
Multi-objective optimisation of grinding process parameters using NSGA-II

P.J. Pawar* and D.P. Rai-Kalal

Production Engineering Department,
K.K. Wagh Institute of Engineering Education and Research,
Nasik, Maharashtra-422003, India
E-mail: pjpawar1@rediffmail.com
E-mail: dhiraj.p.rai@gmail.com
*Corresponding author

Abstract: Selection of appropriate combination of process parameters in any machining process is a crucial task as it significantly affects the process performance. In the present work, an attempt is made to optimise the process parameters of grinding process. A well-known multi-objective optimisation technique known as non-dominated sorting genetic algorithm II (NSGA-II) is applied to obtain the optimum values of process variables such as wheel speed, work-piece speed, depth of dressing, and lead of dressing in order to improve the process performance in terms of production cost, production rate, and surface finish. Various process constraints such as thermal damage of work-piece, wheel wear, and machine tool stiffness are also taken into account.

Keywords: grinding; multi-objective optimisation; non-dominated sorting genetic algorithm II; NSGA-II.

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Biographical notes: P.J. Pawar received his BE from Pune University, his ME from Shivaji University and PhD from SVNIT Surat, India. He has 16 years of teaching and research experience and is currently working as an Associate Professor in the Production Engineering Department of K.K. Wagh Institute of Engineering Education and Research, Nasik. His research interests include advanced manufacturing processes and optimisation techniques.

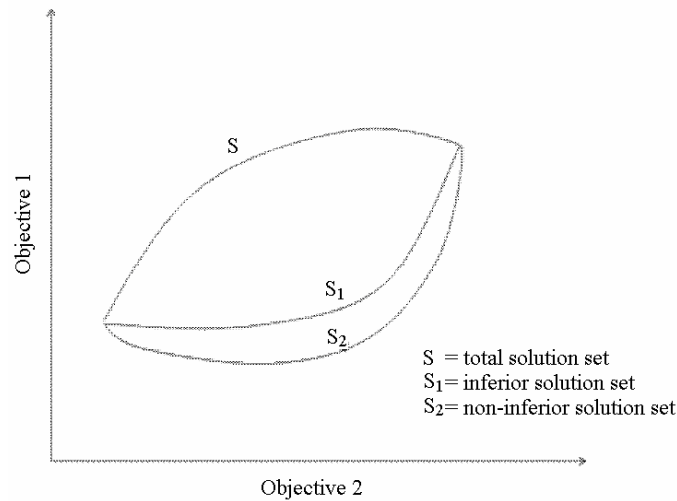
D.P. Rai-Kalal received his BE from Pune University and is currently pursuing an ME programme at K.K. Wagh Institute of Engineering Education and Research, Nasik. His research interests include multi-objective optimisation of manufacturing processes.

1 Introduction

The need of optimisation of machining processes mainly arises due to the various reasons such as to increase the production rate, to increase material removal rate, to improve product quality, to reduce production cost, to reduce tool wear, to reduce operating and maintenance cost, etc. Most of the machining optimisation problems are multi-objective.

Multi-objective optimisation problem deals with two or more objectives usually conflicting in nature. Thus, a multi-objective optimisation problem consists of number of optimal solutions each corresponding to a certain order of importance or preference factor of the objectives (Deb, 2005). Figure 1 shows the set of non-dominated solutions S_2 in the search space S for two objectives with objective 1 to be minimised and objective 2 to be maximised.

Figure 1 Pareto-optimal front



In the present work, an attempt is made to optimise the process parameters of surface grinding operation. Surface grinding is one of the very important machining operations in engineering industries. In this work multi-objective optimisation aspects of grinding process are presented to achieve the better process performance in terms of production cost, productivity and surface roughness. Variables such as wheel speed, work-piece speed, depth of dressing, and lead of dressing which significantly affects the process performance along with various process constraints such as thermal damage of work-piece, wheel wear, and machine tool stiffness are taken into account for parametric optimisation of surface grinding process. Optimisation is then carried out using a well known a posteriori multi-objective optimisation technique known as non-dominated sorting genetic algorithm-II (NSGA-II). The set of non-dominated solution obtained using NSGA-II provides a ready reference to process planner to select the appropriate process parameters of grinding process for specific application scenario.

2 Literature review

Various researchers had applied both traditional and non-traditional methods of optimisation of grinding process parameters. Wen et al. (1992) applied successive quadratic programming (QP) approach for multi-objective optimisation of surface grinding process parameters. However, by this approach the convergence to an optimal solution depends on the chosen initial solution. Also the algorithm tends to get stuck to the local optimal solution.

Saravanan et al. (2002) applied genetic algorithm to optimise the process parameters of surface grinding process considering variables such as wheel speed, work-piece speed, depth of cut lead of dressing, with an objective to minimise the cost of production, maximise production rate and improve surface roughness. However, in case of finish grinding operation, for the optimum parameter combination reported by Saravanan et al. (2002), the value of production cost is wrongly calculated as 6.6 \$/pc, which when calculated correctly is 7.36 \$/pc. This also changes the reported combined objective function value of 0.521 to 0.542.

Gopala (2007) applied differential evolution (DE) algorithm for optimisation of process parameters of grinding operation. However, in case of rough grinding, for the optimum parameter combination reported by Gopala (2007), the value of surface roughness is wrongly calculated as 1.8 μ , which when calculated correctly is 1.87 μ . Thus, the corrected value of surface roughness exceeds the permissible surface roughness value of 1.8 μ . Also in case of finish grinding operation, optimum values of process parameters like wheel speed (V_s) and depth of dressing (doc) reported by Gopala (2007) are laying outside their respective bounds.

Pawar et al. (2010) applied particle swarm optimisation (PSO) algorithm for process parameter optimisation of grinding process considering process parameters such as wheel speed, work piece speed, depth of cut, and lead of dressing. The results of PSO algorithm were compared with the previously published results obtained by using other algorithms.

It is observed from the literature related to parametric optimisation of grinding process that earlier researchers had used mainly a priori approach in which the multi-objective optimisation problem is transformed into a single objective optimisation problem by assigning an appropriate weight to each objective. This ultimately leads to a single optimum solution. Unfortunately, the solution obtained by this process depends largely on weights assigned to various objective functions. This approach does not provide a dense spread of the Pareto points. Also, the process engineer needs to have a complete knowledge of the process in order to determine weights to various objectives. Moreover, in the frequently changing market scenario, the weights of the objective functions needs to be changed frequently which seek different combination of the process variables every time to achieve optimum performance. Hence, a posteriori approach such as non-dominated sorting genetic algorithm (NSGA) is always preferred for solving multi-objective optimisation problems. However, non-dominated sorting genetic algorithm suffers from drawbacks such as high computational complexity of non-dominated sorting, lack of elitism, and need for specifying the sharing parameter (Deb et al., 2002). To overcome the above mentioned limitations, Deb et al. (2002) proposed an improved version of NSGA known as NSGA-II for solving multi-objective optimisation problems. The NSGA and NSGA-II algorithm are found to be suitable in parametric optimisation of several machining processes.

Kuriakose and Shunmugam (2005) applied non-dominated sorting genetic algorithm to optimise the process parameters of wire electric discharge machining (EDM) process with an objective to minimise the surface roughness and maximise the cutting velocity. The process parameters such as machining voltage, ignition pulse current, time between two pulse, and pulse duration were considered. Mitra and Gopinath (2004) used NSGA-II for optimising the industrial grinding operation of a lead-zinc ore beneficiation with objectives as to maximise the grinding product throughput and maximising percentage passing of midsize. Three process parameters are optimised, the solid ore flow rate, primary water flow rate, and secondary water flow rate. The authors compared the spread

of the Pareto front obtained by using NSGA-II and single objective optimisation problem formulation for multi-objective optimisation problem using weighted average approach. It was observed that the quality of the Pareto front obtained by using NSGA-II was better than the other techniques. Mandal et al. (2007) optimised the EDM process using NSGA-II. The authors applied back propagation artificial neural network to model the EDM process. The set of Pareto optimal solutions was then obtained using NSGA-II. The work presents the effectiveness of NSGA-II algorithm for solving complex multi-objective optimisation problem of EDM process. Mitra (2008) provided a comprehensive review on application of genetic algorithms in various polymeric fields, such as in polymer production, polymer scheduling, polymer design, polymer laminates and other polymer-related applications and non-polymeric fields. Kanagarajan et al. (2008) evaluated the effectiveness of the EDM process with tungsten carbide and cobalt composites. The effect of various process parameters of EDM such as pulse current, pulse on time, electrode rotation and flushing pressure on material removal rate and surface roughness was investigated. The optimisation was then carried out by using NSGA-II. Kodali et al. (2008) compared the performance of NSGA-II with other algorithms such as QP, PSO, scatter search (SS), ant colony optimisation (ACO), and DE for multi-objective optimisation of grinding process parameters. It was observed that the results obtained by using NSGA-II outperformed those obtained by using other algorithms. Palanikumar et al. (2009) applied NSGA-II for optimising the cutting conditions for machining of glass fibre reinforced plastic (GFRP) composites.

Yang and Natarajan (2010) compared the performance of the multi-objective differential evolution (MODE) algorithm and NSGA-II for multi-objective optimisation of turning process. It was observed that NSGA-II outperformed MODE in the context of number of solutions and ratio of non-dominated individuals. Senthil Kumar et al. (2010) improved cutting parameters of ECM using NSGA-II to maximise MRR and minimise surface roughness.

It is thus observed that NSGA-II has been successfully applied by the researchers for multi-objective optimisation of various machining processes. It is with this spirit, in the present work, NSGA-II is applied to the multi-objective optimisation of the grinding process parameters. Two cases for optimisation of process parameters using NSGA-II are presented in the next section.

3 Case studies

The optimisation model for grinding process formulated in the present work is based on the analysis given by Wen et al. (1992). The four decision variables considered for this model are, wheel speed ' V_s ' (m/min), work piece speed ' V_w ' (m/min), depth of dressing ' doc ' (mm) and lead of dressing ' L ' (mm/rev).

The three objectives considered in this work are:

- a minimisation of production cost ' C_T ' (\$/pc).
- b maximise the production rate in terms of workpiece removal parameter ' WRP ' ($\text{mm}^3/\text{min.N}$).
- c minimisation of surface roughness ' R_a ' (μm).

However, keeping in view the specific requirement of finish grinding and rough grinding operation, these three objective functions are divided into two groups as follows.

3.1 Case study 1: optimisation model for rough grinding operation

For rough grinding operation following two objective functions are considered with the condition that the surface roughness value should not exceed 1.8 μm .

- a minimisation of production cost (C_T) as given by equation (1)

$$C_T = \frac{M_c}{60p} \left(\frac{L_w + L_e}{V_w 1,000} \right) \left(\frac{b_w + b_e}{f_b} \right) \left(\frac{a_w}{a_p} + S_p + \frac{a_w b_w L_w}{\pi D_e b_s a_p G} \right) + \frac{M_c}{60p} \left(\frac{S_d}{V_r} + t_l \right) + \frac{M_c t_{ch}}{60 N_t} + \frac{M_c \pi b_s D_e}{60 p N_d L V_s 1,000} + C_s \left(\frac{a_w b_w L_w}{p G} + \frac{\pi (doc) b_s D_e}{p N_d} \right) + \frac{C_d}{p N_d} \quad (1)$$

- b maximise the production rate in terms of work-piece removal parameter 'WRP' as given by equation (2)

$$WRP = 94.4 \frac{(1 + (2doc / 3L)) L^{11/19} (V_w / V_s)^{3/19} V_s}{D_e^{43/304} VOL^{0.47} a_g^{5/38} R_c^{27/19}}. \quad (2)$$

3.2 Case study 2: optimisation model for finish grinding operation

For finish grinding operation following two objective functions are considered with the condition that the work-piece removal parameter should not be less than 20 $\text{mm}^3/\text{min.N}$.

- a minimisation of production cost ' C_T ' (\$/pc) as given by equation (1)
b minimisation of surface roughness ' R_a ' (μm) as given by equation (3)

$$R_a = 0.4587 T_{ave}^{0.30} \text{ for } 0 \leq T_{ave} < 0.254 \text{ else,} \quad (3)$$

$$R_a = 0.78667 T_{ave}^{0.72} \text{ for } 0.254 < T_{ave} < 2.54$$

$$T_{ave} = 12.5 \times 10^3 \frac{d_g^{16/27} a_p^{19/27}}{D_e^{8/27}} \left(1 + \frac{doc}{L} \right) L^{16/17} \left(\frac{V_w}{V_s} \right)^{16/27}. \quad (4)$$

Following three constraints are considered for both case studies.

- a *Thermal damage constraint*: The grinding process requires very high energy per unit volume of material removed. Whatever the energy that is concentrated within the grinding zone, it is converted into heat. The high thermal energy causes damage to the work piece, and it leads to the reduced production rate. The specific energy U is calculated by equation (5).

$$U = 13.8 + \frac{9.64 \times 10^{-4} V_s}{a_p V_w} + \left(6.9 \times 10^{-3} \frac{2,102 V_w}{D_e V_s} \right) \times \left(A_0 + \frac{K_u V_s L_w a_w}{V_w D_e^{1/2} a_p^{1/2}} \right) \frac{V_s D_e^{1/2}}{V_w a_p^{1/2}}. \quad (5)$$

The critical specific energy U^* at which burning starts is expressed in terms of the operating parameters as

$$U^* = 6.2 + 1.76 \left(\frac{D_e^{1/4}}{a_p^{3/4} V_w^{1/2}} \right). \quad (6)$$

The thermal damage constraint is then specified as

$$U^* - U \geq 0. \quad (7)$$

- b *Wheel wear parameter constraint:* Wheel wear parameter WWP ($\text{mm}^3/\text{min.N}$) is related directly to the grinding conditions. For single-point diamond dressing, it is given by equation (8).

$$WWP = \left(\frac{k_p a_p d_g^{5/38} R_c^{27/29}}{D_c^{1.2/VOL-43/304} VOL^{0.38}} \right) \times \frac{(1(doc/L)) L^{27/19} (V_s/V_w)^{3/19} V_w}{(1 + (2doc/3L))}. \quad (8)$$

From equations (2) and (8) the wheel wear constraint is obtained as

$$\frac{WRP}{WWP} - G \geq 0. \quad (9)$$

- c *Machine tool stiffness constraint:* Chatter results in poorer surface quality and lowers machining production rate. Chatter avoidance is therefore a significant constraint in selection of machining parameters. The relationship between grinding stiffness K_c (N/mm), wheel wear stiffness K_s (N/mm) and operating parameters during grinding is given below:

$$K_c = \frac{1,000 V_w f_b}{WRP} \quad (10)$$

$$K_s = \frac{1,000 V_s f_b}{WWP}. \quad (11)$$

To avoid chatter during machining, the constraint given by equation (12) has to be fulfilled:

$$MSC - \frac{|R_{em}|}{K_m} \geq 0 \quad (12)$$

where

$$MSC = \frac{1}{2K_c} \left(1 + \frac{V_w}{V_s G} \right) + \frac{1}{K_s}. \quad (13)$$

Values of the constants and parameters used in the process parameter optimisation of grinding process are given in Table 1. The variable bounds for all variables considered in this work are specified in Table 2. The optimisation aspects of grinding process using NSGA-II are briefly discussed in the next section.

Table 1 Constants and parameters used in optimisation of grinding process

<i>Notation</i>	<i>Description</i>	<i>Unit</i>	<i>Value</i>
M_c	Cost per hour labour and administration	\$/hr	30
L_w	Length of work-piece	mm	300
L_e	Empty length of grinding	mm	150
b_w	Width of work-piece	mm	60
b_e	Empty width of grinding	mm	25
f_b	Cross feed rate	mm/pass	2
a_w	Total thickness of cut	mm	0.1
a_p	Down feed of grinding	mm/pass	0.0505
S_p	Number of spark out grinding		2
D_e	Diameter of wheel	mm	355
b_s	Width of wheel	mm	25
G	Grinding ratio		60
S_d	Distance of wheel idling	mm	100
p	Number of work-pieces loaded on the table		1
V_r	Speed of wheel idling	mm/min	254
t_l	Time of loading and unloading work-pieces	min	5
t_{ch}	Time of adjusting machine tool	min	30
N_t	Batch size of the work-pieces		12
N_d	Total number of work-pieces to be ground between two dressings		20
N_{td}	Total number of work-pieces to be ground during the life of dresser		2,000
C_s	Cost of wheel per mm ³		\$0.003
C_d	Cost of dressing		\$25
VOL	Wheel bond percentage		6.99
d_g	Grind size	mm	0.3
R_c	Work-piece hardness	HRC	58
K_u	Wear constant	mm ⁻¹	3.937×10^{-7}
R_{em}	Dynamic machine characteristics		1
K_m	Static machine stiffness	N/mm	100,000
K_a	Constant dependent on coolant and grain type		0.0869

Table 2 Variable bounds of input variables

<i>Parameters</i>	<i>Unit</i>	<i>Lower limit</i>	<i>Upper limit</i>
V_s	m/min	1,000	2,023
V_w	m/min	10	22.7
doc	mm	0.01	0.137
L	mm/rev	0.01	0.137

4 Multi-objective optimisation of grinding process parameters using NSGA-II

The optimisation of grinding process for rough grinding operation and finish grinding operation as discussed in Section 3 is carried out using NSGA-II through following steps.

4.1 Steps of NSGA-II

- Step 1 *Population initialisation*: Initialise the population based on the problem range and constraint.
- Step 2 *Non-dominated sorting*: Sorting process based on non-domination criteria of the population that has been initialised.
- Step 3 *Crowding distance*: Once the sorting is complete, the crowding distance value is assign front wise. The individuals in population are selected based on rank and crowding distance.
- Step 4 *Selection*: The selection of individuals is carried out using a tournament selection with crowded-comparison operator.
- Step 5 *Genetic operators*: Real coded GA using simulated binary crossover (SBX) and polynomial mutation.
- Step 6 *Recombination and selection*: Offspring population and current generation population are combined and the individuals of the next generation are set by selection. The new generation is filled by each front subsequently until the population size exceeds the current population size.

In this work, a penalty parameter less constraint handling technique is used as described below (Deb, 2000).

$$F(x) = \begin{cases} f(x), & \text{if } x \text{ is feasible} \\ f_{\max} + \sum_{j=1}^J \langle g_j(x) \rangle + \sum_{k=1}^K |h_k(x)|, & \text{otherwise} \end{cases} \quad (14)$$

where $f(x)$ is the value of objective function corresponding to feasible a solution, f_{\max} is the objective function value of the worst feasible solution in the population, $g_j(x)$, $h_k(x)$ are the inequality and equality constraints respectively.

One fundamental difference between this approach and the conventional approach is that the objective function value is not computed for any infeasible solution. Since all the feasible solutions have a zero constraint violation only and all infeasible solutions are evaluated according to their constraint violation only. Thus, there is no need to have any penalty parameter for this approach.

4.2 Optimisation of rough grinding process

This case of study presents the multi-objective optimisation of rough grinding process. The two objectives considered in this work are minimisation of cost of production (C_T) and maximisation of work piece removal rate (WRP) as given by equations (1) and (2)

respectively. The constraints are on thermal damage, wheel wear parameter, and machine tool stiffness as given by equations (7), (9) and (12) respectively. Also, for rough grinding operation the surface roughness value given by equation (3) should not exceed $1.8 \mu\text{m}$. The steps of optimisation using NSGA-II algorithm as discussed in the Section 4.1 are applied to obtain the set of Pareto-optimal solutions.

To determine suitable parameters of NSGA-II, the effect of varying the NSGA-II parameters on optimisation of process parameters of rough grinding as well as finish grinding is studied. The population size was reduced in three steps from 80 to 40 and from 40 to 20. It was observed that reduction in population size led to increase in rate of convergence but at the cost of poor spread of solutions on the front. The crossover (p_c) and mutation probabilities (p_m) were changed from $p_c = 0.9, p_m = 0.1$ to $p_c = 0.8, p_m = 0.2$. It was observed that there is no significant effect of varying the crossover probability (p_c) and mutation probability (p_m) in the selected range on the Pareto-optimal front as well as the on the convergence rate. However, when p_c was further reduced to 0.6 and p_m increased to 0.4, it leads to inferior spread of solutions. The distribution index of SBX operator was gradually changed from 1.0 to 5.0 and it was observed that higher value index results in formation of near parent solution whereas small value of distribution index allows distant solutions to be selected as offspring. The distribution index of polynomial mutation operator was gradually changed from 1.0 to 5.0 leading to a marginally better spread of Pareto optimal solutions.

After several trial runs, the following parameters of NSGA-II are selected for the present case study:

- population size (N) = 80
- cross-over probability (p_c) = 0.9
- mutation probability (p_m) = 0.1
- the simulated binary crossover index (η_c) = 0.1
- polynomial mutation index (η_m) = 0.1.

The computer used for computation in the present study is Pentium IV processor with 256 MB RAM. To demonstrate the working of NSGA-II for optimisation of rough grinding process, sample calculations for five initial solutions are presented in Tables 3 to 6. Table 3 gives the real coded values of variables along with the corresponding values of the objective functions. The rank of each solution after assigning penalty to infeasible solutions, non-dominated sorting, and the crowding distance values is also shown in Table 3. Table 4 and Table 5 show the results of simulated binary cross-over and polynomial mutation respectively. Table 6 provides the set of non-dominated solutions. These solutions provide ready reference to the process planner to select the optimal combination of process parameters for rough grinding process as per the objective weights of his/her choice. The Pareto-optimal front is shown in Figure 2.

Table 7 shows the comparison of results obtained for rough grinding operation using NSGA-II with those obtained by earlier researchers using QP, GA, and PSO. As shown in Table 7, it is observed that results obtained by NSGA-II (for same weights of objective functions as considered by earlier researchers) show an improvement of 74.8% over that of QP and 36.19% over that of GA. The results obtained using NSGA-II are close to those obtained by using PSO.

Table 3 Evaluation and non-dominated sorting of solutions

<i>S.N.</i>	V_s <i>m/min</i>	V_w <i>m/min</i>	<i>doc</i> <i>mm</i>	L <i>mm/rev</i>	C_T <i>\$/pc.</i>	WRP <i>mm³/min.N</i>	<i>Rank</i>	<i>Crowding distance</i>
1	1,324.8	10.9397	0.0637	0.0966	7.8187	13.93	3	∞
2	1,323.7	18.688	0.0557	0.0688	6.3372	13.31	3	0.0276
3	1,222.6	15.1103	0.0809	0.0369	6.9353	13.42	3	0.204
4	1,256.8	22.482	0.1043	0.0225	6.212	18.25	2	1.6872
5	1,913.5	15.1077	0.0639	0.1146	6.8524	21.02	3	0.0461

Table 4 Simulated binary cross-over

$S.N.$	V_s		V_w		doc		L	
<i>Parent strings</i>								
1	1,800.9	1,571.1	11.37	16.32	0.073	0.0794	0.116	0.090
2	1,010.0	1,612.6	10.69	18.82	0.065	0.0814	0.051	0.120
3	1,862.6	1,152.3	11.25	20.59	0.136	0.0603	0.08	0.130
4	1,746.3	1,920.4	16.22	17.74	0.113	0.0606	0.134	0.082
5	1,229.4	1,460.8	19.81	17.29	0.072	0.0755	0.08	0.136
<i>Off-springs</i>								
1	1,758.2	1,613.8	12.29	15.4	0.073	0.079	0.1	0.090
2	1,180.2	1,442.3	11.43	18.07	0.069	0.076	0.063	0.100
3	1,797.4	1,217.5	12.98	18.9	0.129	0.067	0.08	0.120
4	1,762.3	1,904.4	16.04	17.9	0.1	0.065	0.14	0.073
5	1,272.4	1,417.8	19.34	17.8	0.072	0.074	0.07	0.130

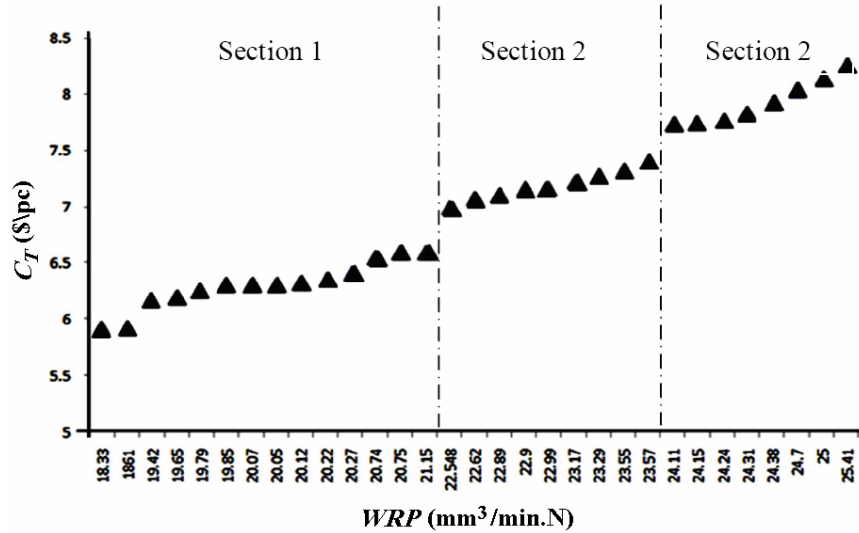
Table 5 Polynomial mutation

$S.N.$	V_s	V_w	doc	L
<i>Parent strings</i>				
1	1,177.9	21.55	0.04	0.105
2	1,026.7	21.54	0.03	0.092
3	1,976.6	17.51	0.13	0.025
4	1,411.1	14.22	0.128	0.074
5	1,983.7	20.83	0.114	0.054
<i>Off-springs</i>				
1	1,365.7	19.21	0.0643	0.0817
2	1,812.9	14.37	0.0772	0.0202
3	1,596.6	10.33	0.0334	0.0968
4	2,018.8	11.88	0.0304	0.0268
5	1,603.7	11.07	0.185	0.0773

Table 6 The best compromised results for rough grinding operation

<i>Sr.</i>	V_s <i>m/min</i>	V_w <i>m/min</i>	<i>doc</i> <i>mm</i>	L <i>mm/rev</i>	C_T <i>\$/pc.</i>	WRP <i>mm³/min.N</i>	R_a <i>μm</i>
1	1,984.48	22.50	0.0390	0.082	5.910	18.33	1.783
2	2,009.2	22.62	0.0420	0.078	5.920	18.61	1.796
3	2,014.6	20.23	0.0491	0.083	6.150	19.42	1.792
4	2,004.4	19.89	0.0470	0.091	6.170	19.65	1.797
5	2,019.5	19.30	0.0500	0.088	6.240	19.79	1.783
6	2,013.3	18.94	0.0520	0.088	6.290	19.85	1.786
7	2,021.0	18.86	0.0543	0.087	6.302	20.07	1.787
8	2,001.8	18.70	0.0500	0.097	6.300	20.05	1.788
9	2,018.8	18.64	0.0550	0.087	6.330	20.12	1.797
10	2,017.7	18.21	0.0530	0.093	6.370	20.22	1.783
11	1,997.7	18.10	0.0519	0.099	6.380	20.27	1.791
12	2,004.7	17.13	0.0580	0.098	6.530	20.74	1.795
13	1,990.3	16.81	0.0568	0.103	6.565	20.75	1.792
14	2,014.6	16.78	0.0567	0.107	6.569	21.15	1.790
15	2,018.7	14.43	0.0680	0.119	6.989	22.548	1.795
16	2,004.1	14.01	0.0678	0.126	7.060	22.62	1.790
17	2,008.5	13.77	0.0719	0.125	7.130	22.89	1.799
18	2,020.7	13.70	0.0700	0.125	7.140	22.9	1.784
19	2,020.8	13.54	0.0750	0.121	7.190	22.99	1.791
20	2,004.8	13.35	0.0720	0.132	7.220	23.17	1.796
21	2,021.8	13.22	0.0760	0.125	7.260	23.29	1.793
22	2,017.1	13.01	0.0762	0.132	7.310	23.55	1.796
23	2,016.6	12.75	0.0760	0.134	7.370	23.57	1.786
24	2,009.4	11.68	0.0880	0.133	7.690	24.11	1.791
25	2,015.8	11.63	0.0928	0.123	7.731	24.15	1.793
26	2,014.4	11.56	0.0910	0.131	7.736	24.24	1.791
27	2,010.0	11.15	0.0953	0.128	7.880	24.31	1.780
28	2,004.1	11.14	0.0970	0.128	7.890	24.38	1.798
29	2,007.3	10.70	0.1020	0.128	8.050	24.70	1.795
30	2,020.8	10.50	0.1031	0.132	8.127	25.00	1.787
31	2,021.5	10.17	0.1070	0.136	8.26	25.41	1.790

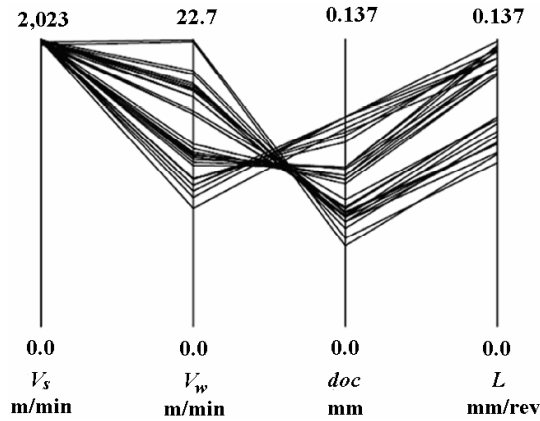
Notes: $a_w = 0.1$ mm, $a_p = 0.050$ mm/pass, $R_a \leq 1.8$ μm. V_s = wheel speed;
 V_w = work piece speed; *doc* = depth of dressing; L = lead of dressing,
 R_a = surface roughness.

Figure 2 Non-dominated front for rough grinding operation**Table 7** Results of optimisation for rough grinding operation using various algorithms

Method	Author and year	V_s m/min	V_w m/min	doc mm	L mm/rev	C_T \$/pc.	WRP mm ³ /min.N	R_a μm	COF
QP	Wen et al. (1992)	2,000.0	19.96	0.055	0.044	6.2	17.47	1.740	-0.127
GA	Saravanan et al. (2002)	1,998.0	11.30	0.101	0.065	7.86	22.25	1.770	-0.163
PSO	Pawar et al. (2010)	2,023	10.00	0.11	0.137	8.33	25.63	1.798	-0.224
NSGA-II		2,021.5	10.17	0.107	0.136	8.26	25.41	1.790	-0.222

Notes: $w_1 = 0.5$, $w_2 = 0.5$.

An investigation is performed to determine whether the obtained Pareto-optimal solutions bear some kind of similarity in terms of the associated decision variables. Figure 3 show a value path plot showing Pareto-optimal solutions for rough grinding operation obtained using NSGA-II. It is observed from Figure 3 that the values of the decision variables which correspond to the obtained solutions show some interesting trends such as the value of the wheel speed (V_s) seems to lie close to the upper bound (i.e., $V_s = 2,023$ m/min) for the entire set of solutions whereas the corresponding value of work piece speed (V_w) seems to be uniformly distributed over its complete range (i.e., from 10 m/min to 22.7 m/min). It is also observed that for the obtained solutions the depth of cut assumes a value between 0.039 mm to 0.107 mm and a higher value of depth of cut is desired to achieve a higher value of work-piece removal parameter. For values of work-piece removal parameter (WRP) < 21 mm³/min.N, the value of lead of dressing seems to lie close to its arithmetic mean (i.e., $L = 0.0735$ mm/rev). However, for higher values of WRP the lead of dressing assumes a value close to its upper bound.

Figure 3 A value path plot showing the best compromised solutions for rough grinding operation obtained using NSGA-II

4.3 Optimisation of finish grinding process

This case of study presents the multi-objective optimisation of finish grinding process. The objectives being minimisation of cost of production (C_T) and minimisation of surface roughness (R_a) given by equation (1) and (3) respectively, satisfying thermal damage, wheel wear parameter and machine tool stiffness constraint given by equations (7), (9) and (12) respectively, with work piece removal rate greater than 20 mm³/min.N. The steps of optimisation using NSGA-II are followed in the same manner as discussed in the previous section.

Table 8 provides the results of optimisation for finish grinding process using NSGA-II. The combination of process parameters shown in Table 8 provides the ready reference to the process planner for finish grinding process as per the objective weights of his choice. The Pareto-optimal front is shown in Figure 4.

Table 8 The best compromised results for finishing grinding operation

S.N.	V_s m/min	V_w m/min	doc mm	L mm/rev	C_T \$/pc	R_a μ m	WRP mm ³ /min.N
1	2,022.7	22.690	0.0110	0.1369	7.1300	0.795	20.070
2	2,022.9	18.190	0.0386	0.1165	8.0070	0.783	20.383
3	2,023.0	17.780	0.0645	0.1247	8.0700	0.775	20.490
4	1,990.0	15.735	0.0427	0.1160	8.6200	0.755	20.031
5	2,023.0	15.325	0.0386	0.1330	8.7200	0.753	20.939
6	1,956.9	14.500	0.0345	0.1370	8.9640	0.739	20.094
7	1,990.1	13.687	0.0468	0.1247	9.3030	0.736	20.470
8	2,023.0	13.277	0.0468	0.1330	9.4600	0.732	21.175
9	2,023.0	12.458	0.0427	0.1370	9.7890	0.706	20.860
10	2,023.0	11.229	0.0632	0.1042	10.488	0.696	20.360
11	2,023.0	10.819	0.0550	0.1247	10.688	0.684	20.701
12	2,022.9	10.000	0.0510	0.1330	11.199	0.659	20.590

Notes: $a_w = 0.055$ mm, $a_p = 0.0105$ mm/pass, $WRP \geq 20$ mm³/min.N.

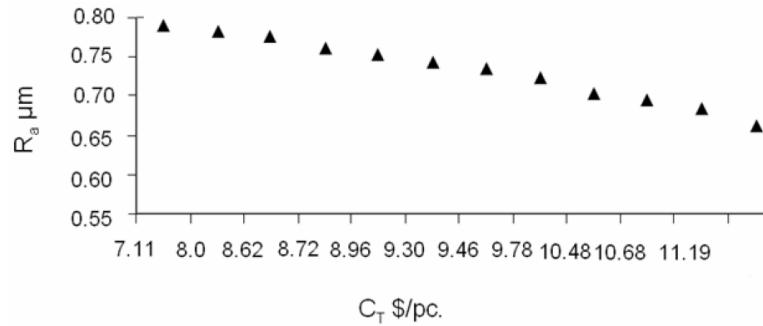
Figure 4 Non-dominated front for finish grinding operation

Table 9 shows the comparison of results obtained for finish grinding operation using NSGA-II with those obtained by earlier researchers using QP, GA, and PSO. As shown in Table 9, it is observed that results obtained by NSGA-II shows an improvement of 5.89% over that of QP and 3.81% over that of GA. The results obtained using NSGA-II are close to those obtained by using PSO.

Table 9 Results of optimisation for finish grinding operation using various algorithms

<i>Method</i>	<i>Author and year</i>	V_s <i>m/min</i>	V_w <i>m/min</i>	<i>doc</i> <i>mm</i>	L <i>mm/rev</i>	C_T <i>\$/pc.</i>	R_a μm	WRP <i>mm³/min</i>	<i>COF</i>
QP	Wen et al. (1992)	2,000.0	19.99	0.052	0.091	7.7	0.83	20.00	0.5547
GA	Saravanan et al. (2002)	1,986.0	21.4	0.024	0.136	6.6 ^a	0.83	20.08	0.521 ^a
GA	Saravanan et al. (2002)	1,986.0	21.4	0.024	0.136	7.36 ^b	0.83	20.08	0.5427 ^b
PSO	Pawar et al. (2010)	2,023.0	22.7	0.01	0.137	7.11	0.79	20.01	0.5200
NSGA-II		2,022.7	22.69	0.011	0.137	7.13	0.795	20.07	0.5220

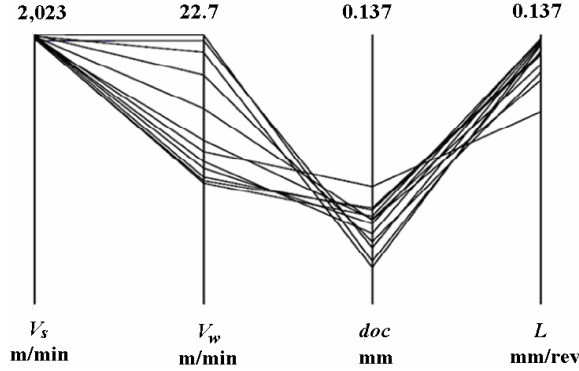
Notes: $w_1 = 0.7$, $w_2 = 0.3$.

^aValues wrongly calculated by Saravanan et al.

^bCorrected values.

An investigation is performed to determine whether the obtained Pareto-optimal solutions bear some kind of similarity in terms of the associated decision variables for finish grinding process. Figure 5 show a value path plot showing Pareto-optimal solutions obtained using NSGA-II for finish grinding operation. It is observed from Figure 5 that the value of decision variables such as wheel speed (V_s) as well as lead of dressing (L) lie close to their respective upper bounds for the entire set of obtained solutions whereas the values of work-piece speed (V_w) seems to be uniformly distributed over its complete range. It is also observed that higher value of work-piece speed results in a poor surface finish. For the entire set of obtained solutions the depth of cut (doc) takes the values close to its lower bound.

Figure 5 A value path plot showing best compromised solutions for finish grinding operation obtained using NSGA-II



A Fritz-John optimality criterion is used to test whether the solutions obtained are optimal. According to this approach, a necessary condition for a solution to be Pareto-optimal is that there exist vectors $\lambda \geq 0$ and $u \geq 0$ such that the conditions given by equation (15), (16), (17) and (18) are true.

$$\sum_{m=1}^M \lambda_m \nabla f_m(x^*) - \sum_{j=1}^J u_j \nabla g_j(x^*) = 0 \quad (15)$$

$$u_j g_j(x^*) = 0 \text{ for all } j = 1, 2, \dots, J \quad (16)$$

$$u \geq 0 \quad (17)$$

$$\lambda \geq 0 \quad (18)$$

where u gradient vector for constraints, λ gradient vector for objectives, ∇f gradient for objectives, ∇g gradient for constraint.

On evaluating the values of constraint given by equations (7), (9), and (12) for the obtained solutions it can be found that $g_1, g_2, g_3, g_4 > 0$. Thus, based on the condition given by equation (16), $u_1, u_2, u_3, u_4 = 0$. It is thus observed that the set of solutions obtained in the present work using NSGA-II satisfy the Fritz-John optimality criterion.

4.4 Characterisation of Pareto-front

For the purpose of characterisation the Pareto front obtained for rough grinding operation is divided into three sections as shown in Figure 2 and the observations are described as follows

- 1 *Section I:* It is observed that the value of work-piece removal parameter between $18.33 \text{ mm}^3/\text{min.N}$ to $21.15 \text{ mm}^3/\text{min.N}$ can be achieved by selecting the process variables in certain range as given below:

- wheel speed (V_s) = 1,984.48 m/min to 2,014.6 m/min
- work-piece speed (V_w) = 12.75 m/min to 14.43 m/min

- depth of dressing (doc) = 0.0678 mm to 0.076 mm
- lead of dressing (L) = 0.078 mm/rev to 0.107 mm/rev.

For the above mentioned ranges of process variables, the cost of production (C_T) lies between 5.910 \$/pc. to 6.569 \$/pc.

- 2 *Section 2:* It is observed that the value of work-piece removal parameter between 23.548 mm³/min.N to 23.57 mm³/min.N can be achieved by selecting the process variables in certain ranges as given below:

- wheel speed (V_s) = 2,004.1 m/min to 2,018.7 m/min
- work-piece speed (V_w) = 10.17 m/min to 11.68 m/min
- depth of dressing (doc) = 0.039 mm to 0.0580 mm
- lead of dressing (L) = 0.119 mm/rev to 0.134 mm/rev.

For the above mentioned ranges of process variables, the cost of production (C_T) lies between 6.989 \$/pc. to 7.37 \$/pc.

- 3 *Section 3:* It is observed that the value of work-piece removal parameter between 24.11 mm³/min.N to 25.41 mm³/min.N can be achieved by selecting the process variables in certain ranges as given below:

- wheel speed (V_s) = 2,004.1 m/min to 2,021.5 m/min
- work-piece speed (V_w) = 11.68 m/min to 10.17 m/min
- depth of dressing (doc) = 0.088 mm to 0.107 mm
- lead of dressing (L) = 0.123 mm/rev to 0.136 mm/rev.

For the above mentioned ranges of process variables, the cost of production (C_T) lies between 7.69 \$/pc. to 8.26 \$/pc.

For the purpose of characterisation, the Pareto front obtained for finish grinding operation is divided into three sections as shown in the Figure 4 and the observations are discussed below:

- 1 *Section 1:* It is observed that the value of surface roughness between 0.795 μ m to 0.755 μ m can be achieved by selecting the process variables in certain ranges as given below:

- wheel speed (V_s) = 1,990 m/min to 2,022.7 m/min
- work-piece speed (V_w) = 15.735 m/min to 22.69 m/min
- depth of dressing (doc) = 0.011 mm to 0.0645 mm
- lead of dressing (L) = 0.1160 mm/rev to 0.1369 mm/rev.

For the above mentioned ranges of process variables, the cost of production (C_T) lies between 7.13 \$/pc. to 8.62 \$/pc.

- 2 *Section 2:* It is observed the value of surface roughness between 0.732 μ m to 0.753 μ m can be achieved by selecting the process variables in certain ranges as given below:

- wheel speed (V_s) = 1,990.1 m/min to 2,023.0 m/min
- work-piece speed (V_w) = 13.277 m/min to 15.325 m/min
- depth of dressing (doc) = 0.0345 mm to 0.0468 mm
- lead of dressing (L) = 0.1247 mm/rev to 0.1370 mm/rev.

For the above mentioned ranges of process variables, the cost of production (C_T) lies between 8.72 \$/pc to 9.46 \$/pc.

- 3 *Section 3:* It is observed that the value of surface roughness between 0.659 μm to 0.706 μm can be achieved by selecting the process variables in certain ranges as given below:

- wheel speed (V_s) = 2,023.0 m/min.
- work-piece speed (V_w) = 10.0 m/min to 12.458 m/min
- depth of dressing (doc) = 0.0427 mm to 0.0632 mm
- lead of dressing (L) = 0.1042 mm/rev to 0.1370 mm/rev

For the above mentioned ranges of process variables, the cost of production (C_T) lies between 9.7890 \$/pc. to 11.199 \$/pc.

5 Conclusions

In the present work, multi-objective optimisation aspects of rough grinding as well as finish grinding process parameters are considered using NSGA-II. The three objectives considered are: minimisation of production cost, maximisation of production rate, and maximisation of surface finish, i.e., minimisation of roughness value subjected to the constraints of thermal damage, wheel wear parameter, and machine tool stiffness. Although various traditional and non-traditional methods have been employed so far by earlier researchers for multi-objective optimisation of grinding process, they converted the multi-objective problem into single objective problem by assigning weights to the different objectives. However, such approach does not provide a dense spread of the Pareto points. Also, the process engineer needs to have a comprehensive knowledge of the process to assign the appropriate weight to each objective. To overcome this drawback of a priori approach, a posterior approach namely NSGA-II is used in this work. The non-dominated solutions obtained in this work using NSGA-II algorithm for rough grinding as well as finish grinding operations provides a ready reference to the process engineer. The results obtained using NSGA-II shows an improvement of about 74.8% in overall objective function over those obtained by using QP and about 36.19% over the results obtained by using genetic algorithm for rough grinding operation. For finish grinding operation the results obtained using NSGA-II shows an improvement of 5.89% in overall objective function over those obtained by using QP and 3.81% improvement over the results obtained by using genetic algorithm.

The algorithm can also be applied effectively to multi-objective optimisation of process parameters of other machining processes such as milling, turning, drilling, etc.

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