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S.E. (Chemical) (I Sem.) EXAMINATION, 2019

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours**Maximum Marks : 50****N.B. :—** (i) Answer Q. No. 1 or 2, Q. No. 3 or 4, Q. No. 5 or 6,

Q. No. 7 or 8.

(ii) Figures to the right indicate full marks.

(iii) Assume suitable data, if necessary.

(iv) Neat diagrams must be drawn wherever necessary.

(v) Use of electronic non-programmable calculator is allowed.

1. (a) Solve any two : [8]

(i) $(D^2 + 5D + 6)y = e^{e^x}$

(ii) $(D^2 - 4D + 4)y = e^{2x} \sin 3x$

(iii) $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 2 \sin [\log (1+x)]$

(b) Find the Fourier cosine transform of the function : [4]

$$f(x) = \begin{cases} \cos x & 0 < x < a \\ 0 & x > a \end{cases}$$

P.T.O.

Or

2. (a) A body weighing 20 kg is hung from a spring. A pull of 40 kg will stretch the spring to 10 cms. The body is pulled down to 20 cms below the static equilibrium, position. Find the displacement in time t seconds and period of oscillation. [4]

(b) Find the Fourier sin transformation of $\frac{e^{-ax}}{x}$ and hence evaluate : [4]

$$\int_0^a \tan^{-1} \frac{x}{a} \cdot \sin x \, dx.$$

(c) Solve the integral equation : [4]

$$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}; \lambda > 0.$$

3. (a) Attempt any one : [4]

(i) Find the Laplace transform of :

$$f(t) = \int_0^t t e^{-3t} \sin t \, dt.$$

(ii) Find the Inverse Laplace transform of :

$$F(s) = \frac{s+2}{s^2-4s+13}.$$

(b) Find the Laplace transform of : [4]

$$f(t) = \begin{cases} t, & 0 < t < \pi \\ \pi-1, & \pi < t < 2\pi \end{cases}; f(t+2\pi) = f(t).$$

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- (c) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ along the tangent to the curve $x = e^t, \cos t, y = e^t, \sin t, z = e^t$ at $t = 0$. [4]

Or

4. (a) Attempt any one : [4]

(i) $\nabla^4 e^r = e^r + \frac{4}{r} e^r$

(ii) $\nabla \times \left(\frac{\vec{a} \times \vec{r}}{r^3} \right) = -\frac{\vec{a}}{r^3} + \frac{3(\vec{a} \cdot \vec{r})}{r^5} \vec{r}$

- (b) Show that the field given by : [4]

$$\vec{F} = (y \sin z - \sin x) \hat{i} + (x \sin z + 2yz) \hat{j} + (xy \cos z + y^2) \hat{k}$$

is irrotational and hence find the scalar potential ϕ such that

$$\vec{F} = \nabla \phi.$$

- (c) Solve the differential equation by using Laplace transform method : [4]

$$y''(t) + 4y(t) = U(t - 2), \text{ where } U(t - 2) \text{ is a step function}$$

given that $y(0) = 0, y'(0) = 0$.

5. Solve any two :

- (a) Find work done by $\vec{F} = x^2\vec{i} + yz\vec{j} + \frac{z}{x}\vec{k}$ is moving a particle along a straight line joining points $(0, 0, 0)$ and $(1, 2, -1)$. [6]

- (b) Prove that $\int_C (\vec{a} \times \vec{r}) \cdot d\vec{r} = \iint_S 2\vec{a} \cdot d\vec{S}$ where C is boundary of the surface S . [6]

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P.T.O.

- (c) By using Gauss-Divergence theorem, evaluate $\iint_S (x^2y^3\vec{i} + z^2x^3\vec{j} + x^2y^3\vec{k}) \cdot d\vec{S}$ where S is surface of the sphere $x^2 + y^2 + z^2 = 1$. [7]

Or

6. Solve any two :

- (a) By using Green's theorem evaluate $\int_C 4y \, dx + 3x \, dy$ over curve C where C is rectangle form by $x = 0, x = 1, y = 0, y = 2$. [6]

- (b) Use Stokes theorem to evaluate $\oint_C (3y\vec{i} + 2z\vec{j} + 4y\vec{k}) \cdot d\vec{r}$ where C is the curve of intersection of $x^2 + y^2 + z^2 = 9$ and xy -plane. [7]

- (c) Prove that :

$$\iint_S (\phi \nabla \psi - \psi \nabla \phi) \cdot d\vec{S} = \iiint_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dV$$

where V is the volume bounded by closed surface S .

6. Solve any two :

- (a) A string is stretched and fastened to two points 10 meter apart. Motion is stated by displacing the string in the form $u = a \cos (\pi x/10)$ from which it is released at time $t = 0$. [7]

- (b) The temperature at any point of the insulated metal rod of 1 meter length is governed by the equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. Find $u(x, t)$ subject to the following conditions : [6]

(i) $u(0, t) = 0^\circ\text{C}$

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- (ii) $u(1, t) = 0^\circ\text{C}$
- (iii) $u(x, 0) = 50^\circ\text{C}$.
- (c) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ if :
- (i) $u(0, y) = 0, \forall y$
- (ii) $u(\pi, y) = 0, \forall y$
- (iii) $u(x, \infty) = 0$, for $0 < x < \pi$
- (iv) $u(x, 0) = u_0, 0 < x < \pi$.

Or

8. Solve any two :

- (a) If $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$, represents the vibrations of string of length l fixed at both ends. Find the solution with boundary conditions : [7]
- (i) $u(0, t) = 0, \forall y$
- (ii) $u(l, t) = 0$ and initial conditions
- (iii) $\left(\frac{\partial u}{\partial t}\right)$ at $t = 0$ is 0
- (iv) $u(x, 0) = 3(lx - x^2), 0 \leq x \leq l$
- (b) Solve the Laplace equation $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ with condition : [6]
- (i) $v = 0$ when $y \rightarrow \infty, \forall x$
- (ii) $v = 0$ when $x \rightarrow 0, \forall y$

- (iii) $v = 0$ when $x = 10, \forall x$
- (iv) $v = (10 - x)$ when $y = 0$ for $0 < x < 10$
- (c) Using Fourier cosine transform solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < \infty, t > 0$, subject to the following conditions : [6]
- (i) $u(0, t) \neq 0; t > 0$
- (ii) $u(x, 0) = e^{-x}; x > 0$
- (iii) $u(x, t)$ is bounded.