

Seat No.

[5667]-1001

F.E. (I Semester) EXAMINATION, 2019

ENGINEERING MATHEMATICS—I

(Phase-II)

(2019 PATTERN)

Time : 2½ Hours

Maximum Marks : 70

N.B. :— (i) Attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4,

Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.

(ii) Use of electronic pocket calculator is allowed.

(iii) Assume suitable data, if necessary.

(iv) Neat diagrams must be drawn wherever necessary.

(v) Figures to the right indicate full marks.

1. (a) If $z = \tan(y + ax) + (y - ax)^{3/2}$, find the value of

$$\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2}. \quad [6]$$

(b) If $T = \sin\left(\frac{xy}{x^2 + y^2}\right) + \sqrt{x^2 + y^2}$, by using Euler's theorem

$$\text{find } x \frac{\partial T}{\partial x} + y \frac{\partial T}{\partial y}. \quad [6]$$

(c) If $u = x^2 - y^2$, $v = 2xy$ and $z = f(u, v)$, then show that

$$x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 2\sqrt{u^2 + v^2} \frac{\partial z}{\partial u}. \quad [6]$$

P.T.O.

2. (a) If $x = u \tan v$, $y = u \sec v$, prove that : [6]

$$\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial x}\right) = \left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial v}{\partial y}\right).$$

(b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x+y}}\right)$, by using Euler's theorem. prove that : [6]

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4}(\tan^3 u - \tan u).$$

(c) If $x = \frac{\cos \theta}{u}$, $y = \frac{\sin \theta}{u}$ and $z = f(x, y)$, then show that : [6]

$$u \frac{\partial z}{\partial u} - \frac{\partial z}{\partial \theta} = (y - x) \frac{\partial z}{\partial x} - (y + x) \frac{\partial z}{\partial y}.$$

3. (a) If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = xy + yz + zx$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. [6](b) Examine whether $u = \frac{x-y}{1+xy}$, $v = \tan^{-1} x - \tan^{-1} y$ are functionally dependent, if so find the relation between them. [5](c) Find the extreme values of $x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$. [6]

Or

4. (a) If $u = x + y^2$, $v = y + z^2$, $w = x^2 + x^2$, using Jacobian find $\frac{\partial x}{\partial u}$. [6]

2

[5667]-1001

- (b) A power dissipated in a resistor is given by $P = \frac{\varepsilon^2}{R}$. If errors of 3% and 2% are found in ε and R respectively, find the percentage error in P . [5]
- (c) Using Lagrange's method find extreme value of xyz if $x + y + z = a$. [6]

5. (a) Examine for consistency of the system of linear equations and solve if consistent : [6]

$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ -2x_1 + 5x_2 + 2x_3 &= 1 \\ 8x_1 + x_2 + 4x_3 &= -1 \end{aligned}$$

- (b) Examine for linear dependence or independence the vectors $(1, 1, 1, 3), (1, 2, 3, 4), (2, 3, 4, 7)$. Find the relation between them if dependent. [6]
- (c) Determine the values of a, b, c when A is orthogonal where : [5]

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$

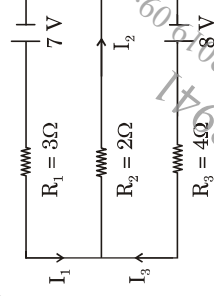
Or

6. (a) Investigate for what values of a and b , the system of equations $2x - y + 3z = 2, x + y + 2z = 2, 5x - y + az = b$ have : [6]
- (1) No solution
- (2) A unique solution
- (3) An infinite number of solutions. [6]

[5667]-1001

P.T.O.

- (b) Examine for linear dependence or independence the vectors $x_1 = (2, 3, 4, -2), x_2 = (1, 1, 2, -1), x_3 = \left(\frac{-1}{2}, -1, -1, \frac{1}{2}\right)$. Find the relation between them if dependent. [6]
- (c) Determine the currents in the network given in figure below : [5]



7. (a) Find the eigen values, and the corresponding eigen vectors for the following matrix : [6]

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

- (b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix}$ and use it to find A^{-1} . [6]

- (c) Find a matrix P that diagonalizes the matrix

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

[5667]-1001

4

8. (a) Find the eigen values and the corresponding eigen vectors for the following matrix : [6]

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}.$$

- Or
- (b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$ and use it to find A^{-1} . [6]

- (c) Reduce the following quadratic form to the sum of the squares form : [6]

$$Q = 2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6xz.$$