y)\vec{i} + (x + yz)\vec{j} - 3xy^2\vec{k}$ and S is the surface of sphere $x^2 + y^2 + z^2 = a^2$ above XOY-plane. [4]

7. (a) If $\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ represents the vibration of a stretched string of length l , fixed at both ends. Find the solution under the conditions : [7]

(i) $y(0, t) = 0; \forall t$

(ii) $y(l, t) = 0; \forall t$

(iii) $\left. \frac{\partial y}{\partial t} \right|_{t=0} = 0 \quad \forall x$

(iv) $y(x, 0) = (lx - x^2); 0 \leq x \leq l.$

- (b) Solve $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$, under the conditions : [6]

(i) $u(0, t) = 0^\circ\text{C}$

(ii) $u(1, t) = 0^\circ\text{C}$

(iii) $u(x, 0) = 50^\circ\text{C}$ in $0 \leq x \leq 1.$

Or

8. (a) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ under the conditions : [7]

(i) $u(x, \infty) = 0; \forall x$

(ii) $u(0, y) = 0; \forall y$

(iii) $u(l, y) = 0; \forall y$

(iv) $u(x, 0) = (x - x^2)$ in $0 \leq x \leq l.$

- (b) Using Fourier sine transform, solve the equation : [6]

$$\frac{\partial u}{\partial t} = 2 \quad \frac{\partial^2 u}{\partial x^2}; 0 < x < \infty, t > 0$$

subject to the following conditions :

(i) $u(0, t) = 0; t > 0$

(ii) $u(x, 0) = e^{-x}; x > 0$

(iii) $u \rightarrow 0$ and $\frac{\partial u}{\partial x} \rightarrow 0$ as $x \rightarrow \infty.$