Total No. of Questions—8] [Total No. of Printed Pages—6

Seat No.	

[5668]-155

S.E. (Ele.) (I Sem.) EXAMINATION, 2019

(Common to Instru. Control)

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Figures to the right indicate full marks.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Use of non-programmable electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve (any *two*) : [8]

(*i*) $\left(D^2 - 4D + 4\right)y = e^{2x} + \sin 2x$

(*ii*) $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = x + x^{-1}$

(*iii*) $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

by variation of parameters method.

(*b*) Solve the differential equation by using Laplace transform : [4]

$$\frac{dy}{dt} + \alpha y(t) + \int_0^t y(t)\,dt = \sin t, y(0) = 1.$$

P.T.O.

Or

2. (*a*) An inductor of 0.5 henry is connected in series with a resistor of 6 ohms, a capacitor of 0.02 farads, a generator having alternative voltage given by $24 \sin 10t$ and a switch k. Find the charge at time t, if the charge on the capacitor is zero when switch k is closed at $t = 0$. [4]

(*b*) Solve (any *one*) : [4]

(*i*) Find :

$$L\left[\int_0^t \frac{e^{-4t}\sin 3t}{t}\,dt\right].$$

(*ii*) Find :

$$L^{-1}\left[\frac{1}{s(s+1)^3}\right].$$

(*c*) Find Laplace transform of : [4]

$$t^2 \, v(t - 4).$$

3. (*a*) Find the Fourier transform of : [4]

$$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0 \quad , & |x| > 1. \end{cases}$$

(*b*) Attempt any *one* : [4]

(*i*) Find z-transform of :

$$f(k) = \frac{2^k}{k!}, k \ge 0.$$

(*ii*) Find inverse z-transform of :

$$F(z) = \frac{z}{z-5}, |z| > 5.$$

(*c*) If the directional derivative of : [4]

$$\phi = a(x + y) + b(y + z) + c(x + z)$$

has maximum value 12 in the direction parallel to

$$6(x - 1) = 3(y - 2) = 2(z - 1),$$

find the values of a, b, c.

Or

4. (*a*) Prove any *one* : [4]

(*i*) $\nabla^2 f(r) = \dfrac{d^2 f}{dr^2} + \dfrac{2}{r}\dfrac{df}{dr}$

(*ii*) $\nabla \times \left[\bar{a} \times \nabla \dfrac{1}{r}\right] = \dfrac{\bar{a}}{r^3} - \dfrac{3(\bar{a} \cdot \bar{r})}{r^5}\,\bar{r}.$

(*b*) Find the angle between tangent at any point on the curve $x = e^{\theta}\cos\theta$, $y = e^{\theta}\sin\theta$, $z = e^{\theta}$ and z-axis. [4]

(*c*) Solve : [4]

$$f(k + 1) + f(k) = 2^k, \; k \geq 0$$

given $f(0) = 1$.

5. Attempt any *two* :

(*a*) Evaluate the line integral [6]

$$\int_C \bar{F} \cdot d\bar{r},$$

where

$$\bar{F} = (2xy + z^3)\,\bar{i} + x^2\,\bar{j} + 3xz^2\,\bar{k},$$

along the st. line joining points (1, –2, 1) and (3, 1, 4).

(*b*) Verify Stokes' theorem for [7]

$$\bar{F} = xy^2\,\bar{i} + y\,\bar{j} + z^2 x\,\bar{k}$$

over a rectangular Lamina bounded by $x = 0$, $y = 0$, $x = 1$, $y = 2$ in X-Y-plane ($z = 0$).

(*c*) Evaluate : [6]

$$\iint_S \left(x\bar{i} + y\bar{j} + z^2\bar{k}\right) \cdot d\bar{S}$$

where 'S' is the surface of cylinder $x^2 + y^2 = 9$, $z = 0$ to $z = 2$ closed at both ends.

Or

6. Attempt any *two* :

(*a*) Find the work done moving the particle along the curve $x = 2t^2$, $y = t$, $z = t^3$, from $t = 0$ to $t = 1$ under the force : [6]

$$\bar{F} = (2y + 3)\,\bar{i} + xz\,\bar{j} + (yz - x)\,\bar{k}.$$

(*b*) Evaluate : [7]

$$\iint_S \nabla \times \overline{F} \cdot d\overline{S},$$

where

$$\overline{F} = \left(x^3 - y^3\right)\overline{i} - xyz\,\overline{j} + y^3\,\overline{k}$$

and 'S' is the surface $x^2 + 4y^2 + z^2 - 2x = 4$, open at $x = 0$.

(*c*) Using Gauss theorem, evaluate : [6]

$$\oiint_S \overline{F} \cdot d\overline{S}$$

where

$$\overline{F} = \left(x + y^2\right)\overline{i} - 2x\,\overline{j} + 2yz\,\overline{k}$$

and volume of the tetrahedron bounded by co-ordinate planes and the plane, $2x + y + 2z = 6$.

7. (*a*) If $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. Find v such that $u + iv$ is analytic function. Write $f(z) = u + iv$ in terms of z. [4]

(*b*) Evaluate : [5]

$$\oint_C \frac{2z^2 + 2z + 1}{(z+1)^2 (z-3)}\, dz,$$

where 'C' is the circle $|z + 1| = 2$. Using Cauchy's formula.

(*c*) Find the bilinear transformation which maps points –1, 0, 1 of z-plane onto points 0, i, $3i$ of w-plane. [4]

Or

8. (*a*) Show that analytic function with constant modulus is constant. [4]

(*b*) Evaluate : [5]

$$\int_0^{2\pi} \frac{\sin^2\theta}{5 + 4\sin\theta}\, d\theta$$

using Residue theorem.

(*c*) Obtain the image of Hyperbola $x^2 - y^2 = 1$, under the transformation $w = \frac{1}{z}$. [4]