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Research Article

Centrifugal pumps (Agriculture) unbalance and shaft Dynamic analysis from the experimental data in a rotor system

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Abstract

The study of torsional vibration of rotor system is very important especially in application of high speed rotor. In the present work is extending the analysis of simple rotor to torsional vibration. The analysis of torsional vibration of the three rotor system of Agricultural pump with the conversional Newton's second of motion or energy method. The analysis is extended to the system having different disc/ rotor dimension and different material. Where system natural frequencies of torsional vibration may be close to, or within, the source frequency range during normal operation, torsional vibrations constitute a potential design problem area. In such case designers should ensure the accurate prediction of design a rotor system based on torsional vibrations.

Keywords: *Three rotor system, Torsional natural frequency, Node, Eigen vector.*

1. Introduction

In rotor-dynamic systems, the rotor can be modeled quite accurately with the present modeling theories and analysis techniques available are well developed. However, reliable estimates of rotor dynamic parameters are difficult to obtain with theoretical models due to the difficulty in accurately modeling the multitude of factors controlling their dynamic behavior.

On the contrary, identification methods based on experimental data from actual test conditions provide reliable dynamic characterization of rotor, which avoids the complexity of exact system modeling and analysis. It is for this reason that designers of rotating machinery mostly rely on the experimentally estimated bearing stiffness and damping parameters in their instability analysis (R. Tiwari , V. Chakravarthy,2009).

Exhaustive research works have been done on the experimental identification of the Natural frequency and node of rotor system (R. Tiwari , V. Chakravarthy, 2009). Most of the work is based on the various theory like Direct approach, Indirect approach, Hamilton's principal, Lagrange's equation, Holzers method , TMM etc., which is given to the system, and corresponding response is measured.

In the present paper the development of three rotor system based on experimental demonstration. These rotor system used in agriculture water pump (Mono-block pump, couple-set pump, submersible pump), as shown in figure 1. Speed and dimension of these rotor are comparatively small than another rotating system. The

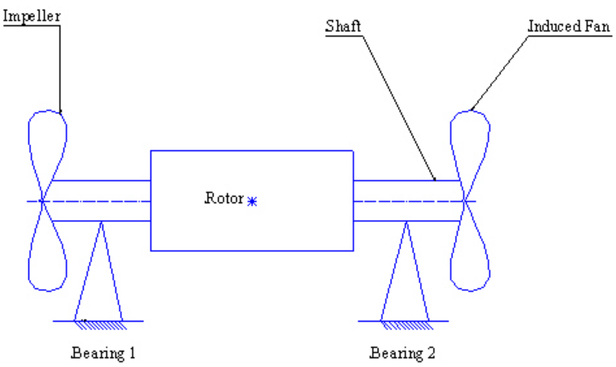

Figure 1 (Theoretical model)

Plate: 1 Practical Model

testing of nodes and natural frequency has been performed, first by the numerical simulation and then by using experimentally measured data. The first method uses

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of measured natural frequency and node corresponding to frequency, which given on the rotor in the horizontal directions. The natural frequency is calculated with the help of the direct and characteristic or frequency equation method. The second method uses Eigen value method measurements of the frequency and node position of rotor system. The experimentally identified parameters are validated by using them in a numerical model of the machine to simulate responses for the calculated node position and frequency.

If we have several discs on a shaft as shown in figure 1, there are several possible nodes and natural frequencies. There is more than one method of solving this system. Three disc rotor system having free-free boundary condition. Two different methods are applied for the free vibration analysis too gets the torsional natural frequencies and corresponding mode shapes by Direct approach & Transfer matrix method (Dr. Rajiv Tiwari, 2010).

2. Mathematical Model

A three-disc rotor system (Impeller, Rotor & Fan) is shown in plate1. Here assumed that there is no friction at support and boundary condition (bearing 1 & bearing 2) are that of the free –free case. Newton's second law method using, with the help of free body diag.1 applied to analyse the three masses rotor system (Dr. Rajiv Tiwari, 2010).

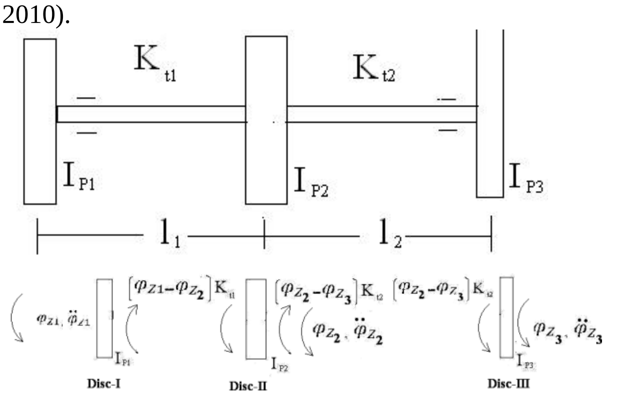


Diagram:1

Equation of motions for free vibrations obtained from F.B.D.in the matrix form.

$$\begin{bmatrix} I_{p1} & 0 & 0 \\ 0 & I_{p2} & 0 \\ 0 & 0 & I_{p3} \end{bmatrix} \begin{Bmatrix} \ddot{\varphi}_{Z1} \\ \ddot{\varphi}_{Z2} \\ \ddot{\varphi}_{Z3} \end{Bmatrix} + \begin{bmatrix} k_{t1} & -k_{t1} & 0 \\ k_{t1} & (k_{t1}+k_{t2}) & -k_{t2} \\ 0 & -k_{t2} & k_{t2} \end{bmatrix} \begin{Bmatrix} \varphi_{Z1} \\ \varphi_{Z2} \\ \varphi_{Z3} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{--(1.1)}$$

For free vibration, the SHM, it in the form

$$\left(-\omega_{nf}^2 \begin{bmatrix} I_{p1} & 0 & 0 \\ 0 & I_{p2} & 0 \\ 0 & 0 & I_{p3} \end{bmatrix} + \begin{bmatrix} k_{t1} & -k_{t1} & 0 \\ k_{t1} & (k_{t1}+k_{t2}) & -k_{t2} \\ 0 & -k_{t2} & k_{t2} \end{bmatrix} \right) \begin{Bmatrix} \varphi_{Z1} \\ \varphi_{Z2} \\ \varphi_{Z3} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{--(1.2)}$$

Where ω_{nf} is the torsional natural frequency of the system.

Natural frequencies finding by two methods: 1. Characteristic (frequency) equation 2. Eigen value problem

2.1. Characteristic (frequency) equations

Equating the determinant to zero in equation (1.1) we get frequency equation in following form

$$\omega_{nf}^2 \left\{ \omega_{nf}^2 - \left(k_{t1} \frac{I_{p1}+I_{p2}}{I_{p1}I_{p2}} + k_{t2} \frac{I_{p2}+I_{p3}}{I_{p2}I_{p3}} \right) \omega_{nf}^2 + \frac{k_{t1}k_{t2}(I_{p1}+I_{p2}+I_{p3})}{I_{p1}I_{p2}I_{p3}} \right\} = 0$$

This gives natural frequencies as

$\omega_{nf1} = 0;$

And

$$\omega_{nf2,3}^2 = \frac{1}{2}\left(k_{t1} \frac{I_{p1}+I_{p2}}{I_{p1}I_{p2}} + k_{t2} \frac{I_{p2}+I_{p3}}{I_{p2}I_{p3}} \right) \pm \sqrt{\frac{1}{4}\left(k_{t1} \frac{I_{p1}+I_{p2}}{I_{p1}I_{p2}} + k_{t2} \frac{I_{p2}+I_{p3}}{I_{p2}I_{p3}} \right)^2 - \left(\frac{k_{t1}k_{t2}(I_{p1}+I_{p2}+I_{p3})}{I_{p1}I_{p2}I_{p3}} \right)} \quad \text{--(1.3)}$$

Mode shape obtained by substituting natural frequencies, one by one into equation (1.3) and obtaining relative amplitudes with the help of any two equations, as

$$\left(k_{t1} - \omega_{nf}^2 I_{p1}\right)\varphi_{Z1} - k_{t1}\varphi_{Z2} = 0$$

$$\Rightarrow \frac{\varphi_{Z1}}{\varphi_{Z2}} = \frac{\left(k_{t1} - \omega_{nf}^2 I_{p1}\right)}{k_{t1}} \quad \text{-- (1.4)}$$

And

$$-k_{t1}\varphi_{Z1} + \left\{(k_{t1}+k_{t2}) - \omega_{nf}^2 I_{p2}\right\}\varphi_{Z2} - k_{t2}\varphi_{Z3} = 0 \quad \text{-- (1.5)}$$

Substituting eq.(1.4) in equation (1.5), we get simplified equation (1.6)

$$\frac{\varphi_{Z3}}{\varphi_{Z2}} = \frac{(I_{p1}I_{p2})\omega_{nf}^4 - \left\{(I_{p1}+I_{p2})k_{t1} + I_{p1}k_{t2}\right\}\omega_{nf}^2 + (k_{t1}k_{t2})}{(k_{t1}k_{t2})} \quad \text{--(1.6)}$$

2.2. Eigen value problem

Is a general method of obtaining of natural frequencies and mode shape is to formulate an Eigen value problem and that will be solved by MATLAB software.

Equation (1.2) can be written as

$$\left(-\omega_{nf}^2[M] + [K]\right)\{\Phi\} = \{0\} \quad \text{-- (2.1)}$$

With

$$[M] = \begin{bmatrix} I_{p1} & 0 & 0 \\ 0 & I_{p2} & 0 \\ 0 & 0 & I_{p3} \end{bmatrix};$$

$$[K] = \begin{bmatrix} k_{t1} & -k_{t1} & 0 \\ k_{t1} & (k_{t1}+k_{t2}) & -k_{t2} \\ 0 & -k_{t2} & k_{t2} \end{bmatrix};$$

$$\{\Phi\} = \begin{Bmatrix} \varphi_{Z1} \\ \varphi_{Z2} \\ \varphi_{Z3} \end{Bmatrix}$$

Table 1 Property value

Property	Value	Unit
Rotor		
Rotor shaft diameter	0.028	m
Rotational speed	1500	r.p.m
Mass	2.9688	Kg
Length of rotor	0.135	m
Polar moment of inertia of shaft	$3.017185*10^{-08}$	m^4
Rigid discs(Impeller)		
Inner diameter	0.028	m
Outer diameter	0.170	m
Thickness	0.032	m
Rigid discs(Induced Fan)		
Inner diameter	0.028	m
Outer diameter	0.180	m
Thickness	0.015	m
d= diameter G= modulus of rigidity Ip= Polar mass moment of Inertia J= Polar second moment of area K= Stiffness [K] = Stiffness matrix [M] = Mass matrix L= length φ_Z= Angular displacement ω_{nf} = Torsional Natural Frequencies [D] = Eigen vector matrix		

Table 2

Sr. No.	Dia. Of shaft (mm)	Diameter of rotor (Disk) (m)			Length between Two rotor (m), L1 & L2		Mass of rotor (Kg)			Length/thickness of disc/rotor(mm)		
	D	D1	D2	D3	1^{st} & 2^{nd}	2^{nd} & 3^{rd}	M1	M2	M3	L1	L2	L3
1	36	0.19	0.095	0.18	0.135	0.0725	8.84161 8.04813 9.97515	6.07861 2.18207 2.18207	2.97578 0.43874 2.70872	40	110	15
2	35	0.19	0.124	0.19	0.303	0.171	8.84161 8.04813 9.97515	8.19081 2.94029 2.94029	7.95745 1.17321 7.24332	40	87	36
3	20	0.16	0.085	0.16	0.182	0.154	3.13498 2.85363 3.5369	5.08745 1.82626 1.82626	2.35123 0.34666 2.14022	20	115	15
4	28	0.17	0.1	0.18	0.193	0.165	5.66255 5.15437 6.38852	8.26605 2.9673 2.9673	2.97578 0.43874 2.70872	32	135	15
5	32	0.2	0.096	0.205	0.265	0.185	8.32728 7.57996 9.39488	7.11013 2.55235 2.55235	4.37442 0.64495 3.98185	34	126	17
6	38	0.18	0.095	0.17	0.275	0.244	5.95156 5.41744 6.71458	16.578 5.95109 5.95109	4.95473 0.73051 4.51008	30	300	28

Actual on field dimensions of three rotor system (water pump) with different manufacturer (pump).

Table 3 Identified (experimental) Natural frequencies and Mode shape for different unbalance configurations with readings

.	Material-I		Material-II		Material-III	
Sr. No.	ω_{nf}-A	Mode Shapes I	ω_{nf}-B	Mode Shapes II	ω_{nf}-C	Mode Shapes III
1	1.0e+03 * 5.8270 0.0000 1.9854	1.0000 -1.4773 -2.4699 1.0000 -20.3391 8.2623 1.0000 1.0000 1.0000	1.0e+04 * 1.1269 0 + 0.0000i 0.3884	1.0000 -7.6303 -9.8680 1.0000 -71.6528 78.8270 1.0000 1.0000 1.0000	1.0e+03 * 9.0317 0 + 0.0000i 2.1031	1.0000 -2.1363 -3.6238 1.0000 -56.8391 8.6510 1.0000 1.0000 1.0000
2	1.0e+03 * 2.3415 0.9139 0 + 0.0000i	1.0000 1.0000 1.0000 1.0000 -7.6563 2.2455 1.0000 -0.3188 -0.9713	1.0e+03 * 4.3480 0 + 0.0000i 1.6112	1.0000 -2.7307 -3.9450 1.0000 -26.1695 21.0749 1.0000 1.0000 1.0000	1.0e+03 * 3.6687 0 + 0.0000i 0.9167	1.0000 -0.4969 -1.2912 1.0000 -22.9739 2.5950 1.0000 1.0000 1.0000
3	1.0e+03 * 1.6480 0.0000 0.7456	1.0000 -0.2466 -1.1827 1.0000 -5.0909 1.7755 1.0000 1.0000 1.0000	1.0e+03 * 3.0619 0 + 0.0000i 1.3396	1.0000 -2.6630 -4.2725 1.0000 -18.1372 18.7351 1.0000 1.0000 1.0000	1.0e+03 * 2.5536 0 + 0.0000i 0.7496	1.0000 -0.4215 -1.5511 1.0000 -15.4985 2.0798 1.0000 1.0000 1.0000
4	1.0e+03 * 2.1605 0.0000 1.0536	1.0000 -0.4009 -1.3536 1.0000 -4.8910 2.4960 1.0000 1.0000 1.0000	1.0e+03 * 0.0000 1.8528 4.1987	1.0000 -19.2529 29.7099 1.0000 1.0000 1.0000 1.0000 -2.9436 -4.3346	1.0e+03 * 3.2833 0.0000 1.0719	1.0000 -0.6361 -1.8886 1.0000 -14.3499 2.7480 1.0000 1.0000 1.0000
5	1.0e+03 * 2.6358 0.0000 0.8824	1.0000 -0.6058 -1.5960 1.0000 -13.3277 2.9387 1.0000 1.0000 1.0000	1.0e+03 * 4.7665 0 + 0.0000i 1.7328	1.0000 -4.6364 -7.1627 1.0000 -41.6479 24.9582 1.0000 1.0000 1.0000	1.0e+03 * 4.2044 0.0000 0.8987	1.0000 -0.8791 -2.1222 1.0000 -40.1279 3.3950 1.0000 1.0000 1.0000
6	1.0e+03 * 2.5464 1.3991 0 + 0.0000i	1.0000 1.0000 1.0000 1.0000 -3.0396 1.8294 1.0000 -0.2195 -1.1173	1.0e+03 * 0 2.3241	1.0000 -13.2587 25.4163 1.0000 1.0000 1.0000 1.0000 -2.0632 -3.0653	1.0e+03 * 3.7916 0 + 0.0000i 1.4073	1.0000 -0.3920 -1.5076 1.0000 -9.1050 2.0827 1.0000 1.0000 1.0000

Table: 4 Material for three disc rotor (shown in plate 2) system,

Graph No.[**] & Sr.No		**Disc-I Impeller**	**Disc-II Rotor**	**Disc-III Fan**
1 A, 2A, 3A, 4A, 5A, 6A	Material-I	Steel	Steel	Steel
1B, 2B, 3B, 4B, 5B, 6B	Material-II	Cast Iron	Aluminium	Plastic
1C, 2C, 3C, 4C, 5C, 6C	Material-III	Bronze	Aluminium	Cast Iron

**Graph 1A-4A, 1B-4B & 1C-4C, calculated but not present (Display) in paper. Nature of these graphs observed, same as the graphs which are shown in paper with same conclusion.

By multiplying both sides by inverse of mass matrix with above equation (2.1) we get a standard Eigen value problem of the following form

$$\left(-\omega_{nf}^2[I] + [D]\right)\{\Phi\} = \{0\} \quad -(2.2)$$

With

$$[D] = [M]^{-1}[K]$$

The Eigen value and Eigen Vector of the matrix $[D]$ obtained & square root of Eigen value gives the natural frequencies and corresponding Eigen vectors as mode shapes.

3. Details of the rotor model for the numerical example

If we have several discs on a shaft as shown in plate 1, there are different material and dimension are used for discs. Table (2) shows the on field details dimension of disc and its positions from the end and corresponding masses. The dimension and masses of rotor system base on the manufacturer and H.P capacity of centrifugal pump. If the material of pump rotor and shaft are Aluminium, then efficiency of pump are maximum (M.Shelar and V.Patil, 2012). The Masses of rotor based on the material of disc, like steel, cast Iron, Aluminium, bronze and Plastic. There is more than one method of solving this system. Table (1) shows the property value of one of the rotor system and Nomenclature used in paper.

3.1. Details of the rotor model for the numerical example#

The detail is given in table 2.

4. Experimental results of the rotor model

Table (3) identified several possible nodes and natural frequencies of rotor system, three mode shapes corresponding to three torsional natural frequencies. ω_{nf}-A for the Mode shapes I where Material of disc & shaft are Steel. ω_{nf}-B for the Mode shapes II, where Material of Three discs are Cast iron, Aluminium, and Plastic respectively. ω_{nf}-C for the Mode shapes III, where Material of three discs is Bronze,Aluminium,and Castiron respectively.

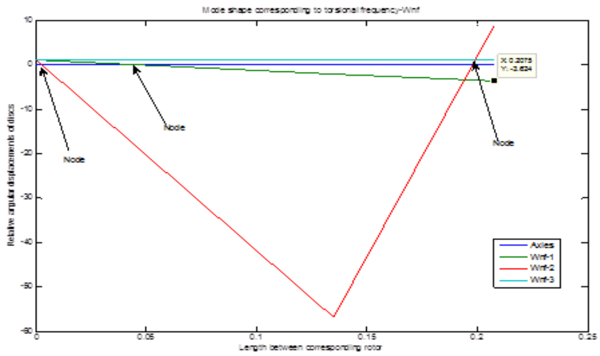
From equation (1.6&2.2) it can concluded that the first root equation represent the case when both discs rolls together in phase with each other as shown in fig. The representation of the relative angular displacements of two discs in this form is called the mode shape. Mode shapes Wnf-1 are the rigid body mode, and which significance that their no stresses develops in the shaft (Bevan,T., 1984), (Timoshenko S.P. and Young D.H, 1968), (Tuplin, W.A., 1966). Another value of root equation represent ,When the both masses are vibrating in the opposite direction, there must be a point on the shaft where torsional vibration not present means the angular displacement is zero.

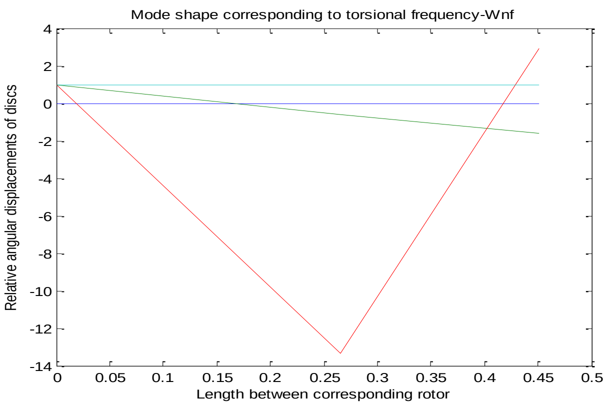
This point is called Node point and noted that the shear stress would be maximum at the node point (Bevan,T.,1984), (Timoshenko S.P. and Young D.H , 1968), (Tuplin, W.A., 1966). In the graph of Sr. No. 1 to 6 for Table 3 (Graph 1 A- Graph 6 C) where the elastic line crosses zero axis line, point called node. Following graph 5A-5C & 6A-6C represent the mode shape and node point for respective natural frequencies of calculation of Sr. No. 5 & 6 from table 3.It is observed that torsional natural frequency, mode shape & node point are different for rotor whose dimensions are same but disc material is different.

Impeller

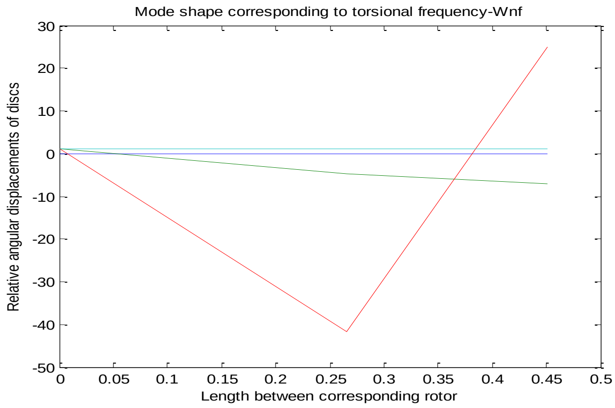
Rotor

Fan

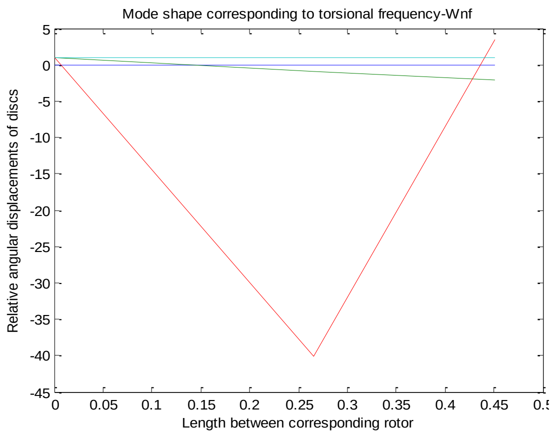




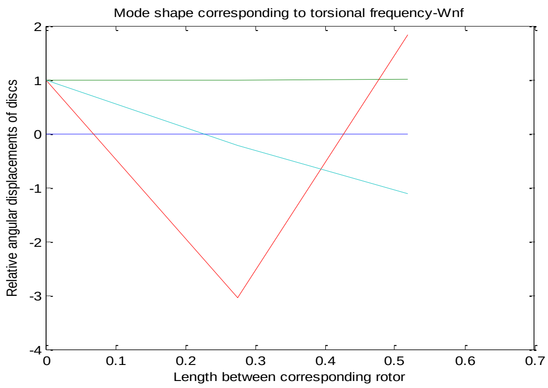
Graph 5A



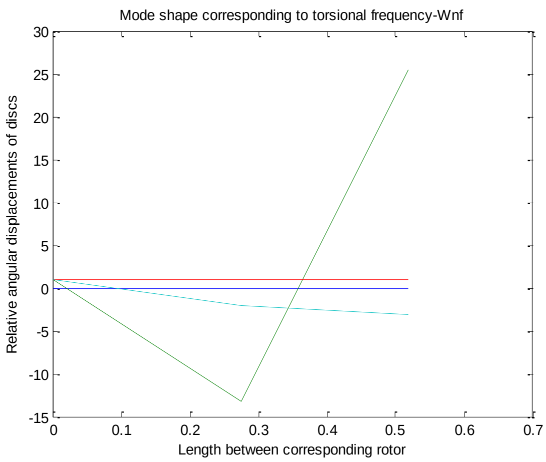
Graph 5B



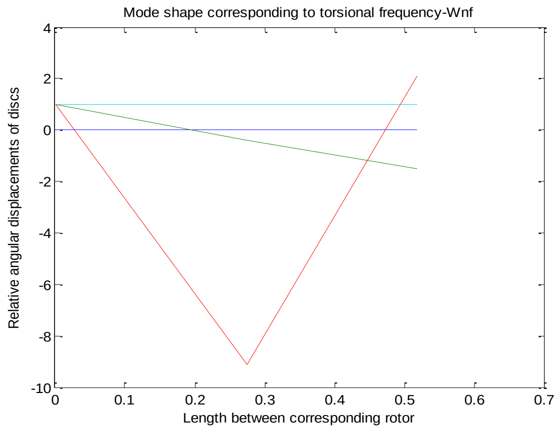
Graph 5C



Graph 6A



Graph 6B



Graph 6C

5. Conclusions

In the present paper, to summaries now the idea about the torsional natural frequency and mode shape for simple rotors which are used for agricultural centrifugal water pump. We have obtained torsional natural frequency and mode shapes using Newton's second law motion, and using the direct approach, characteristic equation and Eigen value problem method. The location of node present at each end of the real system and this point is treated where the shaft is rigidly fixed and maximum shear stress present at the node. Designer to select the appropriate material and dimensions of disc for design the rotor system for agriculture pump, hence improve the efficiency of these systems.

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References

R. Tiwari , V. Chakravarthy,(2009) Simultaneous estimation of the residual unbalance and bearing dynamic parameters from the experimental data in a rotor-bearing system, Mechanism and Machine Theory 44, 792–812.

Rajiv Tiwari,(2010) Analysis and Identification in Rotor-Bearing System, edition 271-296.

M.Shelar, V.Patil,(2011) Field Testing of Agricultural pumps for performance assessment and policy intervention, Prayas Energy Group, Pune.

Tiwari, (2005) Conditioning of regression matrices for simultaneous estimation of the residual unbalance and bearing dynamic parameters, Mechanical System and Signal Processing 19 (5) 1082–1095.

Bevan,T., (1984),The Theory of Machines, Third Edition, CBS Publishers and Distributors, New Delhi.imoshenko S.P. and Young D.H ,(1968), Elements of strength of Materials, Fifth edition, Affiliated East-West Press Private Limited, New Delhi.

Tuplin, W.A., (1966), Torsional Vibrations, Pitman, London.