

Seat No.
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**[5668]-191****S.E. (Comp. & IT) (Second Semester) EXAMINATION, 2019****ENGINEERING MATHEMATICS—III****(2015 PATTERN)****Time : Two Hours****Maximum Marks : 50****N.B. :-** (i) Neat diagrams must be drawn wherever necessary.

(ii) Figures to the right indicate full marks.

(iii) Use of electronic pocket calculator is allowed.

(iv) Assume suitable data, if necessary.

**1. (a) Solve any two differential equations : [8]**

(i)  $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 2y = e^{4x} \cosh 2x$

(ii)  $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 3y = x^2 \sin(\log x)$

(iii)  $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = \frac{e^{4x}}{x^6}$ , by using the method of variation of parameters.

**(b) Solve the integral equation : [4]**

$$\int_0^x f(x) \sin \lambda x \, dx = \begin{cases} 1 - \lambda, & 0 \leq \lambda \leq 1 \\ 0, & \lambda > 1 \end{cases}$$

P.T.O.

Or

**2. (a)** A capacitor of  $10^{-3}$  farads and inductor of  $(0.4)$  henries are connected in series with an applied emf 20 volts in an electrical circuit. Find the current and charge at any time  $t$ . [4]

**(b)** Solve any one of the following : [4]

(i) Obtain  $z[k e^{-k}]$ ,  $k \geq 0$

(ii) Obtain  $z^{-1} \left[ \frac{8z}{(z-1)(z-2)} \right]$ ,  $|z| > 2$ ,  $k \geq 0$ .

**(c)** Solve the difference equation : [4]

$$y_{k+1} + \frac{1}{2} y_k = \left( \frac{1}{2} \right)^k$$

where  $y_0 = 0$ ,  $k \geq 0$ .

**3. (a)** The first three moments of a distribution about the value 2 are 1, 16 and  $-40$ . Find the first three central moments, standard deviation and  $\beta_1$ . [4]

**(b)** Fit a straight line of the form  $X = aY + b$  to the following data by the least square method : [4]

X	Y
2	2
5	3
8	4
11	5
17	7
20	8

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- (c) On an average, there are 2 printing mistakes on a page of a book. Using Poisson distribution, find the probability that a randomly selected page from the book has at least one printing mistake. [4]

Or

4. (a) 200 students appeared for an examination. Average marks were 50% with standard deviation 5%. How many students are expected to score at least 60% marks assuming that marks are normally distributed. [Given :  $Z = 2$ ,  $A = 0.4772$ ]. [4]

- (b) On an average, a box containing 10 articles is likely to have 2 defectives. If we consider a consignment of 100 boxes, how many of them are expected to have at the most one defective ? [4]

- (c) Find the regression equation of Y on X for a bivariate data with the following details.  $n = 25$ ,  $\sum_{i=1}^n x_i = 75$ ,  $\sum_{i=1}^n y_i = 100$ ,  $\sum_{i=1}^n x_i^2 = 250$ ,  $\sum_{i=1}^n y_i^2 = 500$ ,  $\sum_{i=1}^n x_i y_i = 325$ . [4]

5. (a) Find the directional dervative of  $\phi(x, y, z) = xy^3 + yz^3$  at the point  $(2, -1, 1)$  in the direction of vector  $\vec{i} + 2\vec{j} + 2\vec{k}$ . [4]
- (b) Show that  $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$  is irrotational. Hence find the scalar potential  $\phi$  such that  $\vec{F} = \nabla\phi$ . [4]

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P.T.O.

- (c) Evaluate  $\oint_C \vec{F} \cdot d\vec{r}$  where  $\vec{F} = \sin z \vec{i} + \cos x \vec{j} + \sin y \vec{k}$  and C is the boundary of the rectangle  $0 \leq x \leq \pi$  and  $0 \leq y \leq 1$  and  $z = 3$ . [5]

Or

6. (a) Show that (any one) : [4]

$$(i) \nabla \left[ r \nabla \left( \frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$$

$$(ii) \nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r).$$

- (b) Find the directional derivative of  $\phi = xy^2 + yz^3$  at  $(1, -1, 1)$  towards the point  $(2, 1, -1)$ . [4]

- (c) If : [5]

$$\vec{F} = (2xy + 3z^2)\vec{i} + (x^2 + 4yz)\vec{j} + (2y^2 + 6xz)\vec{k}$$

Evaluate :

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is the curve  $x = t$ ,  $y = t^2$ ,  $z = t^3$  joining the points  $(0, 0, 0)$  and  $(1, 1, 1)$ .

7. (a) Determine the analytic function  $f(z) = u + iv$  if  $u = 4xy - 3x + 2$ . [4]
- (b) Find the bilinear transformation which maps the point  $z = i, -1, 1$  into the point  $w = 0, 1, \infty$ . [4]

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(c) Evaluate :

$$\int_C \frac{3z+4}{(z-1)(z-2)} dz,$$

where C is the circle  $|z-1| = \frac{3}{2}$ . [5]

Or

8. (a) Determine the analytic function  $f(z) = u + iv$  if  $u = x^2 - y^2 - 2xy - 2x - y - 1$ . [4]

(b) Under the transformation  $w = \frac{1}{z}$ , find the image of  $|z - 3i| = 3$ . [4]

(c) Evaluate :

$$\int_C \frac{z dz}{(z-1)(z-3)}$$

where C is the circle  $|z| = \frac{3}{2}$ . [5]