Sliding Mode Control for Spatial Stabilization of Advanced Heavy Water Reactor

R. K. Munje, B. M. Patre, S. R. Shimjith, and A. P. Tiwari

***Abstract*—Spatial oscillations in neutron flux distribution resulting from xenon reactivity feedback are a matter of concern in large nuclear reactors. If the spatial oscillations in power distribution are not controlled, power density and rate of change of power at some locations in the reactor core may exceed their respective limits causing increase in chances of fuel failure. Hence, during the design stages of any large nuclear reactor, it is essential to identify the existence of spatial instabilities and to design suitable control strategy for regulating the spatial power distribution. This paper presents a method to design and analyze the effect of sliding mode control (SMC) for spatial control of Advanced Heavy Water Reactor (AHWR). The AHWR model considered here is of 90th order with 5 inputs and 18 outputs. In this paper, numerically ill-conditioned system of AHWR is separated into 73rd order 'slow' subsystem and 17th order 'fast' subsystem and SMC is designed from slow subsystem. Further, using simple linear transformation matrices, SMC for full system is constructed. Also, it is proved that slow subsystem SMC results in a sliding mode motion for full system. Dynamic simulations has been carried out using nodal core model of AHWR to show effectiveness and robustness of proposed method.**

***Index Terms*—Siding mode control, sliding mode motion, spatial oscillations, two-stage decomposition.**

NOMENCLATURE

A	System state matrix.
B	System input matrix.
C	Delayed neutron precursor concentration.
E_{eff}	Average thermal energy liberated per fission, J.
E_n	Identity matrix of dimension n.
H	Position of regulating rod, % in.
I	Iodine concentration.
M	System output matrix.
P	Fission power, W.
V	Volume, m^3.
X	Xenon concentration.
c	Hyperplane matrix.
h	Enthalpy, kJ/kg.
q	Mass flow rate, kg/s.
u	Control input vector.
v	Voltage signal to regulating rod drive, V.
s	Sliding surface.
x	Exit mass quality.
y	Output vector.
z	State vector.
α	Coupling coefficient.
β	Delayed neutron fraction.
γ	Fraction fission yield.
λ	Decay constant.
ℓ	Prompt neutron life-time, s.
ρ	Reactivity, k.
σ_a	Microscopic absorption cross-section, cm^2.
Σ_a	Macroscopic absorption cross-section, cm^{-1}.
Σ_f	Macroscopic fission cross-section, cm^{-1}.
κ	Constant of regulating rod position.
δ	Deviation parameter.
φ	Eigenvalue.
ϵ	Perturbation parameter.
μ	Positive scalar.

As Subscripts:

C	Precursor.
H	Position of regulating rod.
I	Iodine.
L	Left.
P	Power.
R	Right.
X	Xenon.
c	Vaporization.
d	Downcomer.
f	Feed water, Fast, Fission.

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i, j	Node number.
m, n	System dimensions.
s	Steam, Slow, Spatial power.
w	Water.
x	Exit quality.
g	Global power.
eq	Equivalent.

I. Introduction

In India, Advanced Heavy Water Reactor (AHWR), a 920 MW (thermal), vertical pressure tube type nuclear reactor, moderated by heavy water, cooled by boiling light water under natural circulation is designed. The AHWR is fueled with (Th-^{233}U)O_2 pins and (Th-Pu)O_2 pins. The reactivity control devices in AHWR consist of eight absorber rods (ARs), eight shim rods (SRs) and eight regulating rods (RRs). Among these, RRs are used for fine adjustments in reactor power. Out of the eight RRs, four are available for automatic control whereas the remaining four are under manual operation. The neutron flux is measured using out-of-core ion chambers as well as in-core detectors. In-core detectors, however, are provided primarily for monitoring of spatial flux distribution in the core [1]–[3]. The core heat is removed by boiling light water under natural circulation at a pressure of 7 MPa. Reactor consists of 4 interconnected steam drums. Water from each steam drum flows down to a common inlet header. Individual coolant channels of the core are fed from this common header through individual feeder pipes [1], [4]. AHWR has a significant degree of coupling between the neutronics and the two-phase thermal hydraulics. The physical dimensions of AHWR are large compared to the neutron migration length in the core, making it susceptible to xenon-induced spatial oscillations [5]. To suppress these oscillations it is necessary to design controller for stabilization of total power and core-power distribution.

The analysis and control of large scale systems is complicated due to high order nature and interacting dynamic phenomena of widely different speeds, which gives rise to time-scales. Such systems are extensively studied in control theory by singular perturbations and time-scale methods. Excellent survey of such methods carried out first by Kokotovic [6] and later by Saksena [7] are available. These methods work by decoupling the fast and slow varying phenomena. The task of decomposition of the system leads to model order reduction. In two-stage decomposition, higher order system is decomposed into two comparatively lower order subsystems, namely, 'slow' subsystem and 'fast' subsystem. The model order reduction procedure and it's validation along with composite controller design can be found in [8]–[13]. Tiwari *et al.* [14] have developed the non-linear mathematical model of Pressurized Heavy Water Reactor (PHWR) characterized by 56 state variables and 14 inputs. Singularly perturbed structure of linear model is exploited to decompose it into fast subsystem of 14th order and slow subsystem of 42nd order. Separate regulators are designed and finally combined to obtain the near optimal composite control for the original model. Various other techniques are applied to control xenon induced spatial oscillations in PHWR and AHWR. Such techniques can be found in [4], [15]–[27].

Variable structure control (VSC) with sliding mode control (SMC) were first proposed and elaborated in the early 1950's in the Soviet Union by Emelyanov and several co-researchers [28], [29]. Since then the significant interest on variable structure system (VSS) and SMC has been generated in the control research community worldwide. One of the most intriguing aspects of sliding mode is the discontinuous nature of control action whose primary function of each feedback channel is to switch between the two distinctively different system structures such that the new type of system motion, called sliding mode, exists in the manifold. This peculiar system characteristics is claimed to result in superb system performance which includes insensitivity to parameter variations, and complete rejection of disturbance [29], [30]. However, it is not easy to design a sliding mode control law for singularly perturbed systems due to the complication of different time-scale behavior and the discontinuous nature of switching action. Various attempts to apply sliding mode control strategy to singularly perturbed system have been reported by several researches [31]–[38]. A singularly perturbed system is first decomposed into slow and fast subsystems and then a composite control law derived from slow and fast SMC is proposed in [31] in order to stabilize a class of linear time-invariant systems. A similar kind of approach was studied by Li *et al.* [32], in which the upper bound problem of singular perturbation parameter in such a control system is also determined. In [35] global stability of closed loop system reduced in their fast and slow subsystem using singular perturbation with sufficiently small perturbation parameter is addressed. However, in these papers the external disturbances were not considered. Yue and Xu [33] proposed design of SMC for singular perturbation system with parameter uncertainties and external disturbances, but it is difficult to compute some parameters for the control law design. SMC designed for slow subsystem only of singular perturbation system to control the full order model is investigated in [34]. The fast subsystem is considered as unmodeled high frequency dynamics. Recently, Nguyen *et al.* in [35] proposed a method in which a state feedback control law is firstly established to stabilize either slow or fast dynamics and secondly a SMC law is designed for remaining dynamics of the system to ensure stability and disturbance rejection. Further, in [36] the problem of SMC for singularly perturbed systems in the presence of matched bounded external disturbance is discussed. In this, sliding surfaces are designed on the Lyapunov equations for slow and fast subsystems. In [38], Bandyopadhyay *et al.* have proposed sliding mode control design via reduced model approach, in which higher order system is decoupled by similarity transformation into slow and fast subsystems. It is also shown that the SMC designed for slow subsystem alone can result in sliding mode motion for the high order system.

Rest of the paper is organized in the following sequence. In Section II modeling of AHWR is explained along with state-space representation. Control problem of AHWR is discussed in Section III. In Section IV brief overview of sliding mode control is given. Section V presents proposed control law. The application of control strategy to AHWR and transient simulations are illustrated in Section VI followed by the conclusion in Section VII.

II. Mathematical Model of AHWR

A very extensive derivation of AHWR mathematical model is given in [2] and [3] and the same has been used here for the study carried out in this paper. However, for brevity the model is discussed briefly in the following.

A. Core Neutronics Model

The AHWR core is considered to be divided in 17 relatively large nodes as shown in Fig. 1. Based on finite difference approximation of the two group neutron diffusion equations and the associated equations for an effective single group of delayed neutron precursor's concentration, xenon and iodine concentration, the nodal core model has been developed in [2]. The following equations constitute the core neutronics model:

$$\frac{dP_i}{dt} = (\rho_i - \alpha_{ii} - \beta)\frac{P_i}{\ell} + \sum_{j=1}^{17} \alpha_{ji}\frac{P_j}{\ell} + \lambda C_i \tag{1}$$

$$\frac{dC_i}{dt} = \frac{\beta}{\ell}P_i - \lambda C_i \tag{2}$$

$$\frac{dI_i}{dt} = \gamma_I \Sigma_{fi} P_i - \lambda_I I_i \tag{3}$$

$$\frac{dX_i}{dt} = \gamma_X \Sigma_{fi} P_i + \lambda_I I_i - (\lambda_X + \bar{\sigma}_{Xi} P_i) X_i \tag{4}$$

$$\frac{dH_k}{dt} = \kappa v_k \quad k = 2,4,6,8; \quad i = 1,2,\ldots,17 \tag{5}$$

where α_{ji} and α_{ii} denote the coupling coefficients between jth and ith nodes and self coupling coefficients of ith node respectively. $\bar{\sigma}_{Xi} = \sigma_{Xi}/E_{eff}\Sigma_{fi}V_i$ and v_k is control signal applied to the RR drive and κ is a constant having value 0.56. The neutronic parameters, nodal volumes and cross-sections, nodal powers and coolant flow rates under full power operation and coupling coefficients are given in [4].

B. Thermal Hydraulics Model

The thermal hydraulics model derived from [39] is given by

$$e_{vx_i}\frac{dx_i}{dt} = P_i - q_{d_i}(h_w - h_d) - q_{d_i} x_i h_c \tag{6}$$

$$e_{xh}\frac{dh_d}{dt} = q_f(\hat{k}_2 h_f - \hat{k}_1) - q_d(\hat{k}_2 h_d - \hat{k}_1) \tag{7}$$

where, $\hat{k}_2 = (h_s)/(h_c)$ and $\hat{k}_1 = h_w \hat{k}_2$. In [4], values of e_{vx_i} and e_{xh} are given and the same are used here.

C. Reactivity Feedbacks

The reactivity term ρ_i in (1) is expressed as $\rho_i = \rho_{i_u} + \rho_{i_X} + \rho_{i_\alpha}$, where ρ_{i_u} is the reactivity introduced by the control rods, ρ_{i_X} is the reactivity feedback due to xenon and ρ_{i_α} is the reactivity feedback due to coolant void fraction. The reactivity contributed by the movement of the RRs around their equilibrium positions is expressed as

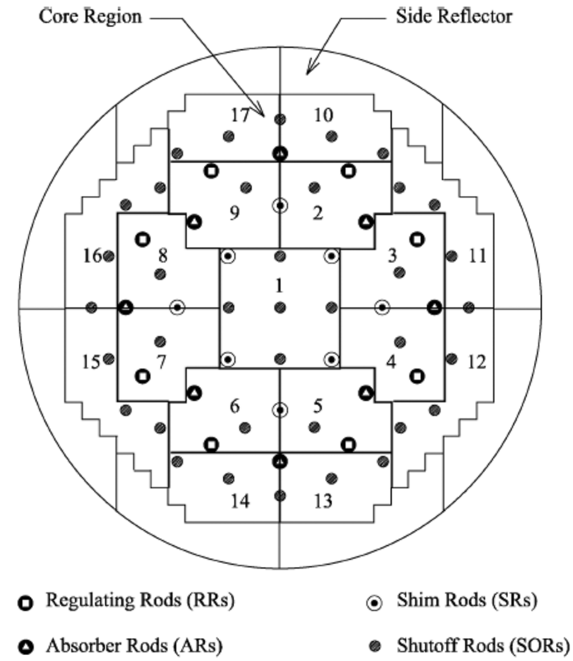

Fig. 1. 17 nodes AHWR scheme.

$$\rho_{i_u} = \begin{cases} (-10.234H_i + 676.203) \times 10^{-6}, & \text{if } i = 2,4,6,8. \\ 0 & \text{elsewhere.} \end{cases}$$

The xenon reactivity feedback in node i can be expressed as

$$\rho_{i_X} = -\frac{\bar{\sigma}_{Xi} X_i}{\Sigma_{ai}}.$$

The reactivity contribution by the coolant void fraction is

$$\rho_{i_\alpha} = -5 \times 10^{-3}\left(9.2832x_i^5 - 27.7192x_i^4 + 31.7643x_i^3 - 17.7389x_i^2 + 5.2308x_i + 0.0792\right).$$

D. Linearization and State-Space Representation

The set of equations given by (1)–(7) can be linearized around steady state operating conditions $(H_{j_0}, X_{i_0}, I_{i_0}, h_{d_0}, C_{i_0}, x_{i_0}, P_{i_0})$ and the linear equations so obtained can be represented in standard state space form. For this, define the state vector as

$$z = \left[z_H^T \; z_X^T \; z_I^T \; \delta h_d \; z_C^T \; z_x^T \; z_P^T\right]^T \tag{8}$$

where $z_H = [\delta H_2 \; \delta H_4 \; \delta H_6 \; \delta H_8]^T$ and the rest $z_\xi = [(\delta\xi_1/\xi_{1_0}) \; \cdots \; (\delta\xi_{17}/\xi_{17_0})]^T$, $\xi = X, I, C, x, P$, in which δ denotes the deviation from respective steady state value of the variable. Likewise define the input vector as $u = [\delta v_2 \; \delta v_4 \; \delta v_6 \; \delta v_8]^T$ and output vector as $y = [y_g \; y_1 \; \ldots \; y_{17}]^T$ where $y_g = \sum_{i=1}^{17}(\delta P_i)/(\sum_{j=1}^{17} P_{j_0})$

and $y_i = (\delta P_i)/(P_{i_0})$ corresponds to normalized total reactor power and nodal powers respectively. Then the system given by (1)–(7) can be expressed in standard linear state space form as

$$\dot{z} = Az + Bu + B_{fw}\delta q_{fw} \quad (9)$$
$$y = Mz \quad (10)$$

where q_{fw} is feed water flow rate. Matrices A, B, B_{fw} and M are given in [4]. Eigenvalues of A fall in two distinct clusters. First cluster has 73 eigenvalues ranging from -1.8395×10^{-1} to 3.9654×10^{-6} and the second one is of 17 eigenvalues ranging from -2.7626×10^{2} to -7.2516. Six eigenvalues of A have their real parts positive while four eigenvalues are at the origin (grouped in first cluster), which indicates instability. Hence, it is necessary to design an effective controller to maintain the total power of the reactor while the xenon induced oscillations are being controlled.

III. Control Problem of AHWR

The control of large nuclear reactors has been a challenging problem. Many authors have addressed the control problem of other reactors in the perspective of modern control methodologies, but very few literatures are available on AHWR. Control of AHWR refers to maintaining the total power constant and nodal power distribution as given in [4]. This is referred as spatial control, by means of which xenon induced oscillations are suppressed from growing. The spatial control problem of AHWR has been attempted by Shimjith *et al.* in [4], [15], [27]. In [4] control strategy based on feedback of total power as well as spatial power distribution signal is suggested. The design of controller utilizing merely the feedback of outputs does not ensure better transient performance and robustness characteristics. In extension to this, the three-time-scale property of AHWR has been directly exploited in [15] to design a spatial control based on state feedback approach. In this, quasi-steady-state method is used to decouple the system in 'slow', 'fast 1' and 'fast 2' subsystems which involves approximation. Further, the practical implementation of such a state feedback controller demands a state observer of large order. To overcome this, Fast Output Sampling (FOS) controller is proposed in [27]. This method is based on multirate output feedback, by which the states of the system can be computed exactly. The method has its own advantages, but these methods do not assure robustness.

In this paper, spatial controller is designed for numerically ill-conditioned system of AHWR based on sliding mode control technique. However, it is not easy and straightforward to synthesize a sliding mode control law for numerically ill-conditioned system due to the complication of different time-scale behavior and the discontinuous nature of switching actions. Hence, the higher order system is first decoupled into slow and fast subsystem and sliding mode control is designed using only slow subsystem. It is also shown that if a SMC is designed using slow subsystem and if applied to full order system by simple transformation, it results in the sliding mode motion of full order system. The controller so designed has been used for simulation of non-linear model and the results have been found to be satisfactory and better than three-time-scale based and fast output sampling feedback based controllers. Moreover, the controller design and simulation results are directly relevant to India's ongoing nuclear power programme.

IV. Brief Overview of Sliding Mode Control

The design of sliding mode control law involves, designing a switching surface $s = 0$ to represent a desired system dynamics, which is of lower order than the given plant and then designing a suitable control, such that any state of the system outside the switching surface is driven to reach the surface in finite time. Consider the linear time-invariant controllable system of order n, as

$$\dot{z} = Az + Bu \quad (11)$$
$$y = Mz \quad (12)$$

where $z \in \Re^n$ is the system state, $u \in \Re^m$ is the control with $1 \leq m < n$ and $y \in \Re^p$ is the system output. The matrices A, B and M are constant matrices of appropriate dimensions. Controllability of (A, B) implies the existence of transformation matrix $T_r \in \Re^{n \times n}$ such that

$$T_r B = \begin{bmatrix} 0 \\ B_0 \end{bmatrix} \quad (13)$$

where $B_0 \in \Re^{m \times m}$ and is nonsingular. Under this transformation, system (11) is transformed into regular form given as

$$\begin{bmatrix} \dot{\bar{z}}_1 \\ \dot{\bar{z}}_2 \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} \bar{z}_1 \\ \bar{z}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B_0 \end{bmatrix} u \quad (14)$$

where $\bar{z}_1$ and $\bar{z}_2$ are of orders $n - m$ and m respectively and

$$\bar{z} = \begin{bmatrix} \bar{z}_1 \\ \bar{z}_2 \end{bmatrix} = T_r z. \quad (15)$$

A. Sliding Surface Design

Consider the sliding surface [28], [29] of form $\bar{c}^T \bar{z} = 0$ with sliding function parameter of form

$$\bar{c}^T = [K \; E_m]. \quad (16)$$

The normal form of system (14), when restricted on sliding surface $\bar{c}^T \bar{z} = 0$, would obey the relationship

$$\bar{z}_2 = -K\bar{z}_1. \quad (17)$$

Thus, the dynamics of $\bar{z}_1$ can be represented as

$$\begin{aligned} \dot{\bar{z}}_1 &= \bar{A}_{11}\bar{z}_1 - \bar{A}_{12}K\bar{z}_1, \\ &= (\bar{A}_{11} - \bar{A}_{12}K)\bar{z}_1. \end{aligned} \quad (18)$$

From (14), variable $\bar{z}_2$ should be regarded as a control input to the dynamic equation of $\bar{z}_1$. The controllability of (A, B) implies controllability of $(\bar{A}_{11}, \bar{A}_{12})$. As a result, from (18) if K is selected such that eigenvalues of $(\bar{A}_{11} - \bar{A}_{12}K)$ are assigned in desired locations, then $\bar{z}_1$ is stabilized when confined to sliding surface. Consequently, due to algebraic relationship (17), $\bar{z}_2$ is also stable confined to sliding surface. Thus, stability requirement of the sliding surface is satisfied and it can be expressed in terms of original state coordinates as

$$s = \bar{c}^T \bar{z} = \bar{c}^T T_r z = c^T z. \tag{19}$$

B. Controller Design

When sliding surface (19) is designed, it is necessary that for all initial conditions, the system states converge towards the switching surface. In other words, if $s < 0$ then $\dot{s} > 0$ and, if $s > 0$ then $\dot{s} < 0$. This may be combined to yield

$$\dot{s}s < 0. \tag{20}$$

This is the existence condition for sliding mode motion. And, when sliding motion takes place after finite time t_s, $s = c^T z = 0$ and $\dot{s} = c^T \dot{z} = 0$ for all $t \geq t_s$. Substituting for $\dot{z}$ from (11) gives equivalent sliding mode control [29] as

$$u_{eq} = -(c^T \ B)^{-1} c^T A z. \tag{21}$$

The control law (21) satisfies only the sliding condition. One must add a regulating control force Δu in order to satisfy the reaching condition. Thus, define

$$u = u_{eq} + \Delta u \tag{22}$$

where Δu can be designed by sigmoid function [32] to eliminate the chattering and is implemented as

$$\Delta u = -(c^T B)^{-1} \mu sig(c^T \cdot z) * sgn(c^T \cdot z) \tag{23}$$

where "$*$" represents the array multiplication, μ is a positive scalar and

$$sig(c^T \cdot z) = \frac{1 - e^{-|c^T \cdot z|}}{1 + e^{-|c^T \cdot z|}} \geq 0,$$
$$sgn(c^T \cdot z) = \begin{cases} 1 & \text{for } c^T \cdot z > 0 \\ -1 & \text{for } c^T \cdot z < 0. \end{cases}$$

The system (11) is asymptotically stable in sliding mode on sliding surface (19) [32].

V. Proposed Control Law

The quasi-steady-state method [8] is a good method for decoupling the full order system for sufficiently small perturbation parameter. However, for large systems like nuclear reactor the perturbation parameter is not zero. As a result, when using quasi-steady state method the eigenvalues of the slow and fast subsystem are no longer in the same position as the eigenvalues of the full order system. This can be avoided by employing two-stage decomposition method [6].

A. Two-Stage Decomposition

Consider the singularly perturbed form of the system (11), rewritten as

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ \frac{A_{21}}{\epsilon} & \frac{A_{22}}{\epsilon} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ \frac{B_2}{\epsilon} \end{bmatrix} u \tag{24}$$

where $z_1 \in \Re^{n_1}$, $z_2 \in \Re^{n_2}$ denote states such that $n_1 + n_2 = n$, the matrices A_{ij} and B_i are of appropriate dimensionality and parameter $\epsilon > 0$ is a scalar representing the speed ratio of the slow versus fast phenomenon. The model represented by (24) is a standard singularly perturbation model extensively studied in the control literature. As the parameter ϵ tends to zero, the solution behaves non-uniformly, producing so called singularly perturbed stiff problem. Suppose $\varphi(A)$ be the eigenvalues of matrix A arranged in increasing order of absolute values as

$$\varphi(A) = \{\varphi_1, \varphi_2, \ldots, \varphi_{n_1}, \varphi_{n_1+1}, \ldots, \varphi_n\}$$

where

$$0 \leq |\varphi_1| < |\varphi_2| < \cdots < |\varphi_{n_1}| \ll |\varphi_{n_1+1}| < \cdots < |\varphi_n|.$$

Thus the system (24) has n_1 dominant (slow) modes and n_2 non-dominant (fast) modes. The basic idea of using the two-stage decomposition approach in generating lower order models is to decouple the dominant modes from non-dominant modes. This is performed through use of two-stage linear transformations [11], [12]. The first stage is to apply the change of variables

$$\begin{bmatrix} z_1 \\ z_f \end{bmatrix} = \begin{bmatrix} E_{n_1} & 0 \\ L & E_{n_2} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = T_1 \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{25}$$

to system (24). Here E_{n_1} and E_{n_2} are respectively n_1 and n_2 identity matrices and $(n_2 \times n_1)$ matrix L satisfies the equation

$$\epsilon L A_{11} + A_{21} - \epsilon L A_{12} L - A_{22} L = 0. \tag{26}$$

Then system (24) transforms into

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_f \end{bmatrix} = \begin{bmatrix} A_s & A_{12} \\ 0 & \frac{A_f}{\epsilon} \end{bmatrix} \begin{bmatrix} z_1 \\ z_f \end{bmatrix} + \begin{bmatrix} B_1 \\ \frac{B_f}{\epsilon} \end{bmatrix} u \tag{27}$$

where

$$A_s = A_{11} - A_{12} L, A_f = A_{22} + \epsilon L A_{12}, B_f = B_2 + \epsilon L B_1.$$

If A_{22} is invertible, unique solution of L in (26) can be determined by iterative procedure. Now the second linear transformation is applied as

$$\begin{bmatrix} z_s \\ z_f \end{bmatrix} = \begin{bmatrix} E_{n_1} & -\epsilon N \\ 0 & E_{n_2} \end{bmatrix} \begin{bmatrix} z_1 \\ z_f \end{bmatrix} = T_2 \begin{bmatrix} z_1 \\ z_f \end{bmatrix} \quad (28)$$

to system (27) and choose $(n_1 \times n_2)$ matrix N such that

$$A_{12} - NA_{22} - \epsilon NLA_{12} + \epsilon(A_{11} - A_{12}L)N = 0. \quad (29)$$

Then (27) transforms into

$$\begin{bmatrix} \dot{z_s} \\ \dot{z_f} \end{bmatrix} = \begin{bmatrix} A_s & 0 \\ 0 & \frac{A_f}{\epsilon} \end{bmatrix} \begin{bmatrix} z_s \\ z_f \end{bmatrix} + \begin{bmatrix} B_s \\ \frac{B_f}{\epsilon} \end{bmatrix} u \quad (30)$$

where

$$B_s = B_1 - NB_f.$$

Thus, the application of two-stage linear transformation results in decoupling of system (24) into separate slow and fast subsystems in (30) from where the slow and fast variables z_s and z_f can be solved independently. Also, the magnitude of the largest eigenvalue of A_s is much smaller than the magnitude of the smallest eigenvalue of $(A_f)/(\epsilon)$, i.e.,

$$\max|\varphi(A_s)| \ll \max\left|\varphi\left(\frac{A_f}{\epsilon}\right)\right|.$$

The transformations (25) and (28) relate the slow and fast variables z_s and z_f with the original variables z_1 and z_2 as

$$\begin{bmatrix} z_s \\ z_f \end{bmatrix} = T_2 T_1 \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$
$$\text{or} \quad \hat{z} = Tz \quad (31)$$

where $\hat{z} = [z_s^T \ z_f^T]^T$, $z = [z_1^T \ z_2^T]^T$ and $T = T_2 T_1$.

B. Sliding Mode Control Law Design Using Slow Subsystem

The system formulation (30) is related to its original form (24) via linear transformation (31). Therefore, controllability of (11) implies the controllability of subsystems i.e., pairs (A_s, B_s) and (A_f, B_f) are controllable. In addition, it is assumed that fast subsystem is asymptotically stable, i.e., $\varphi(A_f/\epsilon) < 0$. Since, in case of majority of physical systems, fast subsystem is stable, therefore, sliding mode control law is designed using slow subsystem alone. From (30), slow subsystem can be written as

$$\dot{z_s} = A_s z_s + B_s u. \quad (32)$$

The relationship between the slow subsystem states (32) and states of system (30) is given as

$$z_s = [E_{n_1} \ : \ 0] \begin{bmatrix} z_s \\ z_f \end{bmatrix} = T_z \hat{z} \quad (33)$$

where $T_z \in \Re^{n_1 \times n}$. Let $s_s = c^T z_s$ be a stable sliding surface for slow subsystem (32). Hence, the motion around s_s can be obtained by setting $\dot{s}_s = 0$. Therefore, equivalent sliding mode control is

$$u_{eq} = -(c^T B_s)^{-1} c^T A_s z_s. \quad (34)$$

Thus, motion along s_s is given by

$$\begin{aligned} \dot{z}_s &= A_s z_s - B_s (c^T B_s)^{-1} c^T A_s z_s, \\ &= (A_s - B_s (c^T B_s)^{-1} c^T A_s) z_s. \end{aligned} \quad (35)$$

As the system (35) is stable by design, eigenvalues of $(A_s - B_s(c^T B_s)^{-1} c^T A_s)$ will be stable.

Lemma 1: If motion around $s_s = c^T z_s$ for system (32) is stable then the motion around

$$s = c^T T_z T z \quad (36)$$

for system (24) is also stable.

Proof: Since, $s_s = c^T z_s$ is a stable sliding surface for system (32), from (33) sliding surface for system (30) can be written as

$$s_{\hat{z}} = c^T T_z \hat{z}. \quad (37)$$

Sliding motion around $s_{\hat{z}}$ for system (30) can be obtained by setting $\dot{s}_{\hat{z}} = 0$. As a result equivalent control becomes

$$u_{eq} = -(c^T B_s)^{-1} c^T [A_s \ 0] \hat{z}. \quad (38)$$

Thus, motion around switching surface $s_{\hat{z}}$ is

$$\dot{\hat{z}} = \begin{bmatrix} A_s - B_s(c^T B_s)^{-1} c^T A_s & 0 \\ \frac{-B_f (c^T B_s)^{-1} c^T A_s}{\epsilon} & \frac{A_f}{\epsilon} \end{bmatrix} \hat{z} \quad (39)$$

which is obtained from (30) by replacing u with u_{eq}. As $(A_s - B_s(c^T B_s)^{-1} c^T A_s)$ is stable by design and $(A_f)/(\epsilon)$ is assumed to be stable, the sliding motion of (30) is stable. System formulation (30) is related to its original form (24) via linear transformation (31). Therefore, $s = c^T T_z T z$ is also stable sliding surface for (24). ■

Now setting $\dot{s} = 0$, equivalent control for system (24) can be obtained from (38) as

$$u_{eq} = -(c^T B_s)^{-1} c^T [A_s(E_{n_1} - \epsilon NL) - \epsilon A_s N] z. \quad (40)$$

The control (40) satisfies only sliding condition for system (24), as proved in Lemma 1, however, reaching condition is satisfied by (23), where total control law is given by (22).

Lemma 2: Full order system (24) is asymptotically stable in sliding mode on sliding surface (36).

Proof: From (31), (33) and (36) sliding surface for full order system (24) can be written as

$$s = c^T[(E_{n_1} - \epsilon NL) - \epsilon N]z.$$

Now, choosing Lyapunov function as

$$V(s) = \frac{1}{2}s^T s,$$
$$\dot{V}(s) = -\mu \cdot s * sig(s) * sgn(s) < 0$$

for all $z \neq 0$. The reaching condition is thus satisfied. ■

VI. Application to AHWR

The linear model of the AHWR given by (9)–(10) presented in Section II-D, is found to be controllable and observable [4]. Small and medium size nuclear reactors are generally controlled based on feedback of total power, however, large reactors, like AHWR, require feedback of spatial power distribution along with the total power feedback for effective spatial control. Hence, conventional controller for total reactor power is designed first.

A. Total Power Feedback

Consider the input u in (9) of the form

$$u = u_g + u_s \tag{41}$$

where, u_g and u_s give respectively the global power and spatial power components. Now, consider the total power feedback as,

$$u = u_g = -K_G y \tag{42}$$

where K_G is (4×18) matrix given by $[\hat{K}_g \; \mathbf{0} \; \cdots \; \mathbf{0}]$, in which $\mathbf{0}$ represents vectors of (4×1) dimension and $\hat{K}_g = [K_g \; K_g \; K_g \; K_g]^T$ such that the feedback gain corresponding to total power is K_g for all RRs and is zero corresponding to nodal powers. Using (42) the state equation (9) becomes

$$\dot{z} = (A - BK_G M)z + B_{fw}u_{fw} = \hat{A}z + B_{fw}u_{fw} \tag{43}$$

where $\hat{A} = (A - BK_G M)$. Submatrices of $\hat{A}$, B and M are given in [15]. The stability characteristic of the system (43) is investigated by varying the value of K_g and for $K_g = 12.5$, the gross behavior of the system seems stable though the system can show spatial instability [15]. This is revealed by simulating transient involving a spatial power disturbance using non-linear model of the reactor given by (1)–(7) using MATLAB/SIMULINK. It was assumed that the reactor was operating initially at full power, with control signal generated by (42). The RR2 which was initially at its equilibrium position was driven out by about 1% by giving proper control signal. Immediately after that, RR2 was driven back to its original position and thereafter left under the influence of controller. The response of the model to this disturbance was investigated in terms of variations in total power and tilts in the first and second azimuthal modes defined as:

$$\text{First azimuthal tilt} = \frac{P_L - P_R}{\sum_{i=1}^{17} \frac{P_i}{2}} \times 100$$
$$\text{where } P_L = \frac{1}{2}P_1 + \sum_{i=6}^{9} P_i + \sum_{i=14}^{17} P_i,$$
$$P_R = \frac{1}{2}P_1 + \sum_{i=2}^{5} P_i + \sum_{i=10}^{13} P_i;$$
$$\text{Second azimuthal tilt} = \frac{P_{q1} - P_{q2}}{\sum_{i=1}^{17} \frac{P_i}{2}} \times 100$$
$$\text{where } P_{q1} = \frac{1}{2}P_1 + P_2 + P_3 + P_6 + P_7 + P_{10} + P_{11} + P_{14} + P_{15},$$
$$\text{and } P_{q2} = \frac{1}{2}P_1 + P_4 + P_5 + P_8 + P_9 + P_{13} + P_{12} + P_{16} + P_{17}.$$

It was observed in the simulation that, in spite of the global power being regulated at full power as shown in Fig. 2(a), the power distribution in the core undergoes oscillations. Within 38 h, the first and second azimuthal modes of oscillations grow to the amplitudes of the order of 1.4% and 0.75% respectively as shown in Fig. 2(b). Period of the oscillations are observed to be 20 h and 12 h respectively for first azimuthal and second azimuthal. These spatial oscillations and subsequent local overpowers pose a potential threat to the fuel integrity of any nuclear reactor, and hence require control. Therefore, it is necessary to devise a suitable spatial power controller for AHWR.

B. Spatial Control for AHWR

After designing the global power control component, we turn our attention to the spatial control component of input in (41). Along with the total power and spatial power feedbacks, state equation (9) becomes

$$\dot{z} = \hat{A}z + Bu_s + B_{fw}u_{fw} \tag{44}$$

where, $\hat{A}$ given by (43) has eigenvalues falling in two different clusters. First cluster of 73 eigenvalues ranging from -1.8402×10^{-1} to -2.6799×10^{-5} with three eigenvalues at the origin and second cluster of 17 eigenvalues range from -7.2484 to -2.7626×10^2. Hence, it is possible to transform model (44) into singularly perturbed form (24).

1) Singularly Perturbed Form of AHWR Model: In case of AHWR, after linearization of set of equations given by (1)–(7),

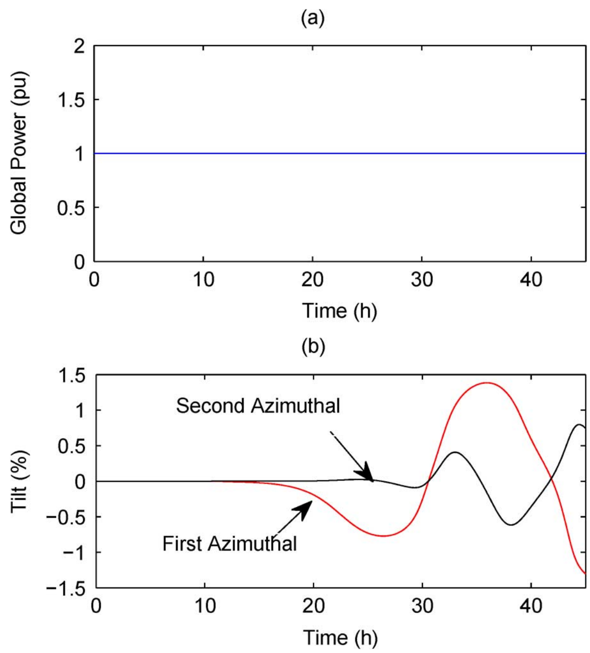

Fig. 2. Unstable modes of spatial instability. (a) Global Power, (b) First and Second Azimuthal Tilts.

it is indeed observed that coefficients in the 17 equations for nodal powers, contain ℓ in their denominator. Its value is 3.6694×10^{-4} s. This parameter can be picked up as ϵ. Therefore, the state variables of system defined by (8) are grouped into slow and fast ones as

$$z_1 = \left[z_H^T \; z_X^T \; z_I^T \; \delta h_d \; z_C^T \; z_x^T\right]^T \tag{45}$$
$$z_2 = z_P. \tag{46}$$

Now, the AHWR model is transformed into standard singularly perturbed form where $n_1 = 73$ and $n_2 = 17$. Submatrices A_{11}, A_{12}, $(A_{21})/(\epsilon)$, $(A_{22})/(\epsilon)$, B_1 and $(B_2)/(\epsilon)$ are respectively of dimensions (73×73), (73×17), (17×73), (17×17), (73×4) and (17×4).

2) Control Law Design: Transformation matrices T_1 and T_2, fast subsystem matrix $(A_f)/(\epsilon)$ and slow subsystems matrix A_s can be obtained very easily by using the procedure explained in Section V-A. The eigenvalues of matrices A_f and A_s are given in Tables (I) and (II) respectively. It is seen that the eigenvalues of $(A_f)/(\epsilon)$ and A_s are in excellent agreement with the last 17 and remaining 73 eigenvalues of matrix $\hat{A}$ respectively. Hyperplane matrix c^T for slow subsystem is determined using the method explained in Section V-B. From Table I it can be observed that the eigenvalues of fast subsystem are asymptotically stable i.e., $\varphi(A_f/\epsilon) < 0$. Hence, sliding mode control law can be constructed using the slow subsystem as given below.

$$u = -(c^T B_s)^{-1} c^T A_s [(E_{n_1} - \epsilon NL) \; - \epsilon N] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - (c^T \, B_s)^{-1} \mu sig(s) * sgn(s) \tag{47}$$

TABLE I
EIGENVALUES OF FAST SUBSYSTEM

Sr. No.	Eigenvalues	Sr. No.	Eigenvalues
1	-7.2484	10	-1.6967 $\times 10^2$
2	-3.2844 $\times 10^1$	11	-1.7568 $\times 10^2$
3	-3.3372 $\times 10^1$	12	-1.9497 $\times 10^2$
4	-6.6599 $\times 10^1$	13	-2.111 $\times 10^2$
5	-6.8323 $\times 10^1$	14	-2.1904 $\times 10^2$
6	-9.3653 $\times 10^1$	15	-2.3591 $\times 10^2$
7	-9.4612 $\times 10^1$	16	-2.7163 $\times 10^2$
8	-1.0868 $\times 10^2$	17	-2.7626 $\times 10^2$
9	-1.1705 $\times 10^2$		

TABLE II
EIGENVALUES OF SLOW SUBSYSTEM

Sr. No.	Eigenvalues	Sr. No.	Eigenvalues
1–3	0	43	-5.773 $\times 10^{-2}$
4	-2.6799 $\times 10^{-5}$	44	-5.7893 $\times 10^{-2}$
5	-3.7781 $\times 10^{-5}$	45	-5.9707 $\times 10^{-2}$
6	-3.7993 $\times 10^{-5}$	46	-5.9723 $\times 10^{-2}$
7	-4.0124 $\times 10^{-5}$	47	-6.0344 $\times 10^{-2}$
8	-4.152 $\times 10^{-5}$	48	-6.0642 $\times 10^{-2}$
9	-4.2044 $\times 10^{-5}$	49	-6.1848 $\times 10^{-2}$
10	-4.4204 $\times 10^{-5}$	50	-6.1942 $\times 10^{-2}$
11	-4.7371 $\times 10^{-5}$	51	-6.22 $\times 10^{-2}$
12	-4.8866 $\times 10^{-5}$	52	-6.238 $\times 10^{-2}$
13–14	(-7.7407 $\pm$ i 2.9929) $\times 10^{-5}$	53	-6.2458 $\times 10^{-2}$
15–16	(-7.3359 $\pm$ i 3.9319) $\times 10^{-5}$	54	-6.2608 $\times 10^{-2}$
17–18	(-6.5952 $\pm$ i 5.4785) $\times 10^{-5}$	55	-6.2865 $\times 10^{-2}$
19–20	(-6.4855 $\pm$ i 5.3109) $\times 10^{-5}$	56	-6.2893 $\times 10^{-1}$
21–22	(-3.9003 $\pm$ i 8.9009) $\times 10^{-5}$	57	-1.1715 $\times 10^{-1}$
23–24	(-3.7785 $\pm$ i 7.6475) $\times 10^{-5}$	58	-1.4712 $\times 10^{-1}$
25–26	(-3.5380 $\pm$ i 7.7343) $\times 10^{-5}$	59	-1.4713 $\times 10^{-1}$
27–28	(8.8268 $\pm$ i 2.1800) $\times 10^{-5}$	60	-1.4809 $\times 10^{-1}$
29–30	(8.0470 $\pm$ i 3.9864) $\times 10^{-5}$	61	-1.485 $\times 10^{-1}$
31	-1.4107 $\times 10^{-4}$	62	-1.5580 $\times 10^{-1}$
32	-1.4532 $\times 10^{-4}$	63	-1.5585 $\times 10^{-1}$
33	-1.5717 $\times 10^{-4}$	64	-1.5662 $\times 10^{-1}$
34	-1.6524 $\times 10^{-4}$	65	-1.5585 $\times 10^{-1}$
35	-1.6653 $\times 10^{-4}$	66	-1.5662 $\times 10^{-1}$
36	-1.7308 $\times 10^{-4}$	67	-1.5761 $\times 10^{-1}$
37	-1.8807 $\times 10^{-4}$	68	-1.6325 $\times 10^{-1}$
38	-1.8870 $\times 10^{-2}$	69	-1.6405 $\times 10^{-1}$
39	-5.2501 $\times 10^{-2}$	70	-1.8037 $\times 10^{-1}$
40	-1.5867 $\times 10^{-2}$	71	-1.8049 $\times 10^{-1}$
41	-5.0954 $\times 10^{-2}$	72	-1.8122 $\times 10^{-1}$
42	-5.1159 $\times 10^{-2}$	73	-1.8402 $\times 10^{-1}$

where s is the switching surface given by

$$s = [c^T(E_{n_1} - \epsilon NL)]z_1 - [\epsilon c^T N]z_2.$$

C. Transient Simulations

The reactor was assumed to be initially operating at full power equilibrium condition. Shortly, RR6, originally under auto control was driven out by almost 2% manually by giving proper control signal and simultaneously RR4 was driven in by 2%. Subsequently these regulating rods were left at their new positions under the effect of controller. The control signals to RR drives were generated as per (47). Non-linear model of the reactor given by (1)–(7) is simulated using

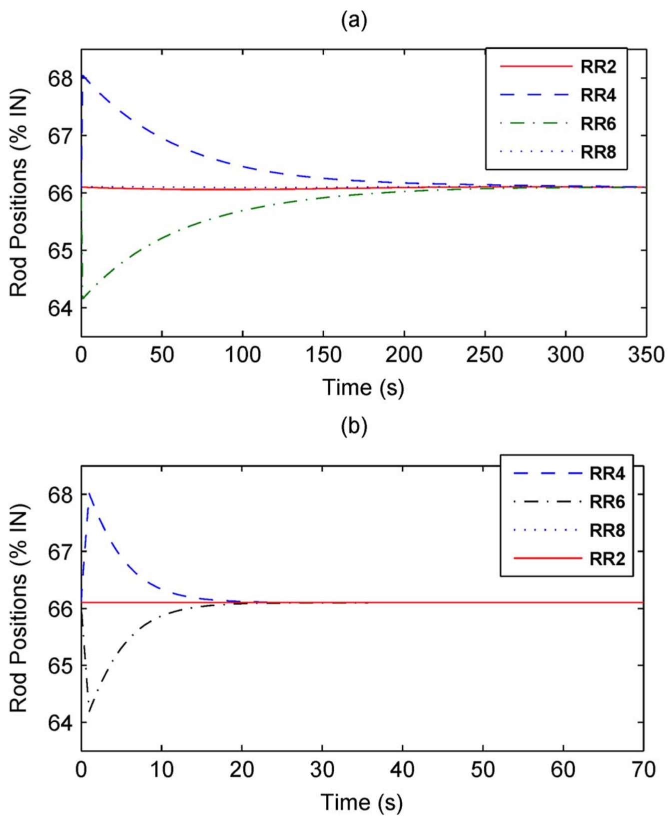

Fig. 3. Variations inRR positions.(a)Three-time-scale approach (Shimjith *et al.* [15]), (b) Proposed approach.

MATLAB/SIMULINK for step size of 10 ms. Generated results are shown in Fig. 3. From the simulations, it was noticed that RRs were driven back to their equilibrium positions by controller (Fig. 3(b)). This result is compared with the result obtained by applying three-time scale control law given in [15]. It is observed that, both the controllers are driving RRs back to their equilibrium positions but time required to do so is much less in the proposed controller.

Fig. 4 shows the closed loop system response during another spatial power transient initiated by a momentary disturbance in positions of RR2 and RR4. RR2 which was initially at its equilibrium position was driven out by 1% and RR4 was parallelly driven in by the same amount. Immediately after that these RRs were driven back to their original positions respectively and thereafter again transferred under the influence of the controller (42). As a consequence of this disturbance and as the control is based only on feedback of the total power, the tilts started picking up. After about 16 hours and 40 minutes the spatial control component was introduced in the control signal. Fig. 4(a) shows the variations in first and second azimuthal and Fig. 4(b) shows a zoomed version of the former, focusing the region near the introduction of spatial control. It was observed that tilts are controlled within 50 seconds and completely suppressed in 2 hours. Furthermore, when control signal is given to the rods as shown in Fig. 5(a), the variations in control rod positions is observed, as depicted in Fig. 5(b). This resulted in the perturbation in spatial power distribution, which were suppressed by the spatial controller within about 70 s, as shown in Fig. 6(a)–(d).

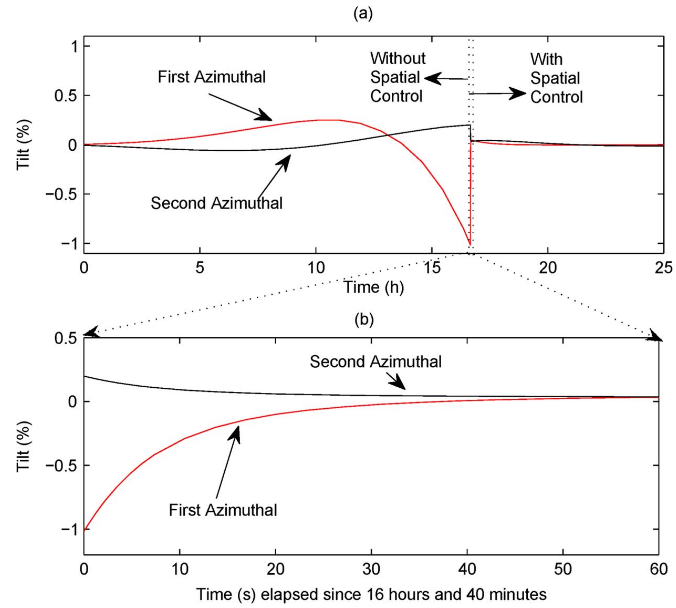

Fig. 4. Suppression of tilts after introduction of spatial control. (a) First and Second Azimuthal Tilts, (b) First and Second Azimuthal Tilts.

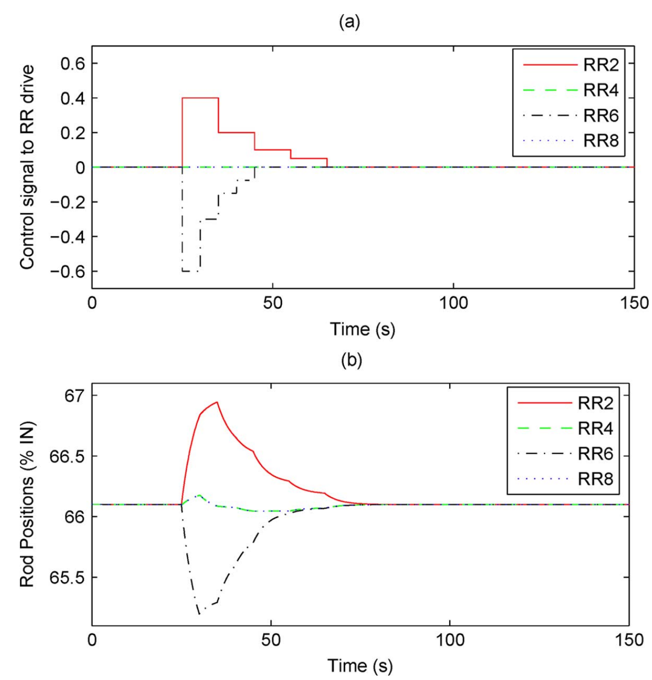

Fig. 5. Effect of spatial control (change in RR2 and RR6). (a) Control signal to RR drive, (b) Rod Positions.

VII. Conclusion

In this paper, sliding mode control law is proposed for spatial stabilization of AHWR. The original numerically ill-condition system is decomposed into two subsystems. Sliding mode control law is then designed for slow subsystem. Subsequently, SMC law for full order system is designed using linear transformation matrices. Performance of the proposed control law is judged via simulations carried out under various transient conditions with step time of 10 ms. It is observed that the controller is stabilizing the spatial oscillations and nodal power variations in very small time. The control strategy for AHWR, presented

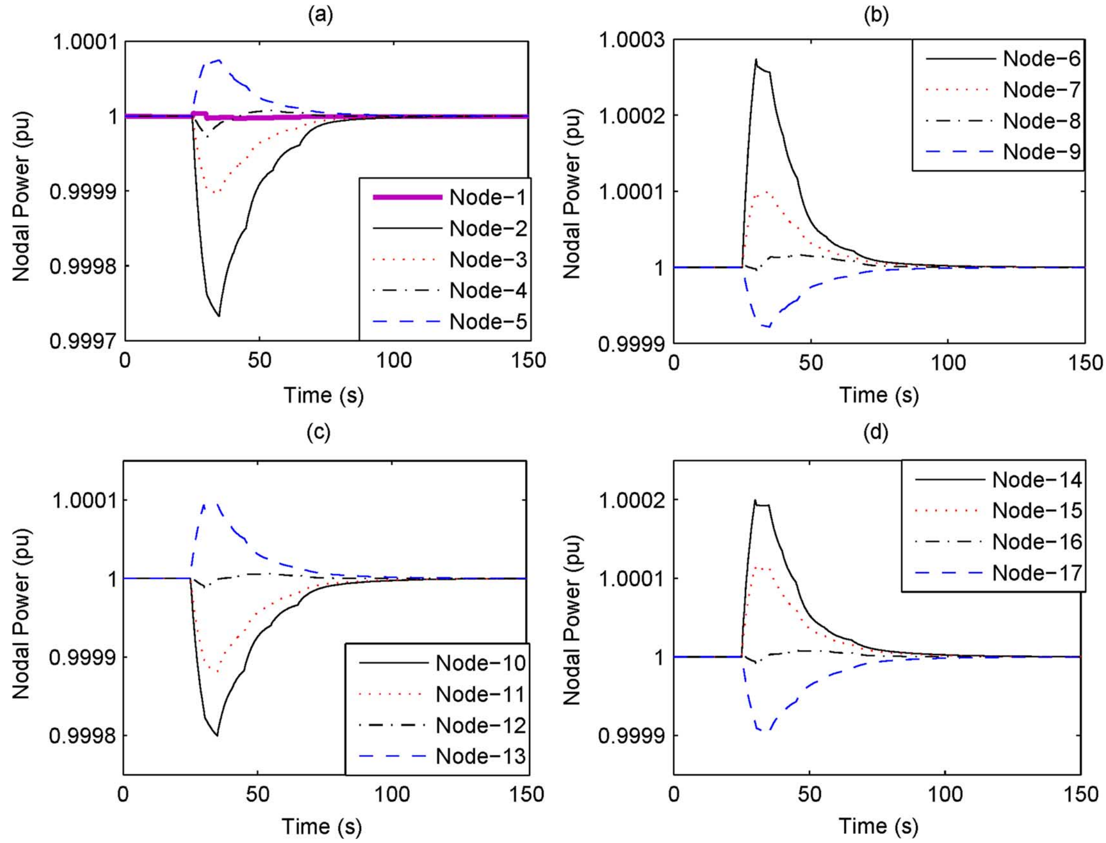

Fig. 6. Variations in nodal powers. (a) Node-1, 2, 3, 4 and 5 Powers, (b) Node-6, 7, 8 and 9 Powers, (c) Node-10, 11, 12 and 13 Powers, (d) Node-14, 15, 16 and 17 Powers.

here, utilizes the feedback of nodal powers, regulating rods' positions and xenon and iodine concentrations. For the latter two variables, it would be necessary to employ an observer or estimator.

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