

Seat No.

[5668]-191**S.E. (Comp. & IT) (Second Semester) EXAMINATION, 2019****ENGINEERING MATHEMATICS—III****(2015 PATTERN)****Time : Two Hours****Maximum Marks : 50****N.B. :-** (i) Neat diagrams must be drawn wherever necessary.

(ii) Figures to the right indicate full marks.

(iii) Use of electronic pocket calculator is allowed.

(iv) Assume suitable data, if necessary.

1. (a) Solve any *two* differential equations : [8]

(i) $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 2y = e^{4x} \cosh 2x$

(ii) $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 3y = x^2 \sin(\log x)$

(iii) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = \frac{e^{4x}}{x^6}$, by using the method of variation of parameters.

(b) Solve the integral equation : [4]

$$\int_0^x f(x) \sin \lambda x \, dx = \begin{cases} 1 - \lambda, & 0 \leq \lambda \leq 1 \\ 0, & \lambda > 1 \end{cases}$$

P.T.O.

Or

2. (a) A capacitor of 10^{-3} farads and inductor of (0.4) henries are connected in series with an applied emf 20 volts in an electrical circuit. Find the current and charge at any time t . [4]

(b) Solve any *one* of the following : [4]

(i) Obtain $z[k e^{-k}]$, $k \geq 0$

(ii) Obtain $z^{-1} \left[\frac{8z}{(z-1)(z-2)} \right]$, $|z| > 2$, $k \geq 0$.

(c) Solve the difference equation : [4]

$$y_{k+1} + \frac{1}{2} y_k = \left(\frac{1}{2} \right)^k$$

where $y_0 = 0$, $k \geq 0$.

3. (a) The first three moments of a distribution about the value 2 are 1, 16 and -40 . Find the first three central moments, standard deviation and β_1 . [4]

(b) Fit a straight line of the form $X = aY + b$ to the following data by the least square method : [4]

X	Y
2	2
5	3
8	4
11	5
17	7
20	8

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- (c) On an average, there are 2 printing mistakes on a page of a book. Using Poisson distribution, find the probability that a randomly selected page from the book has at least one printing mistake. [4]

Or

4. (a) 200 students appeared for an examination. Average marks were 50% with standard deviation 5%. How many students are expected to score at least 60% marks assuming that marks are normally distributed. [Given : $Z = 2$, $A = 0.4772$]. [4]
- (b) On an average, a box containing 10 articles is likely to have 2 defectives. If we consider a consignment of 100 boxes, how many of them are expected to have at the most one defective ? [4]

- (c) Find the regression equation of Y on X for a bivariate data with the following details. $n = 25$, $\sum_{i=1}^n x_i = 75$, $\sum_{i=1}^n y_i = 100$, $\sum_{i=1}^n x_i^2 = 250$, $\sum_{i=1}^n y_i^2 = 500$, $\sum_{i=1}^n x_i y_i = 325$. [4]

5. (a) Find the directional dervative of $\phi(x, y, z) = xy^3 + yz^3$ at the point $(2, -1, 1)$ in the direction of vector $\vec{i} + 2\vec{j} + 2\vec{k}$. [4]
- (b) Show that $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ is irrotational. Hence find the scalar potential ϕ such that $\vec{F} = \nabla\phi$. [4]

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P.T.O.

- (c) Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = \sin z \vec{i} + \cos x \vec{j} + \sin y \vec{k}$ and C is the boundary of the rectangle $0 \leq x \leq \pi$ and $0 \leq y \leq 1$ and $z = 3$. [5]

Or

6. (a) Show that (any one) : [4]

$$(i) \nabla \left[r \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$$

$$(ii) \nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r).$$

- (b) Find the directional derivative of $\phi = xy^2 + yz^3$ at $(1, -1, 1)$ towards the point $(2, 1, -1)$. [4]

- (c) If : [5]

$$\vec{F} = (2xy + 3z^2)\vec{i} + (x^2 + 4yz)\vec{j} + (2y^2 + 6xz)\vec{k}$$

Evaluate :

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is the curve $x = t$, $y = t^2$, $z = t^3$ joining the points $(0, 0, 0)$ and $(1, 1, 1)$.

7. (a) Determine the analytic function $f(z) = u + iv$ if $u = 4xy - 3x + 2$. [4]
- (b) Find the bilinear transformation which maps the point $z = i, -1, 1$ into the point $w = 0, 1, \infty$. [4]

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(c) Evaluate :

$$\int_C \frac{3z+4}{(z-1)(z-2)} dz,$$

where C is the circle $|z-1| = \frac{3}{2}$. [5]

Or

8. (a) Determine the analytic function $f(z) = u + iv$ if $u = x^2 - y^2 - 2xy - 2x - y - 1$. [4]

(b) Under the transformation $w = \frac{1}{z}$, find the image of $|z - 3i| = 3$. [4]

(c) Evaluate :

$$\int_C \frac{z dz}{(z-1)(z-3)}$$

where C is the circle $|z| = \frac{3}{2}$. [5]