Total No. of Questions—8]

Seat
No.

[5668]-142

S.E. (Elect/E.&TC) (Second Semester) EXAMINATION, 2019

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours   **Maximum Marks : 50**

N.B. :— (*i*) Attempt question Nos. **1** *or* **2**, **3** *or* **4**, **5** *or* **6**, **7** *or* **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^3 + 6D^2 + 12D + 8)y = e^{-2x} + 3^x + \cos 2x$

(*ii*) $x^2 \dfrac{d^2 y}{dx^2} - x \dfrac{dy}{dx} + 4y = \cos(\log x) + \sin(\log x)$

(*iii*) $\dfrac{d^2 y}{dx^2} + 4y = \tan 2x$ (By method of variation of parameter).

(*b*) Find Fourier sine transform of : [4]

$$f(x) = \begin{cases} \dfrac{\pi}{2} & 0 < x < \pi \\ 0 & x > \pi \end{cases}$$

P.T.O.

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2. (*a*) A 0.1 henry inductor, a 0.004 farad capacitor and a generator having e.m.f. given by 180 cos 40*t*, $t \geq 0$ are connected in series. Find the instantaneous charge Q and current I, if [4]

I = Q = 0 at *t* = 0.

(*b*) Solve any *one* : [4]

(*i*) Find Z-transform of $\dfrac{2^k}{k}, k \geq 1$.

(*ii*) Find inverse *z*-transform of $\dfrac{1}{(z-3)(z-2)}$ for

$2 < |z| < 3$.

(*c*) Find *f*(*k*), given that : [4]

$12 f(k+2) - 7 f(k+1) + f(k) = 0, k \geq 0, f(0) = 0, f(1) = 3.$

3. (*a*) Given : [4]

$$\dfrac{dy}{dx} = \dfrac{1}{x^2} - \dfrac{y}{x}, \quad y(1) = 1.$$

Evaluate *y*(1.1) by Euler's modified method (Take $h_0 = 0.1$).

(*b*) Find Lagrange's interpolating polynomial passing through set of points : [4]

x	0	1	2
y	2	3	6

Use it to find *y* at *x* = 1.5 and $\int_0^2 y \, dx$.

(c) Find directional derivative of $\phi = xy^2 + yz^3$ at $(1, -1, 1)$ along the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. [4]

Or

4. (a) Show that (any *one*) : [4]

(i) $\nabla^2[\nabla.(\bar{r}/r^2)] = 2/r^4$

(ii) $\nabla\left(\dfrac{\bar{a}.\bar{r}}{r^2}\right) = \dfrac{\bar{a}}{r^2} - \dfrac{2(\bar{a}.\bar{r})}{r^4}\bar{r}$

(b) Show that the vector field $\bar{F} = (y^2\cos x + z^2)\,\bar{i} + (2y\sin x)\bar{j} + 2xz\bar{k}$ is irrotational and find scalar potential ϕ such that $\bar{F} = \nabla\phi$. [4]

(c) Evaluate $\int_0^3 \dfrac{dx}{1+x}$ by dividing the interval into six parts using Simpson's $\dfrac{3}{8}$ th rule correct upto four decimal places. [4]

5. (a) Evaluate the integral $\int_c \bar{F}.d\bar{r}$, where $\bar{F} = (3x^2 + 6y)\bar{i} - 14yz\bar{j} + 20\,xz^2\bar{k}$ and C is the curve $x = t$, $y = t^2$, $z = t^3$ from $t = 0$ to $t = 1$. [4]

(b) Verify Green's lemma for $\bar{F} = x^2\bar{i} + xy\bar{j}$ over the region bounded by the boundaries :

$$x = 0, y = 0, x = 1, y = 1$$ [5]

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(c) Use Stokes theorem to evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F} = xy^2i + yj + xz^2k$ over the surface bounded by $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 2$. [4]

Or

6. (a) Use Green's Lemma to evaluate $\int_C (3ydx + 2xdy)$ where C is the boundary of the region bounded by $y = 0$, $y = \sin x$ for $0 \leq x \leq \pi$. [4]

(b) Find the work done by force field $\bar{F} = 2xy^2i + (2x^2y + y)j$ in taking a particle from $(0, 0, 0)$ to $(2, 4, 0)$ along the parabola $y = x^2$, $z = 0$. [4]

(c) Maxwell's equations are given by:

$$\nabla.\bar{E} = 0, \nabla.\bar{H} = 0, \nabla\times\bar{H} = \frac{\partial\bar{E}}{\partial t}, \nabla\times\bar{E} = -\frac{\partial\bar{H}}{\partial t}$$

then show that $\bar{E}$ satisfies the equation $\nabla^2 u = \dfrac{\partial^2 u}{\partial t^2}$. [5]

7. (a) If $f(s) = u + iv$ is analytic function with constant amplitude, show that $f(z)$ is constant. [4]

(b) Evaluate :

$$\oint_C \cot z\, dz$$

where 'c' is the circle $|z| = 4$. [5]

(c) Find the bilinear transformation, which maps the points 1, i, -1 from z-plane onto the points i, 0, $-i$ of the w-plane respectively. [4]

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8. (*a*) If :

$$u = \frac{1}{2}\log(x^2 + y^2)$$

find *v* such that $f(z) = u + iv$ is analytic. [4]

Or

(*b*) Evaluate : [4]

$$\oint_C \frac{z^2 + 1}{z - 2}\, dz$$

where :

(*i*) C is the circle $|z - 2| = 1$,

(*ii*) C is circle $|z| = 1$.

(*c*) Show that under the transformation $w = z + \frac{1}{z}$ family of circles $r = c$ are mapped onto family of ellipses. What happens if $r = 1$. [5]