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Nonlinear transient stability study of two lobe symmetric hole entry worn hybrid journal bearing operating with non-Newtonian lubricant

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ABSTRACT

The present paper, deals with a theoretical study concerning the effect of non-Newtonian lubricant on the stability of a two lobe symmetric hole entry worn hybrid journal bearing, compensated with constant flow valve restrictor. Trajectories of journal center motion have been obtained by solving the nonlinear equation of motion for the journal center with the Runge–Kutta method. The numerically simulated results indicate that the non-Newtonian behavior of the lubricant has a profound influence on the journal trajectories and stability of a worn hybrid journal bearing. A proper selection of parameters such as offset factor, wear depth parameter and the non-linearity factor may provide better bearing stability.

1. Introduction

The non circular hydrostatic/hybrid journal bearings have been extensively used in many high speed engineering applications such as turbo machinery, cryogenic and test equipments. The non circular journal bearing configurations have attracted a considerable attention of many researchers because of their improved bearing stability. In recent years, an enormous number of studies pertaining to non circular journal bearings, both analytical and experimental have been reported in literature [1–10]. During operation, these bearings normally run over a number of cycles and are subjected to frequent rotor rubs when the machine undergoes start-up and stop-down operations. During these transient periods, the bearing bush slowly wears out owing to abrasive action. As a result of this, the bearing geometry gets altered thereby changing its performance. Thus, it becomes imperative to consider the influence of wear in the analysis of this bearing. Many experimental and analytical studies pertaining to worn out journal bearings have been reported in the literature [8–18]. A study by Redcliff and Vohr [12] reported the first prevalence of wear in the circular hydrostatic journal bearing. Dufrane et al. [13] studied the wear mechanism in fluid film bearings owing to frequent start/stop operations. They reported that the wear damage will occur symmetrically every time on the bottom surface. The worn out bottom surface region of the bearing was first modeled and then experimentally validated by them. A number of studies have also been reported in the literature referring to Dufrane's model to investigate the influence of wear on the bearing performance [8–11,15–18].

In order to enhance the performance characteristics of lubricating oil, additives are often mixed with lubricant so as to fulfill the precise needs of various engineering applications. Several tribologists have claimed that the bearing performance could be improved by usage of high molecular weight polymers [19–23] as additives. The addition of these high molecular weight polymer additives causes industrial lubricants to behave as non-Newtonian fluids. The viscosities of these lubricants follow non-linear relationships between shear stress and shear strain rate [19–22].

The stability of rotating system is an integral facet within the analysis and design of the fluid film bearing systems. Rotor bearing system instability has gained significant importance and extensive investigations have been carried out to improve the stability of hydrodynamic journal bearing [24–38]. In 1925, based on experimental study, Newkirk and Taylor [24] reported a new kind of self-excited rotor dynamic instability in hydrodynamic journal bearings. They observed that the instability occurred, due to the action of oil film and christened this self-excited rotor dynamic instability as oil whirl. Pinkus [25], through experiments investigated the influence of load, viscosity, flexibility, unbalance and external excitation on oil whirl and reported that the stability of the rotor bearing system depends on low lubricant temperature. Choy et al. [27] and Braun et al. [28] studied the non-linear characteristics and their effects on the transient response of a plain journal bearing. Further, they compared the linear and non-linear bearing characteristics for various operating conditions of bearing. Malik et al. [29] carried out a theoretical study dealing with the transient response of a

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Nomenclature

a_b	bearing land width, mm
c	radial clearance, mm
e	journal eccentricity, mm
E	Young's modulus of elasticity, $N\,mm^{-2}$
F	fluid film reaction$(\partial h/\partial t \neq 0)$, N
F_x, F_z	components of fluid film reactions in X and Z direction $(\partial h/\partial t \neq 0)$, N
F_o	fluid film reaction $(\partial h/\partial t = 0)$, N
C_1	clearance due to circumscribed circle on the bearing, mm
C_2	clearance due to inscribed circle on the bearing, mm
g	acceleration due to gravity, $m\,s^{-2}$
h	nominal fluid-film thickness, mm
$h_{\min}$	minimum fluid film thickness, mm
L	bearing length, mm
R_J, R_L, R_b	radius of journal, lobe and bearing, mm
p	pressure, $N\,mm^{-2}$
Q	bearing flow, $mm^3\,s^{-1}$
S_{ij}	stiffness coefficients $(i,j = X,Z)$, $N\,mm^{-1}$
C_{ij}	damping coefficients $(i,j = X,Z)$, $N\,s\,mm^{-1}$
t	time, s
δ_W	wear depth, mm
n	number of rows of holes
K	nonlinearity factor for cubic shear law
τ	shear stress in lubricant film, $N\,mm^{-2}$
$\dot{\gamma}$	shear strain rate, s^{-1}
μ_a	apparent viscosity, $N\,s\,m^{-2}$
ω_I	$(g/c)^{1/2}$, rad s^{-1}
D	journal diameter, mm
W_o	external load, N
X, Y, Z	Cartesian coordinates
X_J, Z_J	coordinates of steady state equilibrium journal center from geometric center of bearing, mm

Greek symbols

$\lambda = L/D$,	aspect ratio
ϕ	attitude angle
μ	dynamic viscosity of lubricant, $N\,s\,m^{-2}$
μ_r	dynamic viscosity of lubricant at reference inlet temperature and ambient pressure, $N\,s\,m^{-2}$
ρ	density of the lubricant $kg\,mm^{-3}$
O_j, O_{Li}	journal centre, lobe centre
ω_J	journal rotational speed, rad s^{-1}
ω_{th}	threshold speed, rad s^{-1}
p_s	lubricant supply pressure $N\,mm^{-2}$

Non-dimensional parameters

$\overline{a}_b = a_b/L$,	land width ratio
$\beta^* = p^*/p_s$,	concentric design pressure ratio
$\overline{Q}_R =$	$\overline{Q}_C$,
$\overline{Q}_C =$	$Q(\mu_r/C^3 p_s)$
$\overline{C}_{ij} =$	$C_{ij}(c^3/\mu R_J^4)$
$(\overline{F}, \overline{F}_o) =$	$(F, F_o)/p_s R_J^2$
$(\overline{h}) =$	$(h)/c$
$\overline{\delta}_W =$	δ_W/c
$\overline{p}, \overline{p}_c, \overline{p}_{\max} =$	$(p, p_c, p_{\max})/p_s$
$\overline{Q} =$	$Q(\mu/c^3 p_s)$
$\overline{S}_{ij} =$	$S_{ij}(c/p_s R_J^2)$
$\overline{W}_o =$	$W_o/p_s R_J^2$;
$(\overline{X}_J, \overline{Z}_J) =$	$(X_J, Z_J)/c$
$\overline{t} =$	$t(c^2 p_s/\mu R_J^2)$
$\overline{X}^i{}_L, \overline{Z}^i{}_L =$	$(X^i{}_L, Z^i{}_L)/c$
$\alpha, \beta = (X, Y)/R_J$,	circumferential and axial coordinates;
$\varepsilon =$	e/c, eccentricity ratio
$\alpha_b, \alpha_e =$	start and end of the worn region
$\delta =$	C_1/C_2, offset factor
$\overline{\delta}_W =$	δ_W/c
$\overline{\mu} =$	μ/μ_r
$\overline{\tau} =$	$(\tau/(cp_s/R_J))$
$\overline{\dot{\gamma}} =$	$(\dot{\gamma}/(cp_s/\mu_r R_J))$
$\overline{K} =$	$(cp_s/R_J)^2 K$
$\overline{\omega}_{th} =$	ω_{th}/ω_I
$\Omega =$	$\omega_J(\mu R_J^2/c^2 p_s)$, speed parameter

Matrices

$N_i, N_j =$	shape functions
$[\overline{F}]$	assembled fluidity matrix
$\{\overline{p}\}$	nodal pressure vector
$\{\overline{Q}\}$	nodal flow vector
$\{\overline{R}_H\}$	column vectors due to hydrodynamic terms
$\{\overline{R}_{Xj}\}, \{\overline{R}_{Zj}\}$	global right hand side vector due to journal center velocities.

Subscripts and superscripts

b :	bearing
J :	journal
R :	restrictor
s :	supply
l:	lobe
i:	lobe number
min:	minimum
max:	maximum
x, y, z :	components in X, Y, and Z directions
.:	first derivative w.r.t. time
r:	reference value
*:	concentric operation
..:	second derivative w.r.t. time
–:	non dimensional parameter

journal in short bearings during acceleration and deceleration periods. Chandrawat and Sinhasan [30] and Jain et al. [31] studied the effect of bearing shell flexibility on transient response of a hydrodynamic journal bearing system. They reported that the motion trajectory obtained by nonlinear analysis provides a much higher stability margin in terms of critical mass, than those obtained through linearized analysis. Raghunandana et al. [32] and Sinhasan and Goyal [33], carried out the study, for rigid hydrodynamic circular and two-lobe journal bearings operating with non-Newtonian lubricant respectively, they showed that the non-linear analysis predicts a lower critical journal mass parameter than that predicted by linear analysis. Both studies [32,33] indicate a considerable reduction in the stability of journal bearing system operating with non-Newtonian lubricant as compared to the bearing lubricated with Newtonian lubricant. The influence of 3D surface roughness of the transient non-Newtonian response of a dynamically loaded journal bearing under

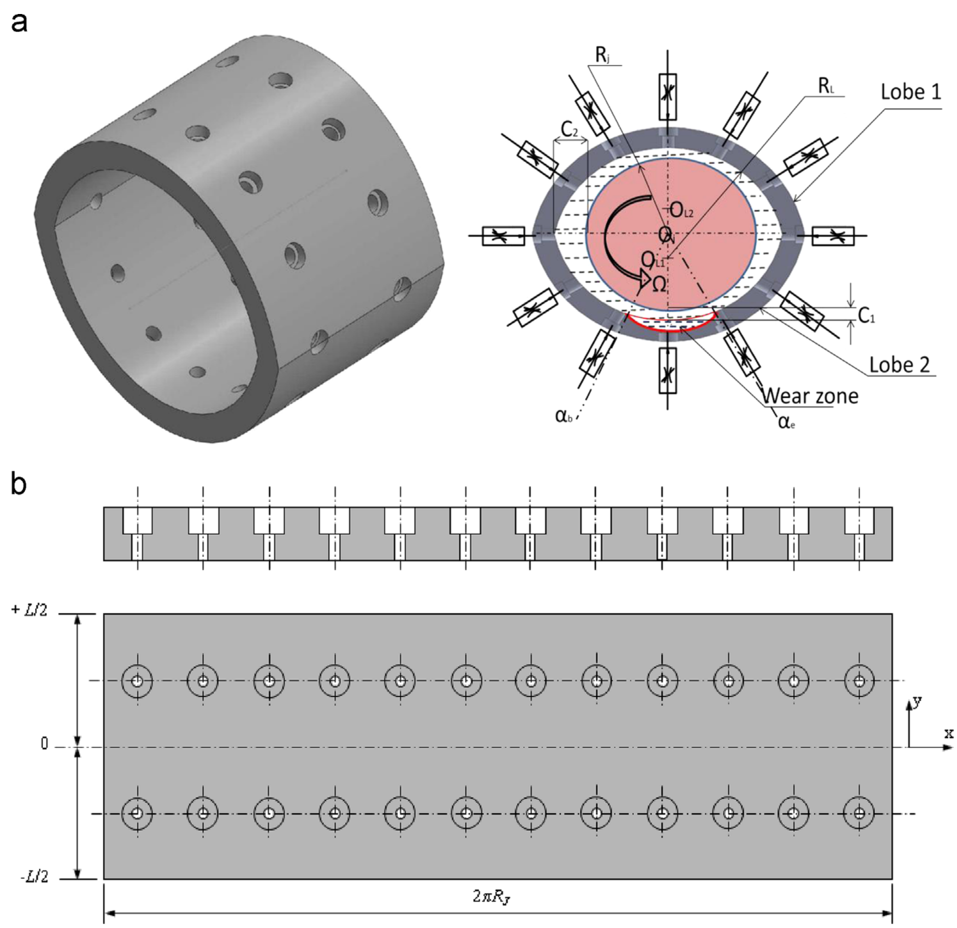

Fig. 1. (a) Two lobe symmetric hole entry hybrid journal bearing system and (b) Geometry of two lobe symmetric hole entry hybrid journal bearing system.

mixed lubrication was studied by Jagadeesha et al. [34]. It was reported that the bearing transient response is significantly affected by the pattern of surface roughness.

Moreover, very limited studies, concerning the transient analysis of hydrostatic/hybrid bearings, have been carried out and reported in the literature [35–38]. Leonard and Rowe [35] carried out the theoretical and experimental work. They highlighted the mechanism of whirl instability in hydrodynamic bearings by showing how the hydrostatic stiffness raises the frequency of whirl onset. Zhicheng Pang et al. [36] studied the transient characteristics of a hydrostatic bearing under a step load, by considering the compressibility of oil. San Andres [37] presented a linear and non-linear transient response study of rectangular recessed shaped hydrostatic journal bearing system. He compared the transient response of linear to non-linear model for various external loads. Yoshimoto and Kikuchi [38] studied the step response characteristics of hydrostatic journal bearings. They theoretically investigated the influence of design parameters such as supply pressure, viscosity, the magnitude of the step load on the step response characteristics.

A thorough scan of the literature indicates that the values of critical mass and threshold speed obtained from linear analysis do not set correct stability margins and nonlinear analysis of transient response generally predicts higher values of stability parameters, compared to those obtained from linear analysis [26,29,31–33,37]. Therefore, to predict the accurate response of journal motions, nonlinear model for motion trajectories have been used. Moreover, to the best of the author′s knowledge, no study is yet available in the literature that addresses a non linear transient stability study of non-Newtonian two lobe symmetric hole entry worn hybrid journal bearing compensated with constant flow valve. Therefore, the present study is aimed to investigate the combined influence of wear and non-Newtonian lubricant on the stability of a two lobe constant flow valve compensated symmetric hole-entry journal bearing as shown in Fig. 1(a). The novel feature of the present work is to obtain the nonlinear trajectories of the journal centre motion from the stability point of view. In the present study, symmetric two lobe hole entry worn journal bearing configurations having 12 holes per row as shown in Fig. 1(b) is considered. The change in the bearing geometry has been accounted by defining a non dimensional offset factor (δ). The results presented in the paper are expected to be quite useful to the bearing designers and the academic community.

2. Governing equations

The generalized Reynolds equation governing the flow of a variable viscosity lubricant in the clearance space of a hole entry symmetric hybrid journal bearing system is expressed in non-dimensional form as [21,22,40]:

$$\frac{\partial}{\partial\alpha}\left(\overline{h}^3\overline{F}_2\frac{\partial\overline{p}}{\partial\alpha}\right)+\frac{\partial}{\partial\beta}\left(\overline{h}^3\overline{F}_2\frac{\partial\overline{p}}{\partial\beta}\right)=\Omega\left[\frac{\partial}{\partial\alpha}\left\{\left(1-\frac{\overline{F}_1}{\overline{F}_0}\right)\overline{h}\right\}\right]+\frac{\partial\overline{h}}{\partial\overline{t}} \tag{1}$$

where $\overline{F}_0,\overline{F}_1$, and $\overline{F}_2$ are the viscosity functions and given by the following relations:

$$\overline{F}_0=\int_0^1\frac{1}{\overline{\mu}}d\overline{z},\overline{F}_1=\int_0^1\frac{\overline{z}}{\overline{\mu}}d\overline{z},\overline{F}_2=\int_0^1\frac{\overline{z}}{\overline{\mu}}\left(\overline{z}-\frac{\overline{F}_1}{\overline{F}_0}\right)d\overline{z} \tag{2}$$

2.1. Fluid film thickness

The fluid-film thickness expression for a multilobe hole entry hybrid journal bearing system, in non dimensional form can be expressed as [6,8,9,20]:

$$\bar{h}_0 = \frac{1}{\delta} - (\bar{X}_j + \bar{x} - \bar{X}_L^i)\cos\alpha - (\bar{Z}_j + \bar{z} - \bar{Z}_L^i)\sin\alpha \quad (3)$$

where $\bar{X}_j$ and $\bar{Z}_j$ are the equilibrium co-ordinates of the journal center and $\bar{X}_L^i$ and $\bar{Z}_L^i$ are the lobe center coordinates of the lobe.

The geometry of the worn zone is shown schematically in Fig. 1 (a). Worn region of the bearing is modeled using abrasive wear model of Dufrane et al. [13]. Dufrane et al. [13] assumed the worn arc at a radius larger than the journal and the footprint created by the shaft is symmetrical at the bottom of the bearing and the wear pattern is uniform along the axial length of the bearing. The change in the bush geometry is expressed as [11,13,15–18]

$$\partial\bar{h} = \bar{\delta}_w - 1 - \sin\alpha; \quad \text{for } \alpha_b \le \alpha \le \alpha_e \quad (3.1)$$

$$\partial\bar{h} = 0; \quad \text{for } \alpha < \alpha_b \text{ or } \alpha > \alpha_e \quad (3.2)$$

α_b and α_e are the angles at the beginning and at the end of the footprint, respectively.

The defect value $\partial\bar{h}$ is added to the nominal fluid-film thickness for rigid multi lobe journal bearing, which is expressed as Eq. (3). The value of fluid-film thickness for symmetric hole entry worn hybrid journal bearing is defined as:

$$\bar{h} = \bar{h}_0 + \partial\bar{h} \quad (4)$$

2.2. Restrictor flow equation

The flow of lubricant through a constant flow valve restrictor ($\bar{Q}_R$) is expressed as [39]:

$$\bar{Q}_R = \bar{Q}_C \quad \text{(Constant flow valve)} \quad (5)$$

where $\bar{Q}_c$ is the specified flow rate for the constant flow valve restrictor.

2.3. Cubic shear stress law model

The flow behavior of most of the polymer-thickened oils is adequately represented by the cubic shear stress law model [21,22,26] and is expressed in non-dimensional form as:

$$\bar{\tau} + \bar{K}\bar{\tau}^3 = \bar{\dot{\gamma}} \quad (6)$$

where $\bar{K}$ is known as a non-linearity factor. When $\bar{K} = 0$, the above relation reduces to the constitutive relation for Newtonian lubricant.

The viscosity of a non-Newtonian lubricant is represented by apparent viscosity ($\bar{\mu}_a$) which is defined as [21,22,26]

$$\bar{\mu}_a = \bar{\tau}/\bar{\dot{\gamma}} \quad (7)$$

Typically, the viscosity of a non-Newtonian fluid is known as a function of the second invariant of shear strain rate, I_2 which can be simplified [21,22]:

$$I_2 = \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 \quad (8)$$

For an incompressible non-Newtonian fluid, the shear strain rate ($\dot{\gamma}$) is expressed as [21,22]:

$$\dot{\gamma} = \sqrt{I_2} = \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]^{1/2} \quad (9)$$

where $(\partial u/\partial z)$ and $(\partial v/\partial z)$ are the velocity gradients and are expressed as [21,22]:

$$\frac{\partial\bar{u}}{\partial\bar{z}} = \left\{\frac{\bar{h}\partial\bar{p}}{\bar{\mu}\partial\alpha}\left(\bar{z} - \frac{\bar{F}_1}{\bar{F}_0}\right) + \frac{\Omega}{\bar{\mu}\bar{h}\bar{F}_0}\right\} \quad (9.1)$$

$$\frac{\partial\bar{v}}{\partial\bar{z}} = \left\{\frac{\bar{h}\partial\bar{p}}{\bar{\mu}\partial\beta}\left(\bar{z} - \frac{\bar{F}_1}{\bar{F}_0}\right)\right\} \quad (9.2)$$

2.4. Finite element formulation

The lubricant flow field in the clearance space of two lobe hole entry symmetric journal bearing has been discretized by using a four-noded isoparametric element. Using Galerkin's orthogonality conditions and following the usual assembly procedure, the global system of equation is obtained as [9,21,23]:

$$[\bar{F}]\{\bar{P}\} = \{\bar{Q}\} + \Omega\{\bar{R}_H\} + \dot{\bar{X}}_J\{\bar{R}_{xj}\} + \dot{\bar{Z}}_J\{\bar{R}_{zj}\} \quad (10)$$

2.5. Stability parameters

As the journal centre is disturbed from its static equilibrium position, the change in hydrodynamic forces occurs. Due to these unbalanced forces, the journal centre starts to whirl around its static equilibrium position.

The linearized equation of motion for the journal in the non-dimensional form is given by [26,31,33]:

$$[\bar{M}_J]\{\ddot{\bar{q}}\} + [\bar{C}]\{\dot{\bar{q}}\} + [\bar{S}]\{q\} = 0 \quad (11)$$

Equation of motion in matrix form may be written as:

$$\begin{bmatrix} \bar{M}_J & 0 \\ 0 & \bar{M}_J \end{bmatrix}\begin{Bmatrix} \ddot{\bar{X}}_j \\ \ddot{\bar{Z}}_j \end{Bmatrix} + \begin{bmatrix} \bar{C}_{xx} & \bar{C}_{xz} \\ \bar{C}_{zx} & \bar{C}_{zz} \end{bmatrix}\begin{Bmatrix} \dot{\bar{X}}_j \\ \dot{\bar{Z}}_j \end{Bmatrix} + \begin{bmatrix} \bar{S}_{xx} & \bar{S}_{xz} \\ \bar{S}_{zx} & \bar{S}_{zz} \end{bmatrix}\begin{Bmatrix} \bar{X}_j \\ \bar{Z}_j \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (11.1)$$

To obtain $\bar{x}_j$, $\bar{z}_j$, $\dot{\bar{x}}_j$ and $\dot{\bar{z}}_j$ which gives the locus of the journal, Eq. (11.1) is integrated for certain initial disturbance $\bar{X}_j$, $\bar{Z}_j$, $\dot{\bar{X}}_j$ and $\dot{\bar{Z}}_j$.

Using Rouths' criteria, the stability parameters of journal bearing system are defined subsequently.

The non-dimensional critical mass $\bar{M}_c$ of the journal is expressed as:

$$\bar{M}_c = \frac{\bar{G}_1}{\bar{G}_2 - \bar{G}_3} \quad (12)$$

where

$$\bar{G}_1 = [\bar{C}_{xx}\bar{C}_{zz} - \bar{C}_{zx}\bar{C}_{xz}] \quad (12.1)$$

$$\bar{G}_2 = \frac{[\bar{S}_{xx}\bar{S}_{zz} - \bar{S}_{zx}\bar{S}_{xz}][\bar{C}_{xx} + \bar{C}_{zz}]}{[\bar{S}_{xx}\bar{C}_{zz} + \bar{S}_{zz}\bar{C}_{xx} - \bar{S}_{xz}\bar{C}_{zx} - \bar{S}_{zx}\bar{C}_{xz}]} \quad (12.2)$$

$$\bar{G}_3 = \frac{[\bar{S}_{xx}\bar{C}_{xx} + \bar{S}_{xz}\bar{C}_{xz} + \bar{S}_{zx}\bar{C}_{zx} + \bar{S}_{zz}\bar{C}_{zz}]}{[\bar{C}_{xx} + \bar{C}_{zz}]} \quad (12.3)$$

Threshold speed can be obtained using the relation given below:

$$\bar{\omega}_{th} = [\bar{M}_c/\bar{F}_o]^{1/2} \quad (13)$$

where $\bar{F}_0$ is the resultant fluid-film force or reaction. $((\partial\bar{h}/\partial\bar{t}) = 0)$.

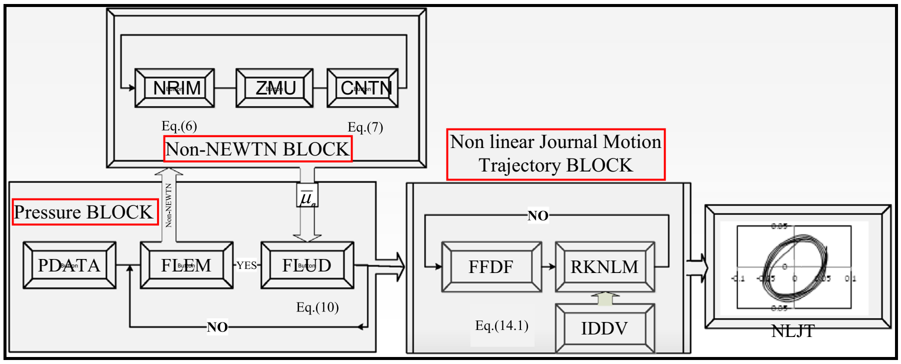

Fig. 2. Overall iterative solution procedure.

Table 1
Geometric and operating parameters of two lobe symmetric hole-entry hybrid journal bearing [9–11,13,15–17,19–23].

Operating parameters			Geometric parameters	
Parameters		**Value/range**	**Parameters**	**Value**
External load ($\overline{W}_o$)	Symmetric	**0.5–4.0**	No. of holes per row (k)	**12**
Symmetric bearing configuration			No. of rows (n)	**02**
Speed parameter (Ω)		**1.0**	Constant flow valve restrictor	
Restrictor flow parameter ($\overline{Q}_R$)		**0.0438**	Aspect ratio (λ)	**1.0**
Non-linearity factor ($\overline{K}$)		**0.0, 0.1, 0.58 and 1.00**		
Wear depth parameter ($\overline{\delta}_w$)		**0.0–0.5**	Land width ratio ($\overline{a}_b$)	**0.25**
Offset factor (δ)		**0.75, 1.00 and 1.25**	Clearance ratio ($\overline{c}$)	**0.001**

2.6. Nonlinear model for motion trajectories

The nonlinear equation for disturbed motion of the journal may be written in terms of instantaneous fluid-film force components $\overline{F}_x$ and $\overline{F}_z$. At any time $\overline{t}$, the nonlinear equation of motion is written as [26,31,33,34]:

$$\begin{bmatrix} \overline{M}_J & 0 \\ 0 & \overline{M}_J \end{bmatrix} \begin{Bmatrix} \ddot{\overline{X}}_J \\ \ddot{\overline{Z}}_J \end{Bmatrix} = \begin{Bmatrix} \Delta\overline{F}_x \\ \Delta\overline{F}_z \end{Bmatrix} = \begin{Bmatrix} \overline{F}_x - \overline{F}_{xo} \\ \overline{F}_z - \overline{F}_{zo} \end{Bmatrix} \tag{14}$$

where

$\overline{F}_x$ and $\overline{F}_z$ are the instantaneous hydrodynamic force components in the transient state of the journal.
$\overline{F}_{xo}$ and $\overline{F}_{zo}$ are the fluid-film force components at the static equilibrium position. The fluid-film force components ($\overline{F}_x$ and $\overline{F}_z$) at time $\overline{t}$, are evaluated at each time step after establishing the pressure field corresponding to the position of the journal at that particular time.

2.7. Boundary conditions

The boundary conditions used in the lubricant flow field analysis are as follows [18,21,23]:

(a) Nodes situated on the external boundary of the bearing have zero relative pressure with respect to ambient temperature; $\overline{p}|_{\beta = \mp 1.0} = 0.\ 0.$
(b) All nodes situated on entry holes have equal pressure.
(c) Flow of lubricant through the restrictor is equal to the bearing input flow.
(d) At the trailing edge of the positive region,

$$\overline{p} = \frac{\partial \overline{p}}{\partial \alpha} = 0.0 \quad \text{(Reynolds boundary condition)}$$

3. Solution scheme

The overall iterative solution procedure shown in Fig. 2 is used to obtain the nonlinear motion trajectories of journal center. The solution of global system of Eq. (10) provides the nodal pressure and velocity components of the lubricant flow field. The shear strain rate ($\overline{\dot{\gamma}}$) is obtained by using Eq. (9). Using Newton–Raphson method, the shear stress ($\overline{\tau}$) is calculated from Eq. (6). The apparent viscosity ($\overline{\mu}_a$) is then computed at each Gauss point using Eq. (7) in non-NEWTN block. Eq. (10) is solved again in unit FLUD using this computed value of apparent viscosity and the nodal pressures and velocity components are obtained. An iterative procedure is repeated till the desired convergence is achieved. Using these converged nodal pressures, fluid film force components are calculated in unit RKNLM. In NLJT unit, the nonlinear motion trajectories of journal center are obtained by integrating the equations of motion (14) with the initial values of disturbances $\overline{X}$, $\dot{\overline{X}}$, $\overline{Z}$ and $\dot{\overline{Z}}$ by incorporating the fourth order Runge–Kutta method.

4. Result and discussions

The nonlinear journal center motion trajectories of two lobe symmetric hole entry worn hybrid journal bearing system compensated with constant flow valve restrictor have been computed using the solution procedure as mentioned in the earlier section. The bearing geometric and operating parameters are the most judiciously selected values based on the published literature [17,26] and is shown in Table 1. As stated earlier, no transient journal trajectory motion response studies for bearing stability are available in the published literature for a two lobe symmetric hole entry worn hybrid journal bearing. Therefore, in order to check the validity of the developed numeric model, the simulated results from the present study have been validated with the already reported results of Sinhasan and Goyal [26], Hashimoto et al. [15]

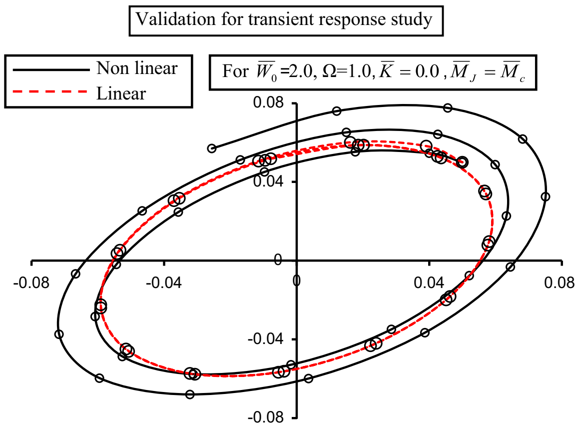

Fig. 3. Journal centre motion trajectories for circular hydrodynamic journal bearing.

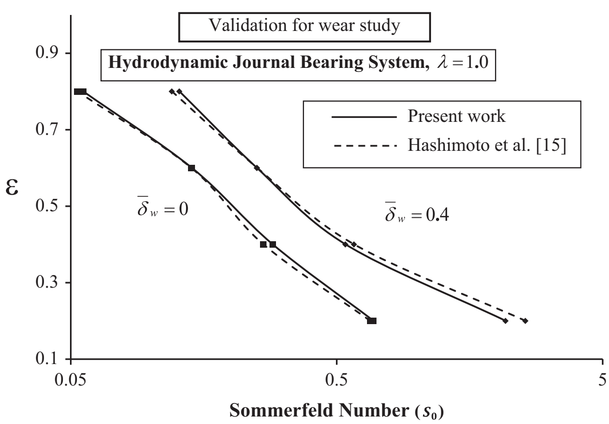

Fig. 4. Variation of eccentricity ratio (ε) with Sommerfeld number (S_0).

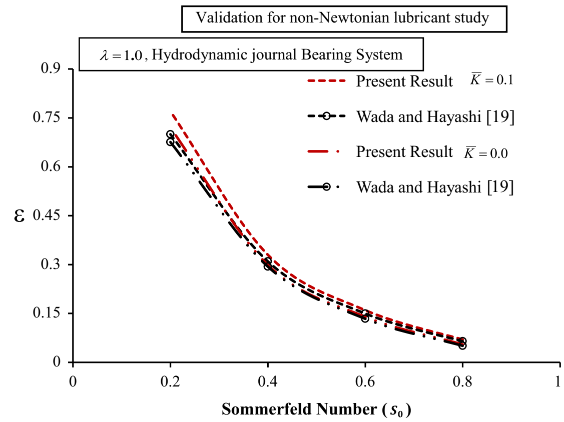

Fig. 5. Variation of eccentricity ratio (ε) with Sommerfeld number (S_0).

and Wada and Hayashi [19]. For the transient response study, the computed journal trajectories have been compared with the published results [26]. The linear and non-linear journal center motion trajectories for a circular hydrodynamic journal bearing for $\overline{W}_0 = 2.0$ and $\overline{K} = 0.0$ are presented in Fig. 3. It is observed that the linear equations of motion predict a limiting cycle and nonlinear equation of motion predict unstable motion when the journal mass equals the critical mass. The linear and non-linear trajectories for hydrodynamic journal bearing lubricated with Newtonian lubricant agree quite well with the published result [26]. For the influence of wear, the computed results have been compared with the published results of Hashimoto et al. [15] as shown in Fig. 4. The present results for worn and unworn hydrodynamic journal bearing are in good agreement with the available published results [15]. Further, the results of the plain hydrodynamic journal bearing lubricated with non-Newtonian lubricants have been computed and compared with the available results of Wada and Hayashi [19] and is shown in Fig. 5. The computed result shows excellent agreement with the reported results of Wada and Hayashi [19].

The nonlinear journal center motion trajectories have been computed and presented for different values of offset factor (δ), depth parameter ($\overline{\delta}_W$) and non-linearity factor ($\overline{K}$) for the three different cases when, $\overline{M}_J = \overline{M}_c$, $\overline{M}_J = 0.8\overline{M}_c$ and $\overline{M}_J = 1.1\overline{M}_c$. Further, the numerically simulated results for threshold speed margin ($\overline{\omega}_{th}$) have been presented for a two lobe symmetric hole entry hybrid journal bearing configuration as a function of an external load ($\overline{W}_0$) and wear depth parameter ($\overline{\delta}_w$) in graphical form.

The results of the nonlinear transient analysis of two lobe symmetric hole entry unworn hybrid journal bearing systems are presented in Figs. 6–11 in the form of journal centre motion trajectories.

Nonlinear journal motion trajectories for $\overline{M}_J = \overline{M}_c$ and $\overline{W}_0 = 2.0$ has been shown in Fig. 6(a)–(h) for two lobe symmetric hole entry journal bearing compensated with constant flow valve. From Fig. 6 (a), it is observed that when the unworn bearing ($\overline{\delta}_W = 0.0$) operates with Newtonian lubricant ($\overline{K} = 0.0$) at an external load $\overline{W}_0 = 2.0$, nonlinear equation of motion predicts a stable cycle for $\delta = 0.75, 1.0$ and $\delta = 1.25$ when the journal mass equals the critical mass. The noncircular bearing ($\delta = 1.25$) operates at a minimum fluid film thickness than that of the circular journal bearing during the entire load. The noncircular journal bearing with offset of $\delta = 0.75$ traces a larger orbit for the same operating condition. As the lubricant become non-linear ($\overline{K} = 0.1$) for unworn hole entry hybrid journal bearing, the maximum value of minimum fluid film thickness ($\overline{h}_{\min}$) gets reduced and journal traces a larger orbit and position of its maximum values shifts towards the left. Further it may be observed that the viscosity of the lubricant decreases as the lubricant becomes more nonlinear, this results in shear thinning of the lubricant and bearing operates at lower values of minimum fluid film thickness (Fig. 6(a)–(d)). It may be noticed that for $\delta = 1.25$, bearing operates at lower values of minimum fluid film thickness when compared to other bearing geometries. Fig. 6(e) depicts that, when worn ($\overline{\delta}_W = 0.5$) bearing operates with Newtonian lubricant ($\overline{K} = 0.0$) at $\overline{W}_0 = 2.0$,non-linear equation of motion predicts a stable motion for $\delta = 1.0$ and $\delta = 1.25$ and form limit cycle for $\delta = 0.75$.The combined influence of damaged bearing i.e.; wear depth parameter ($\overline{\delta}_W = 0.5$) and non-Newtonian lubricant ($\overline{K} \neq 0.0$) on the stability response of symmetric hole entry hybrid journal bearing have been analyzed and presented in Fig. 6(f)–(h). The plot for nonlinear trajectories for worn out bearing ($\overline{\delta}_W = 0.5$) at $\overline{W}_0 = 2.0$ and $\overline{K} = 0.1$ is shown in Fig. 6(f). According to nonlinear analysis, system remains stable for $\overline{M}_J = \overline{M}_c$. The figure further indicates that when non-linearity factor increases from $\overline{K} = 0.1$ to $\overline{K} = 0.58$, nonlinear journal motion trajectory provides unstable motion for an offset factor of $\delta = 0.75$ and at $\overline{K} = 1.0$ and $\overline{\delta}_W = 0.5$ forms limit cycle motion. It may be observed that journal traces the small size circular orbit for an offset factor $\delta = 1.25$. The hydrodynamic forces causes unbalance in the journal and also affects the size and shape of the orbit. Hydrodynamic stability of the journal improves as the bearing becomes more noncircular ($\delta = 1.25$). Also increase in 50% of radial clearance due to wear ($\overline{\delta}_W = 0.5$) provides more eccentricity for the operation of bearing. Fig. 6(g) shows the nonlinear journal motion trajectories for $\overline{K} = 0.58$, $\overline{\delta}_W = 0.5$ and $\overline{W}_0 = 2.0$. The journal motion predicts stable motion for $\overline{M}_j = \overline{M}_c$ at an offset

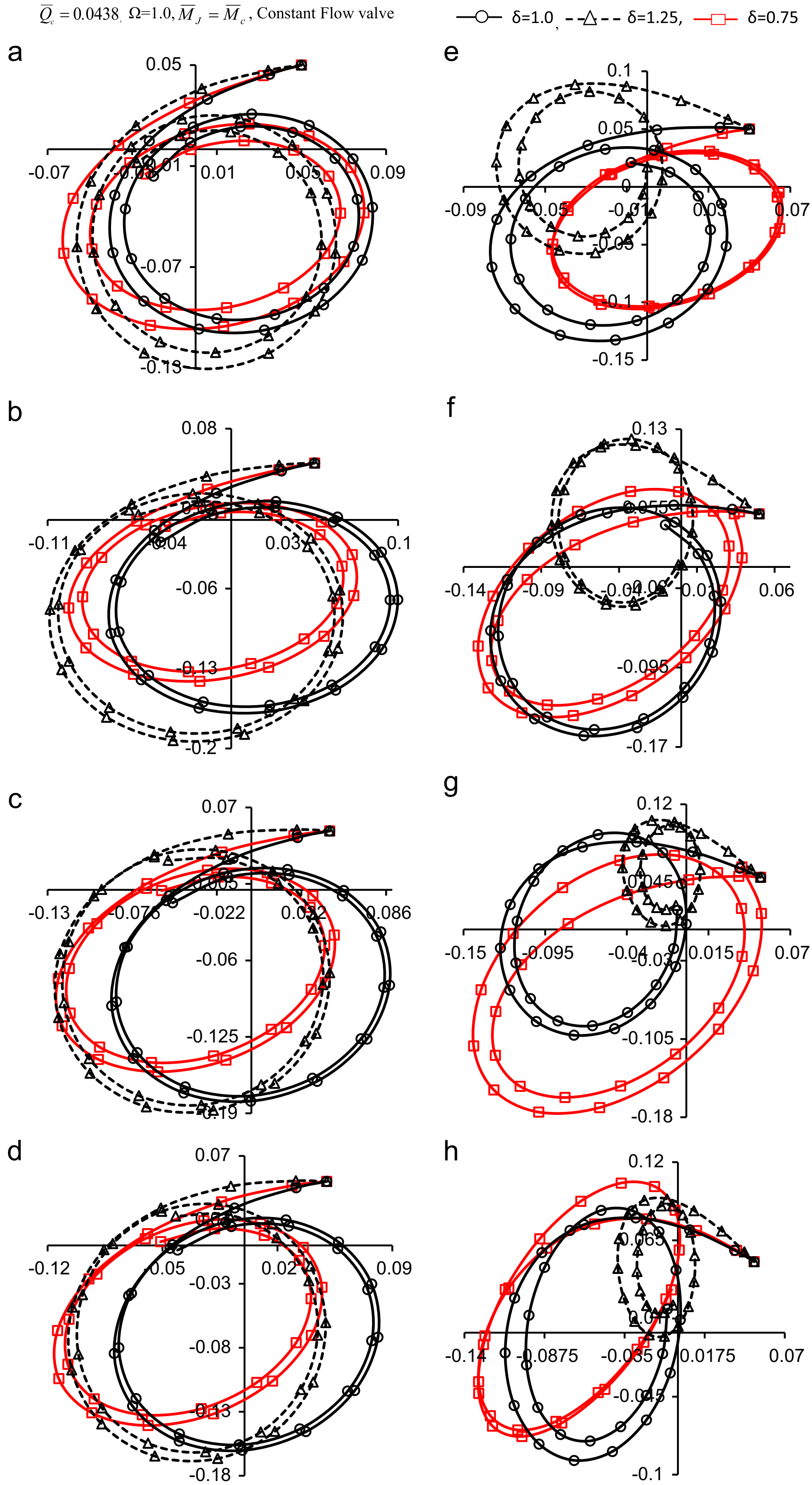

Fig. 6. (a)–(h) Trajectories of journal centre motion for $\overline{W}_0 = 2.0$ and $\overline{M}_J = \overline{M}_c$. (a) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 0.0$, $\overline{M}_J = \overline{M}_c$, (b) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 0.1$, $\overline{M}_J = \overline{M}_c$, (c) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 0.58$, $\overline{M}_J = \overline{M}_c$, (d) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 1.0$, $\overline{M}_J = \overline{M}_c$, (e) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.0$, $\overline{M}_J = \overline{M}_c$, (f) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.1$, $\overline{M}_J = \overline{M}_c$, (g) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.58$, $\overline{M}_J = \overline{M}_c$ and (h) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.5$ and $\overline{K} = 1.0$, $\overline{M}_J = \overline{M}_c$.

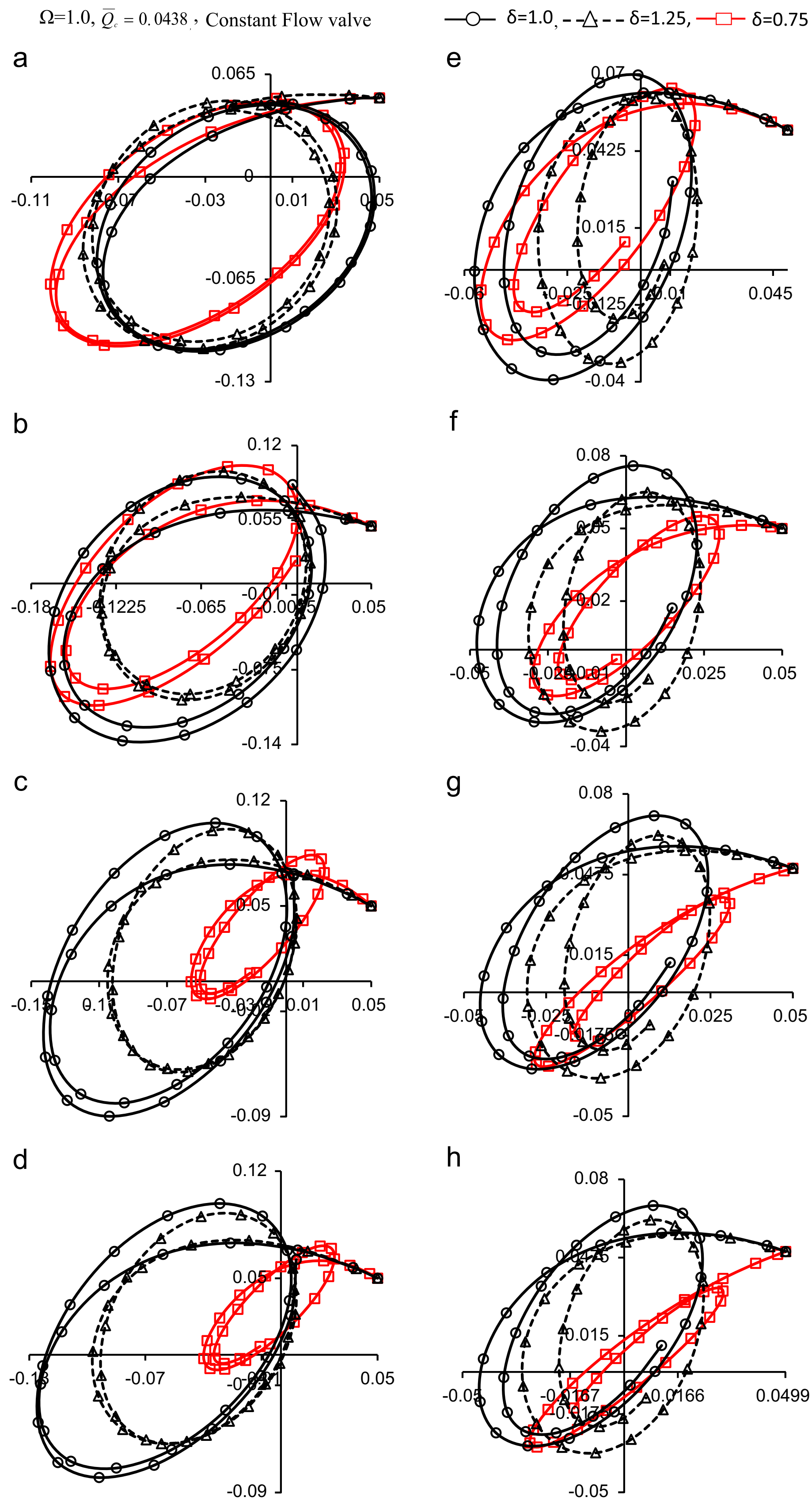

Fig. 7. (a)–(h) Trajectories of journal centre motion for $\overline{W}_0 = 4.0$ and $\overline{M}_J = \overline{M}_c$. (a) $\overline{W}_0 = 4.0$ $\overline{\delta}_W = 0.0$ $\overline{K} = 0.0$, $\overline{M}_J = \overline{M}_c$, (b) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 0.1$, $\overline{M}_J = \overline{M}_c$, (c) $\overline{W}_0 = 4.0$ $\overline{\delta}_W = 0.0$ $\overline{K} = 0.58$, $\overline{M}_J = \overline{M}_c$, (d) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 1.0$, $\overline{M}_J = \overline{M}_c$, (e) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.0$, $\overline{M}_J = \overline{M}_c$, (f) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.1$, $\overline{M}_J = \overline{M}_c$, (g) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.58$, $\overline{M}_J = \overline{M}_c$ and (h) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 1.0$, $\overline{M}_J = \overline{M}_c$.

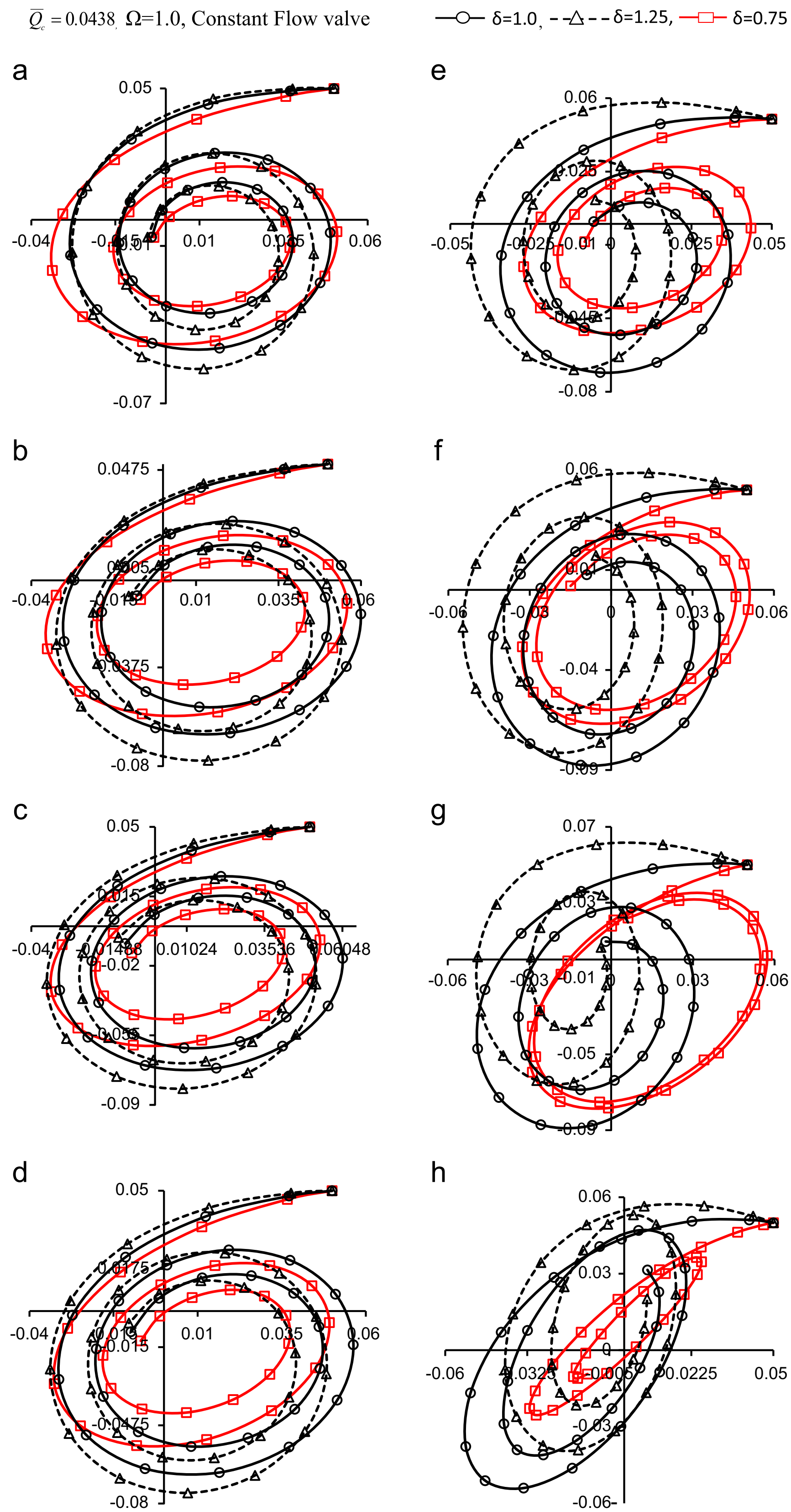

Fig. 8. (a)–(h) Trajectories of journal centre motion for $\overline{W}_0 = 2.0$ and $\overline{M}_J = 0.8\overline{M}_c$. (a) $\overline{W}_0 = 2.0$ $\overline{\delta}_W = 0.0$ $\overline{K} = 0.0$, $\overline{M}_J = 0.8\overline{M}_c$, (b) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 0.1$, $\overline{M}_J = 0.8\overline{M}_c$, (c) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.0$ $\overline{K} = 0.58$, $\overline{M}_J = 0.8\overline{M}_c$, (d) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 1.0$, $\overline{M}_J = 0.8\overline{M}_c$, (e) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.0$, $\overline{M}_J = 0.8\overline{M}_c$, (f) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.1$, $\overline{M}_J = 0.8\overline{M}_c$, (g) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.58$, $\overline{M}_J = 0.8\overline{M}_c$ and (h) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 1.0$, $\overline{M}_J = 0.8\overline{M}_c$.

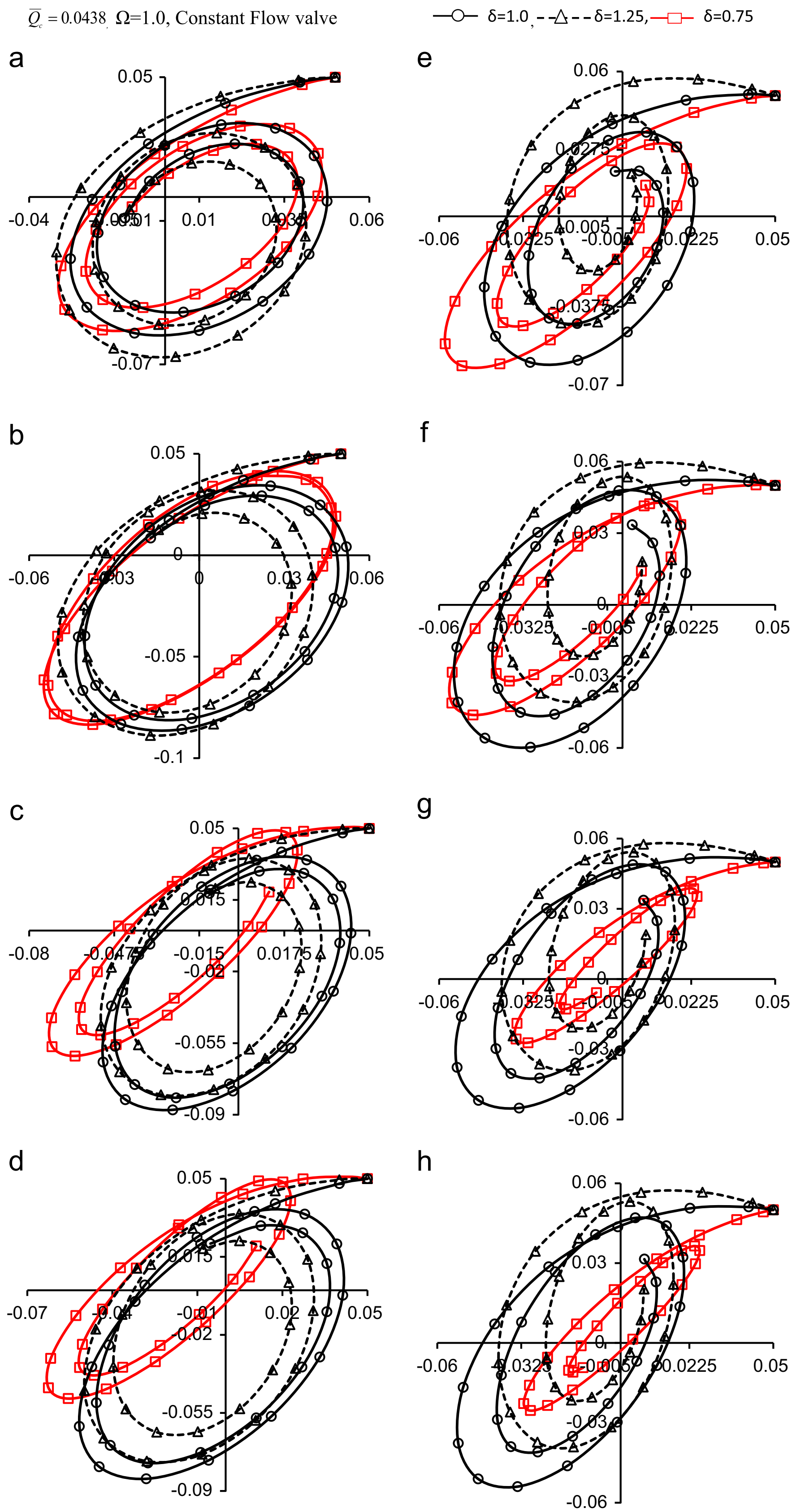

Fig. 9. (a-h) Trajectories of journal centre motion for $\overline{W}_0 = 4.0$ and $\overline{M}_J = 0.8\overline{M}_c$. (a) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.0$ $\overline{K} = 0.0$, $\overline{M}_J = 0.8\overline{M}_c$, (b) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 0.1$, $\overline{M}_J = 0.8\overline{M}_c$, (c) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.0$ $\overline{K} = 0.58$, $\overline{M}_J = 0.8\overline{M}_c$, (d) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 1.0$, $\overline{M}_J = 0.8\overline{M}_c$, (e) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.0$, $\overline{M}_J = 0.8\overline{M}_c$, (f) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.1$, $\overline{M}_J = 0.8\overline{M}_c$, (g) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.58$, $\overline{M}_J = 0.8\overline{M}_c$ and (h) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 1.0$, $\overline{M}_J = 0.8\overline{M}_c$.

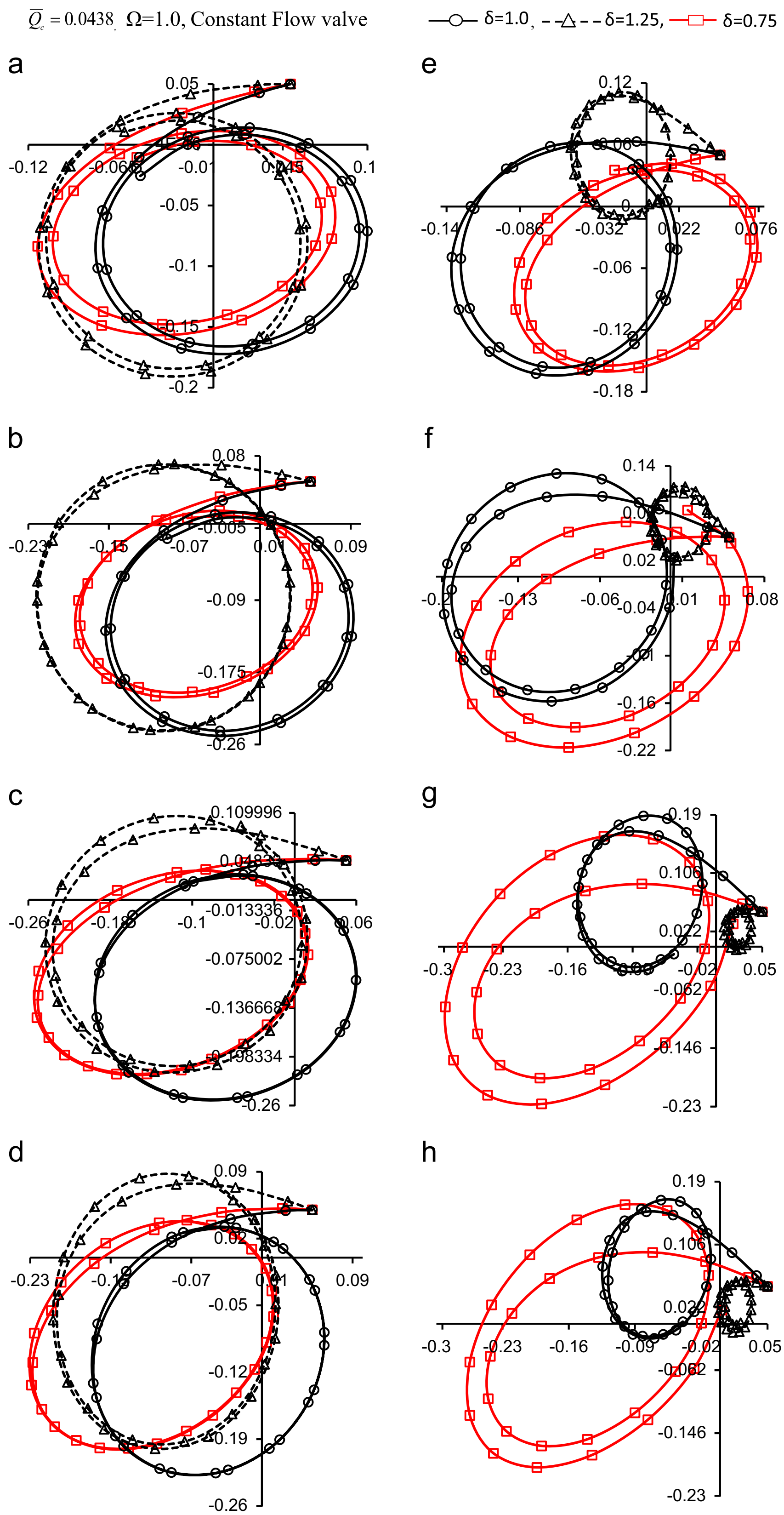

Fig. 10. (a)–(h) Trajectories of journal centre motion for $\overline{W}_0 = 2.0$ and $\overline{M}_J = 1.1\overline{M}_c$. (a) $\overline{W}_0 = 2.0$ $\overline{\delta}_W = 0.0$ $\overline{K} = 0.0$, $\overline{M}_J = 1.1\overline{M}_c$, (b) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 0.1$, $\overline{M}_J = 1.1\overline{M}_c$, (c) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.0$ $\overline{K} = 0.58$, $\overline{M}_J = 1.1\overline{M}_c$, (d) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 1.0$, $\overline{M}_J = 1.1\overline{M}_c$, (e) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.0$, $\overline{M}_J = 1.1\overline{M}_c$, (f) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.1$, $\overline{M}_J = 1.1\overline{M}_c$, (g) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.58$, $\overline{M}_J = 1.1\overline{M}_c$ and (h) $\overline{W}_0 = 2.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 1.0$, $\overline{M}_J = 1.1\overline{M}_c$.

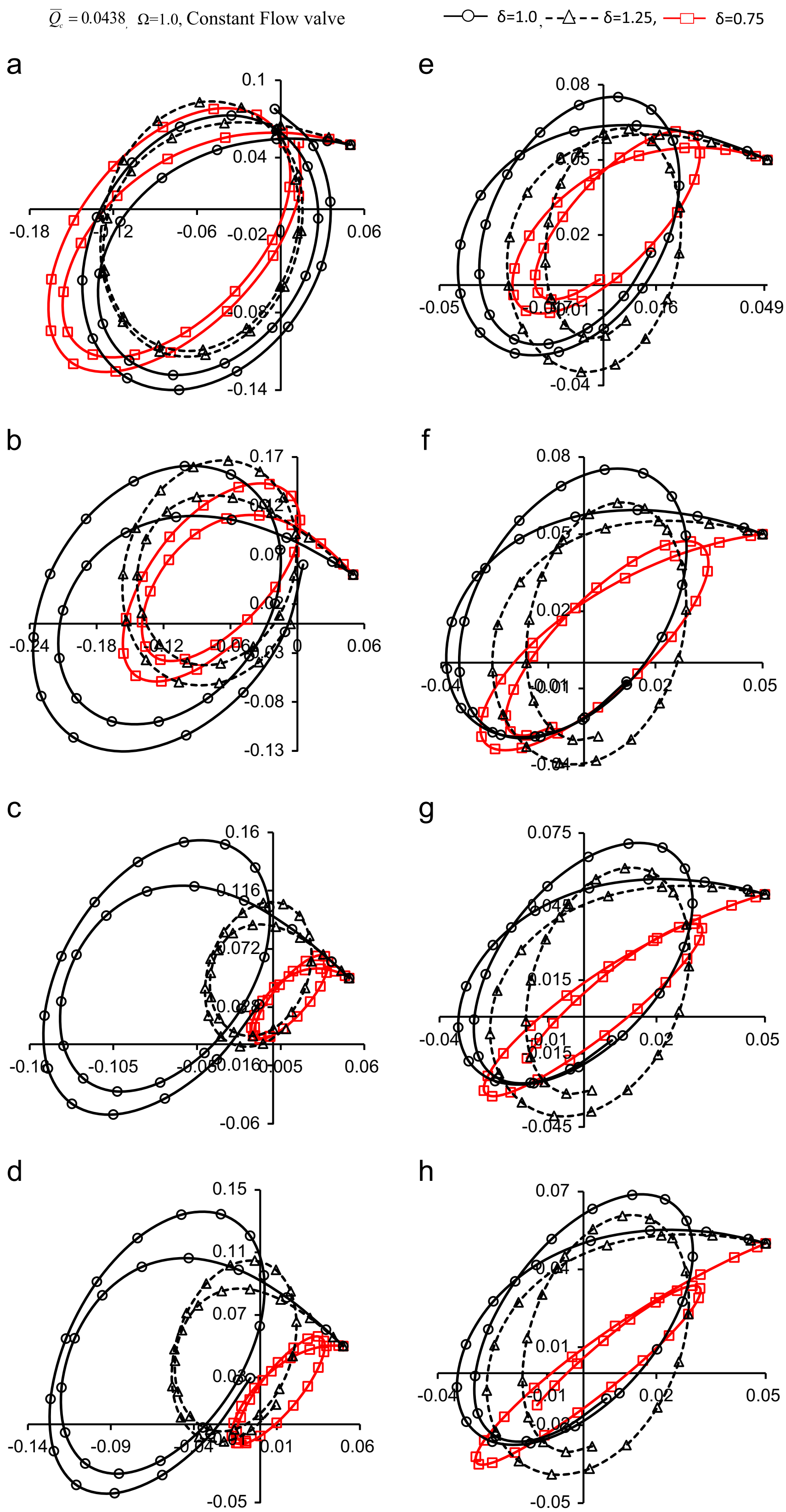

Fig. 11. (a)–(h) Trajectories of journal centre motion for $\overline{W}_0 = 4.0$ and $\overline{M}_J = 1.1\overline{M}_c$. (a) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.0$ $\overline{K} = 0.0$, $\overline{M}_J = 1.1\overline{M}_c$, (b) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 0.1$, $\overline{M}_J = 1.1\overline{M}_c$, (c) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.0$ $\overline{K} = 0.58$, $\overline{M}_J = 1.1\overline{M}_c$, (d) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.0$, $\overline{K} = 1.0$, $\overline{M}_J = 1.1\overline{M}_c$, (e) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.0$, $\overline{M}_J = 1.1\overline{M}_c$, (f) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.1$, $\overline{M}_J = 1.1\overline{M}_c$, (g) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 0.58$, $\overline{M}_J = 1.1\overline{M}_c$ and (h) $\overline{W}_0 = 4.0$, $\overline{\delta}_W = 0.5$, $\overline{K} = 1.0$, $\overline{M}_J = 1.1\overline{M}_c$.

factor of 1.0 and 1.25 whereas at $\delta=0.75$, journal motion predicts unstable motion. As a consequence of this, the bearing operates with high radial clearance and more lubricant is pumped in. Further It may be noticed that at lower values of external load, the non Newtonian lubricant has a marginal influence on the bearing stability when $\overline{M}_J=\overline{M}_c$. The combined influence of higher values of wear depth parameter ($\overline{\delta}_W=0.5$) and non-linearity ($\overline{K}=1.0$) have predominant effect on journal bearing stability.

The nonlinear journal motion trajectories for $\overline{M}_J=\overline{M}_c$ is shown in Fig. 7(a)–(h). Fig. 7(a)–(d) shows the non linear trajectories for an unworn two lobe symmetric hole entry journal bearing operated with Newtonian lubricant and non-Newtonian lubricants for $\overline{W}_0=4.0$. When bearing operates with Newtonian lubricant ($\overline{K}=0.0$) and wear depth parameter $\overline{\delta}_W=0.0$, non-linear equation of motion predicts a stable cycle for $\delta=1.25$ and the unstable motion for $\delta=0.75,\ 1.0$. The non-circular journal bearing with an offset $\delta=0.75$ yields unstable motion with larger orbit and diverged trajectory, i.e. instability. Furthermore, from Figs. 7 and 6(a), it may be observed that, at high values of the external load $\overline{W}_0=4.0$, the journal trajectory forms unstable motion compared to the low value of the external load $\overline{W}_0=2.0$ for an offset factor $\delta=0.75$ and $\delta=1.0$, for unworn journal bearing lubricated with non-Newtonian lubricant. This is because of the variation in the viscosity of the lubricant, as it becomes more nonlinear, the viscosity of the lubricant decreases and lubricant becomes thinner and thinner. It reduces the load carrying capacity of the bearing and pressure developed is not sufficient to balance the fluid film forces. Fig. 7(b) indicates that the nonlinear journal motion trajectory forms unstable motion at an offset factor of $\delta=0.75, 1.0$ for $\overline{K}=0.1$ and limit cycle motion for $\delta=1.25$ however, as the lubricant becomes more nonlinear ($\overline{K}=0.58$), the journal provides unstable motion for $\delta=1.0, 1.25$ and limit cycle motion for $\delta=0.75$ in case of unworn circular and non circular journal bearing as shown in Fig. 7(c).

For limit cycle condition, the locus of journal in the bearing neither converges nor diverges. A small change in $\overline{K}$ or $\overline{W}_0$ or both can make the bearing system either unstable or stable. This type of journal trajectory, characterizes the threshold state of the bearing at that instant. The increase of a non-linearity factor ($\overline{K}$), which attributes to the variation in the apparent viscosity of the lubricant, results into decrease in the bearing load capacity. Therefore, in order to support the external load ($\overline{W}_0$), the bearing needs to be pumped with more quantity of lubricant. As a result of this, the journal tends to glide over the lubricant and journal forms a smaller size orbit compared to low value of an external load ($\overline{W}_0=2.0$). Further it may be noticed that from Figs. 7 and 6 (e) that, increase in radial clearance due to wear ($\overline{\delta}_W=0.5$) may enhance the stability of the bearing at higher load when bearing operates with Newtonian lubricant ($\overline{K}=0.0$). As a consequence of this, journal predicts stable motion for all different bearing geometries. It may be noticed that for an unworn journal bearing, at lower values of an external load, $\overline{W}_0=2.0$ journal motion locus predicts stable motion whereas for higher values of external load $\overline{W}_0=4.0$, nonlinear journal trajectories predict limit cycle or unstable motion for the same operating condition $\overline{M}_j=\overline{M}_c$. Further, it may be observed that two lobe unworn ($\overline{\delta}_W=0.0$) journal bearing with an offset factor greater than one also gives stable motion when operating at an external load $\overline{W}_0=4.0$ with Newtonian lubricant ($\overline{K}=0.0$) for $\overline{M}_j=\overline{M}_c$. As the offset factor increases, it provides maximum bearing clearance than that of a circular bearing; the pressure distribution is concentrated in the narrow region of the gap. As a result of this, the tangential component of fluid film force is balanced by the applied load. Due to this reason, two lobe bearing shows stable motion when bearing operates at an offset factor greater than one. Fig. 7(f)–(h) shows the motion trajectory for $\overline{K}\neq 0.0$ and $\overline{\delta}_W=0.5$at $\overline{W}_0=4.0$. Nonlinear journal motion trajectory traces stable motion for all the cases of $\overline{M}_j=\overline{M}_c$, at, $\overline{\delta}_W=0.5$ for $\delta=1.0, 1.25$. When $\overline{M}_j=\overline{M}_c$, at $\overline{W}_0=4.0$ and $\overline{K}=0.58$, the nonlinear journal motion predicts the stable motion for an offset factor greater than or equal to one as shown in Fig. 7(g). From Fig. 7(e)–(h), it is quite interesting to observe that journal motion predicts stable motion for $\overline{M}_j=\overline{M}_c$ at $\delta\geq 1.0$. The journal operates at higher eccentricity due to increased radial clearance and more lubricant needs to be pumped to support the external load $\overline{W}_0=4.0$ and journal predicts stable motion.

The nonlinear journal motion trajectories for $\overline{M}_J=0.8\overline{M}_c$ is shown in Fig. 8(a)–(h). Fig. 8(a)–(d) shows the non linear trajectories for an unworn journal bearing for $\overline{W}_0=2.0$. Fig. 8(a) depicts that, when unworn ($\overline{\delta}_W=0.0$) bearing operates with Newtonian lubricant ($\overline{K}=0.0$) at $\overline{W}_0=2.0$, nonlinear equation of motion predicts a stable motion for $\delta=0.75, 1.0$ and $\delta=1.25$ when $\overline{M}_J=0.8\overline{M}_c$. It is obvious that, the system is asymptotically stable when $\overline{M}_J<\overline{M}_c$. It may be noticed that the bearing with an offset factor greater than one ($\delta>1.0$) operates at a minimum fluid film thickness when $\overline{M}_J=0.8\overline{M}_c$ at $\overline{W}_0=2.0$. Fig. 8(e)–(h) shows the journal motion path for worn bearing operated with Newtonian and non-Newtonian lubricant. The combined influence of the wear depth parameter ($\overline{\delta}_W$) and non-linearity factor ($\overline{K}$) alters the size and shape of trajectory within the minimum clearance of the bearing when $\overline{M}_J=0.8\overline{M}_c$. The combined effect of wear depth parameter ($\overline{\delta}_W=0.5$) and non-linearity parameter ($\overline{K}=0.1$) have a significant influence at $\overline{W}_0=2.0$ on journal motion stability when bearing operates at an offset factor $\delta=0.75$ and 1.0. The plot for nonlinear trajectories for $\overline{W}_0=2.0$ and $\overline{K}=0.1$ are shown in Fig. 8 (f). According to nonlinear analysis, system remains stable for $\overline{M}_J=0.8\overline{M}_c$. As anticipated, when the journal mass is less than the critical journal mass, the journal trajectory predicts the stable motion.

The same trend has been observed for $\overline{M}_J=0.8\overline{M}_c$ when journal bearing operates in non-Newtonian lubricant ($\overline{K}=1.0$) as shown in Fig. 8(h).

The plot for the nonlinear journal motion trajectory for $\overline{M}_J=0.8\overline{M}_c$ and $\overline{W}_0=4.0$ is shown in Fig. 9(a)–(h). Fig. 9(a) shows the journal motion trajectory for unworn journal bearing lubricated with Newtonian lubricant. It may be observed that journal motion predicts the stable motion for $\delta=0.75, 1.0$ and $\delta=1.25$. As the lubricant becomes more nonlinear, at higher load $\overline{W}_0=4.0$, the journal motion trajectory shows the limit cycle motion for an offset factor less than one ($\delta=0.75$). Further it may be observed that from Fig. 9(b), journal traces stable motion when $\delta\geq 1.0$ for$\overline{M}_J=0.8\overline{M}_c$ at $\overline{W}_0=4.0$. This is due to change in bearing geometry and higher value of external load ($\overline{W}_0=4.0$). A small change in $\overline{K}$ or $\overline{W}_0$ or both can make the bearing system either unstable or stable. Non-linear equation of motion predicts the stable motion for the high value of an external load ($\overline{W}_0$) when $\overline{K}=0.58$ and $\overline{K}=1.0$ for unworn journal bearing. This results are anticipated when the journal mass is less than the critical mass ($\overline{M}_J<\overline{M}_c$). The nonlinear journal trajectory motion for damaged out journal bearing operated with Newtonian and non-Newtonian lubricant is shown in Fig. 9(e)–(h).

For a worn out bearing, as the wear parameter increases to 50% of radial clearance i.e. ($\overline{\delta}_W=0.5$), the stability also increases. Further it may be noticed that an increase in non-linearity factor ($\overline{K}$) decreases the lubricant viscosity and traces a small orbit when $\overline{M}_J<\overline{M}_c$. Fig. 9(e) shows the motion trajectory for $\overline{K}=0.0$ and $\overline{\delta}_W=0.5$ at $\overline{W}_0=4.0$. Nonlinear journal motion trajectory traces stable motion for all the cases of $\overline{M}_J=0.8\overline{M}_c$. Increase in radial clearance due to wear ($\overline{\delta}_W=0.5$) may enhance the stability of the bearing at higher load when bearing operates with Newtonian lubricant ($\overline{K}=0.0$). A similar stability trend was reported in the study of [10] for worn out journal bearing.

From Figs. (8) and (9), it may be noticed that, at an external load $\overline{W}_0 = 2.0$ and $\overline{W}_0 = 4.0$, the journal motion gives stable motion for $\overline{M}_J = 0.8\overline{M}_c$ when bearing operates with nonlinearity factor $\overline{K} = 0.58$ and $\overline{K} = 1.0$. Furthermore, it may be observed that, at higher values of external load $\overline{W}_0 = 4.0$, the journal trajectory traces stable motion when $\overline{M}_J = 0.8\overline{M}_c$ It may be noticed that the bearing with an offset factor greater than one $(\delta > 1.0)$ operates at a minimum fluid film thickness when $\overline{M}_J = 0.8\overline{M}_c$ at $\overline{W}_0 = 2.0$and 4.0.

Fig. 10(a)–(h) shows the trajectories of journal centre motion for $\overline{W}_0 = 2.0$ and $\overline{M}_J = 1.1\overline{M}_c$.

The journal motion trajectory for unworn journal bearing at $\overline{K} = 0.0$ is shown in Fig. 10(a) for $\overline{M}_J = 1.1\overline{M}_c$, journal motion shows a stable motion. This is due to the change in bearing geometry at lower values of external load $\overline{W}_0 = 2.0$.The change in bearing geometry helps to provide high pressure in the convergent region and stabilizes a journal motion by balancing the dynamic fluid film forces in the angular direction. Further it may be noticed that the journal traces larger orbit when compared to a journal trajectory of $\overline{M}_J = \overline{M}_c$ and $\overline{M}_J = 0.8\overline{M}_c$ for the same operating parameter (Ref. Figs. 6 and 8(a)).

As the lubricant behavior become nonlinear, the value of $\overline{h}_{\min}$ is reduced and journal traces a larger orbit and position of its maximum values shifts towards the left. Further it may be observed that the viscosity of the lubricant decreases as the lubricant become more nonlinear, this results in shear thinning of the lubricant and bearing operates at lower values of minimum fluid film thickness. Therefore, for $\overline{M}_J = 1.1\overline{M}_c$, the journal predicts limit cycle motion. Fig. 10(c) shows the nonlinear journal motion trajectories for $\overline{W}_0 = 2.0$ and $\overline{K} = 0.58$. At lower value of external load $(\overline{W}_0 = 2.0)$, journal trajectories predict limit cycle motion for$\delta = 0.75, 1.0$ and the unstable motion for $\delta = 1.25$when $\overline{M}_J = 1.1\overline{M}_c$. Further Fig. 10(c) indicates that as the non-linear behavior of the lubricant increases $(\overline{K} = 0.58)$, the journal traces a larger orbit for $\overline{M}_J = 1.1\overline{M}_c$. This could be due to a reduction in the value of lubricant viscosity. Journal trajectory yields unstable motion when $\overline{M}_j = 1.1\overline{M}_c$ at an offset factor $\delta = 1.25$whereas, form limit cycle motion for the same operating condition at $\delta \leq 1.0$ as shown in Fig. 10(d).

The combined influence of damaged bearing i.e.; wear depth parameter $(\overline{\delta}_W = 0.5)$ and non-Newtonian lubricant $(\overline{K})$ on the stability response of symmetric hole entry hybrid journal bearing have been analyzed and presented in Fig. 10(f)–(h). The trajectories for $\overline{W}_0 = 2.0$, $\overline{K} = 0.0$ and$\overline{\delta}_W = 0.5$ are shown in Fig. 10(e). When the journal bearing system operates with Newtonian lubricant $(\overline{K} = 0.0)$ for an offset factor $\delta \geq 1.0$ and $\overline{W}_0 = 2.0$, journal trajectory yields stable motion, when $\overline{M}_j = 1.1\overline{M}_c$. However, journal motion trajectory gives unstable motion for an offset factor $\delta = 0.75$. From Fig. 10(e), it may be observed that journal traces the small size circular orbit for an offset factor $\delta = 1.25$.This is because of an increase in wear parameter to 50% of radial clearance $(\overline{\delta}_W = 0.5)$. As consequence of this, the bearing runs at high radial clearance and more lubricant is pumped. The combined influence of the wear depth parameter and non linearity factor alters the size and shape of trajectory within the minimum clearance of the bearing when $\overline{M}_J = 1.1\overline{M}_c$.

The combined influence of wear depth parameter $(\overline{\delta}_W = 0.5)$ and non-linearity parameter $(\overline{K} = 0.1)$ have a noteworthy influence at $\overline{W}_0 = 2.0$ on journal motion stability when bearing operates at an offset factor $\delta = 0.75$ and 1.0. The increase in non-linearity parameter and wear depth parameter at $\overline{M}_J = 1.1\overline{M}_c$, makes the system unstable. Further, it may be observed that, the stability of the system obtained by non-linear equation of motion trajectories depend on bearing geometry. From Fig. 10(f) it may be seen that, as the lubricant become more non-linear, provide unstable and limit cycle motion for damaged worn out bearing at $\overline{W}_0 = 2.0$.Further, from Fig. 10(g) and (h), it may be noticed that for an offset factor greater than one, nonlinear journal motion provides the stable motion. This is due to large clearance in the bush. Whereas $\delta \leq 1.0$ predict unstable motion when $\overline{M}_J = 1.1\overline{M}_c$ and $\overline{\delta}_W = 0.5$ at$\overline{K} = 0.58$ and 1.0. Further it can be observed that as the value of the journal mass increases, the size of the journal center increases when bearing operate at low values of an external load.

For the bearing operating at a value of external load $\overline{W}_0 = 4.0$, nonlinear journal motion trajectories for two lobe symmetric hybrid journal bearing is shown in Fig. 11(a)–(h) for $\overline{M}_J = 1.1\overline{M}_c$. Fig. 11(a) shows the non linear trajectories for an unworn two lobe symmetric hole entry journal bearing with Newtonian lubricants for $\overline{K} = 0.0$ and $\overline{W}_0 = 4.0$. Nonlinear equation of motion predicts an unstable motion when the journal mass is greater than the critical mass $(\overline{M}_J = 1.1\overline{M}_c)$. Fig. 11(c) indicates that as the non-linear behavior of the lubricant increases $(\overline{K} = 0.58)$, the journal traces a larger orbit for$\overline{M}_J = 1.1\overline{M}_c$. This is due to decrease in viscosity of the lubricant. As expected, the system is unstable when $\overline{M}_J = 1.1\overline{M}_c$ for higher value of an external load $\overline{W}_0 = 4.0$. Therefore, when the bearing operates at higher values of external load $(\overline{W}_0)$ with non-Newtonian lubricant, more attention may be sought [Ref. Fig. 11(b) and (d)]. Journal trajectory yields unstable motion when $\overline{M}_j = 1.1\overline{M}_c$ at an offset factor $\delta \geq 1.0$ whereas form limit cycle motion for the same operating condition at $\delta \leq 0.75$ as shown in Fig. 11(d). It may be observed that the non- linear behavior of the lubricant have more influence on stability at higher values of an external load. Therefore, for a symmetric hole entry journal bearing, operated by non-Newtonian lubricant, bearing designer may take care of an external load to get improved journal stability. The unworn two lobe symmetric journal bearing may be operated an external load of $\overline{W}_0 \leq 2.0$.

Fig. 11(e) shows the trajectory motion for $\overline{K} = 0.0$ and $\overline{\delta}_W = 0.5$ at $\overline{W}_0 = 4.0$. Nonlinear journal motion trajectory traces stable motion for $\overline{M}_j = 1.1\overline{M}_c$. At, $\overline{\delta}_W = 0.5$ the journal operates at higher eccentricity, the more lubricant is pumped to support the external load $\overline{W}_0 = 4.0$ and journal predicts stable motion. Further it may be noticed that from Fig. 11(e) that, the increase in radial clearance due to worn out $(\overline{\delta}_W = 0.5)$ may enhance the stability of the bearing at higher load when bearing operates with Newtonian lubricant $(\overline{K} = 0.0)$.

The journal motion trajectories are shown in Fig. 11(f) for $\overline{K} = 0.1$, $\overline{\delta}_W = 0.5$ and $\overline{W}_0 = 4.0$. The noteworthy observation may be revealed that journal motion predicts stable motion for an offset factor greater than one when $\overline{M}_J = 1.1\overline{M}_c$. This is due to change in bearing geometry and increase in the wear depth parameter $(\overline{\delta}_W = 0.5)$ at an offset factor, $\delta = 1.25$ which provides large eccentricity. Further it may be noticed that for $\delta = 0.75$ and 1.0; nonlinear journal motion trajectory forms limit cycle motion. In this case of a larger value of an external load, the wear depth parameter is having a significant effect than the non-linear factor $(\overline{K} = 0.1)$ on bearing stability. When $\overline{M}_j = 1.1\overline{M}_c$, at $\overline{W}_0 = 4.0$ and $\overline{K} = 0.58$, the nonlinear journal motion predict stable motion for $\overline{M}_J = 1.1\overline{M}_c$, when an offset factor greater than one and forms limit cycle motion for $\delta = 0.75$ and 1.0 as shown in Fig. 11(g). If the wear defect is significant, i.e. 50% of bearing clearance, the amount of lubricant accumulated in the worn out zone is more due to large eccentricity ratio for an offset factor $\delta = 1.25$.This causes to float journal towards the upward side for an external load off $\overline{W}_0 = 4.0$ and provide more stable motion. Further, it may be observed from Fig. 11(g) and (h) that, an increase in the nonlinear behavior of the lubricant from $\overline{K} = 0.58$ to $\overline{K} = 1.0$ shows marginal effect on the bearing stability and the same trend is observed when worn bearing $(\overline{\delta}_W = 0.5)$ operates with the $\overline{K} = 1.0$. At higher values of wear depth parameter $(\overline{\delta}_W = 0.5)$, the journal shows the improved stability motion and it has significant influence on the bearing stability.

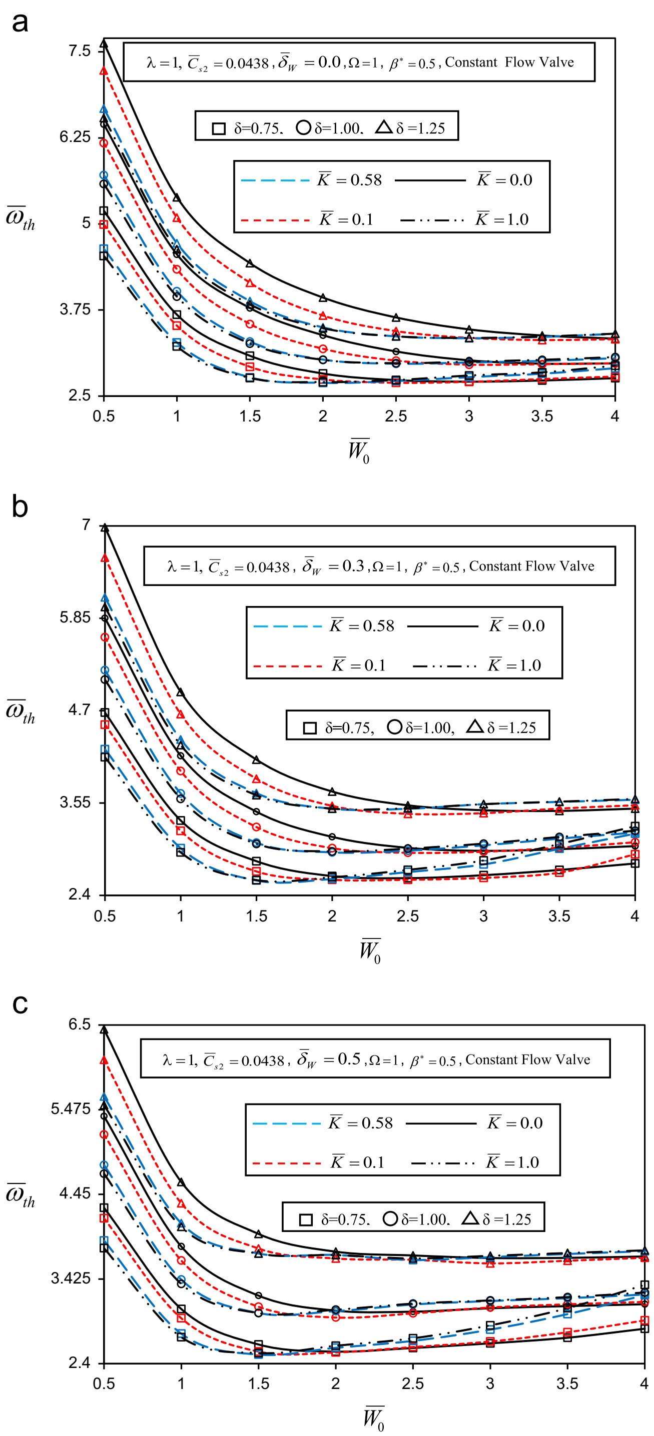

Fig. 12. (a) Variation of $\overline{\omega}_{th}$ with respect to $\overline{W}_0$ for $\overline{\delta}_W = 0.0$, (b) Variation of $\overline{\omega}_{th}$ with respect to $\overline{W}_0$ for $\overline{\delta}_W = 0.3$ and (c) Variation of $\overline{\omega}_{th}$ with respect to $\overline{W}_0$ for $\overline{\delta}_W = 0.5$.

The influence of Newtonian and non-Newtonian lubricant on the threshold speed margin ($\overline{\omega}_{th}$) for different values of an external load ($\overline{W}_0$) and for the various values of an offset factor (δ) has been shown in Fig. 12(a). Fig. 12(a) indicates that threshold speed margin ($\overline{\omega}_{th}$) deteriorates steadily as the lubricant become more nonlinear. As the non-linearity factor $\overline{K}$ increases, the viscosity of the lubricant decreases which causes the change in the dynamic coefficient of the journal bearing for the chosen value of restrictor design parameter ($\overline{C}_{s2}$). Further, the result indicates that, this loss of stability threshold speed margin $\overline{\omega}_{th}$ due to non linear behavior of the lubricant is partially compensated in case of non circular journal bearing for the value of an offset factor greater than one ($\delta = 1.25$). Further, it may be revealed that, there is very marginal change in the value of stability threshold speed margin when bearing operates with $\overline{K} = 0.58$ to $\overline{K} = 1.0$. Further it may be noticed that, for the value of $\delta = 1.25$, the stability threshold speed margin remains higher than that of a circular journal bearing. (i.e. $\delta = 1.0$).It may be observed that at higher values of an external load ($\overline{W}_0 \geq 2.5$), non linearity factors ($\overline{K}$) have marginal effect on the stability threshold speed margin and reduction in the value of $\overline{\omega}_{th}$occurred only due to changes in bearing geometry.

Fig. 12(b) depicts the variation of $\overline{\omega}_{th}$ with respect to$\overline{W}_0$ for the wear depth parameter ($\overline{\delta}_W = 0.3$). It may be observed that stability threshold speed margin ($\overline{\omega}_{th}$) decreases as the non-linear factor ($\overline{K}$) increases. Further it may be seen from Fig. 12(a) and (b), when bearing operates at an offset factor $\delta = 1.25$ with Newtonian lubricant, the stability threshold margin decreased by the order of 7.5% for an external load of $\overline{W}_0 = 2.0$ due to wear depth parameter ($\overline{\delta}_W = 0.3$). Further same trend is observed for the different bearing geometries ($\delta = 1.0$ and 0.75) for non Newtonian lubricant ($\overline{K} \neq 0$). It may be noticed that at an external load $\overline{W}_0 = 2.0$, the stability threshold speed margin gets enhanced for nonlinearity factor $\overline{K} = 0.58$ and $\overline{K} = 1.0$.

Variation of $\overline{\omega}_{th}$ with respect to $\overline{W}_0$ for the wear depth parameter ($\overline{\delta}_W = 0.5$) is shown in Fig. 12(c). From Fig. 12(c), It may be observed that stability threshold speed margin ($\overline{\omega}_{th}$) reduces as the lubricants viscosity decreases due to increase in nonlinearity factor. Further it may be seen, when $\overline{\delta}_W$ is the 50% of the radial clearance and bearing operates at an offset factor $\delta = 1.25$ with non-Newtonian lubricant $\overline{K} = 0.1$, the stability threshold margin gets decreased by the order of 10.0% for an external load of $\overline{W}_0 = 1.5$ when compared to unworn journal bearing operating with non-Newtonian lubricant. Therefore it may be concluded that non-linearity factor ($\overline{K}$) has a normal effect at lower values of an external load ($\overline{W}_0 < 2.0$) and marginal at high value of an external load ($\overline{W}_0 \geq 2.0$). Further it may be noticed that at wear depth parameter ($\overline{\delta}_W$) significantly affects the stability threshold speed margin ($\overline{\omega}_{th}$). The percentage change in $\overline{\omega}_{th}$ due to the combined influence of non-linearity factor ($\overline{K}$) and wear depth parameter $\overline{\delta}_W$ have been shown in Table 2 for the sake of clarity.

Variation of $\overline{\omega}_{th}$ with respect to $\overline{\delta}_W$ for an external load $\overline{W}_0 = 2.0$ is shown in Fig. 13(a)–(c) for different values of an offset factor (δ). It may be observed that the stability threshold speed margin deteriorates significantly as the wear depth parameter increases ($\overline{\delta}_W$) for Newtonian and non-Newtonian lubricant. Further, it may be noticed that the reduction in the value of $\overline{\omega}_{th}$ is observed as when $\overline{\delta}_W$ is the 30% of the radial clearance. As the $\overline{\delta}_W$ increase from 0.3 to 0.5 for an offset factor 1.25, the stability margin gets improved due to nonlinearity factor ($\overline{K} = 0.58$ and 1.0).The same trend is also observed for circular two lobe symmetric hole hybrid journal bearing as shown in Fig. 13(b). Fig. 13

Table 2
Performance change in percentage for $\overline{\omega}_{th}$ for a circular symmetric hole entry journal bearing.

$\overline{W}_0$	$\overline{K} = 0.0$ $\overline{\delta}_W = 0.0$	$\overline{K} = 1.0$ $\overline{\delta}_W = 0.0$	$\overline{K} = 0.0$ $\overline{\delta}_W = 0.3$	$\overline{K} = 1.0$ $\overline{\delta}_W = 0.3$	$\overline{K} = 0.0$ $\overline{\delta}_W = 0.5$	$\overline{K} = 1.0$ $\overline{\delta}_W = 0.5$
0.5	–	**−13.43**	**−9.24**	**−21.10**	**−16.31**	**−27.10**
1	–	**−13.44**	**−9.19**	**−20.97**	**−16.19**	**−26.15**
1.5	–	**−13.65**	**−8.86**	**−19.50**	**−14.63**	**−20.24**
2	–	**−10.55**	**−7.47**	**−12.88**	**−10.05**	**−9.72**
2.5	–	**−5.41**	**−4.89**	**−5.24**	**−3.68**	**−0.635**
3	–	**−0.59**	**−2.05**	**0.86**	**1.78**	**4.90**
3.5	–	**1.91**	**0.264**	**5.321**	**4.34**	**7.98**
4	–	**3.093**	**1.37**	**7.76**	**4.94**	**9.68**

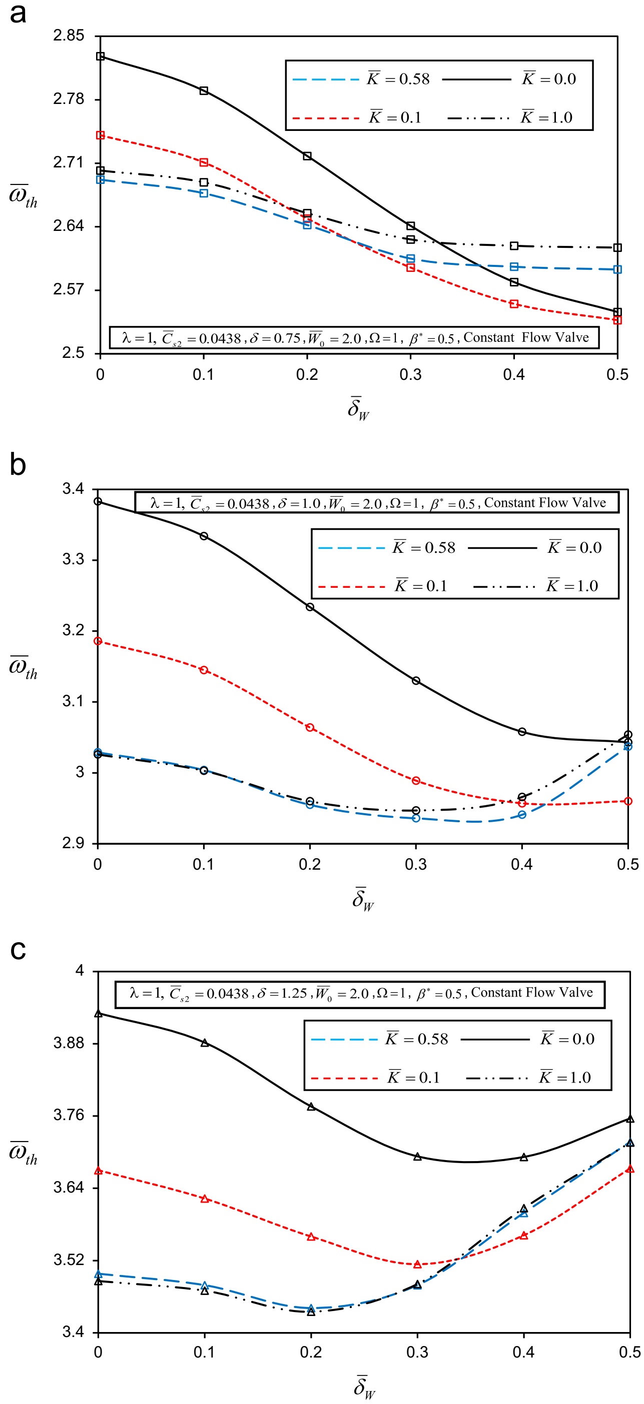

Fig. 13. (a) Variation of $\overline{\omega}_{th}$ with respect to $\overline{\delta}_W$ for $\delta = 0.75$; (b) variation of $\overline{\omega}_{th}$ with respect to $\overline{\delta}_W$ for $\delta = 1.0$ and (c) variation of $\overline{\omega}_{th}$ with respect to $\overline{\delta}_W$ for $\delta = 1.25$.

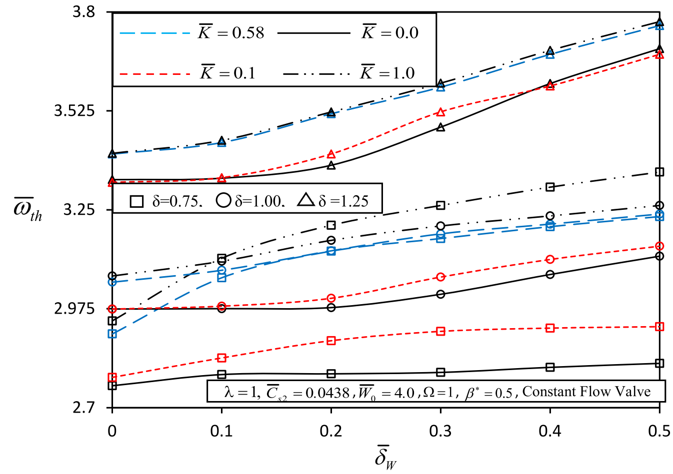

Fig. 14. Variation of $\overline{\omega}_{th}$ with respect to $\overline{\delta}_W$

(a)–(c) depicts that the bearing with an offset factor greater than one provides a higher value of stability threshold speed margin for the same operating parameters.

Variation of $\overline{\omega}_{th}$ with respect to $\overline{\delta}_W$, is shown in Fig. 14 for an external load $\overline{W}_0 = 4.0$ for different values of an offset factor (δ). It may be observed that the stability threshold speed margin increases as the wear depth parameter increases ($\overline{\delta}_W$) for Newtonian and non-Newtonian lubricant for $\delta \geq 1.0$. Further it may be noticed that the decrease in the value of $\overline{\omega}_{th}$ is observed at $\delta = 0.75$ for $\overline{K} = 0.1$ and $\overline{K} = 0.0$. As the $\overline{\delta}_W$ increase from 0.2 to 0.5, the stability margin gets improved due to nonlinearity factor ($\overline{K} = 0.58$ and 1.0).This is due to the high value of an external load $\overline{W}_0 = 4.0$ and more radial clearance. As a consequence of this, more lubricant is needed to operate the bearing. It may be observed that the bearing with $\delta \geq 1.0$ provides a higher value of stability threshold speed margin when $\overline{\delta}_W$ is more than 20% of the radial clearance.

To have a better physical insight on journal motion, the nonlinear stability threshold speed margin ($\overline{\omega}_{th}$) have been presented for circular symmetric hole entry hybrid journal bearing at which the system reaches a critical state or nonlinear journal motion trajectories shows the limit cycle is shown in Fig. 15(a) and (b).

From Fig. 15(a) and (b), it may be revealed that the stability of the journal bearing system may be enhanced by either by choosing the appropriate wear depth parameter ($\overline{\delta}_W$) and non-linearity factor ($\overline{K}$) for various values of an external load.

5. Conclusion

The present study investigated the combined influence of wear and non-Newtonian lubricant on the stability of a two lobe constant flow valve compensated symmetric hole-entry journal bearing. The non-linear trajectories of the journal centre motion have been obtained to address the stability of the bearing system. Based on the numerically simulated results obtained from the study, the following silent conclusions have been drawn:

The combined influence of non-linearity factor ($\overline{K}$) and wear depth parameter ($\overline{\delta}_W$) have a profound effect on nonlinear journal center motion.

On the stability threshold speed margin ($\overline{\omega}_{th}$), the non-linearity factor ($\overline{K}$) have a noteworthy effect at lower values of an external load ($\overline{W}_0 < 2.0$) and marginal effect on the high value of an external load ($\overline{W}_0 \geq 2.0$) for worn hybrid journal bearing.

It may be observed that the stability threshold speed margin increases as the wear depth parameter increases ($\overline{\delta}_W$) for Newtonian and non-Newtonian lubricant for offset factor $\delta \geq 1.0$ at a high value of an external load ($\overline{W}_0 \geq 2.0$).

The non linear analysis predicts higher safety margin(the stability point of view) for offset factor greater than one for higher values of wear depth parameter ($\overline{\delta}_W$) and an external load ($\overline{W}_0$).

The notable observation made from the nonlinear journal motion study, when $\overline{M}_J = 1.1\overline{M}_c$ and an offset factor greater than one ($\delta > 1.0$), the journal center trajectories predict stable motion for $\overline{\delta}_W = 0.5$ at higher values of an external load and for non-Newtonian lubricant.

The influence of wear depth parameter ($\overline{\delta}_W$) and non-linearity factor ($\overline{K}$) on the stability of the journal bearing system can be

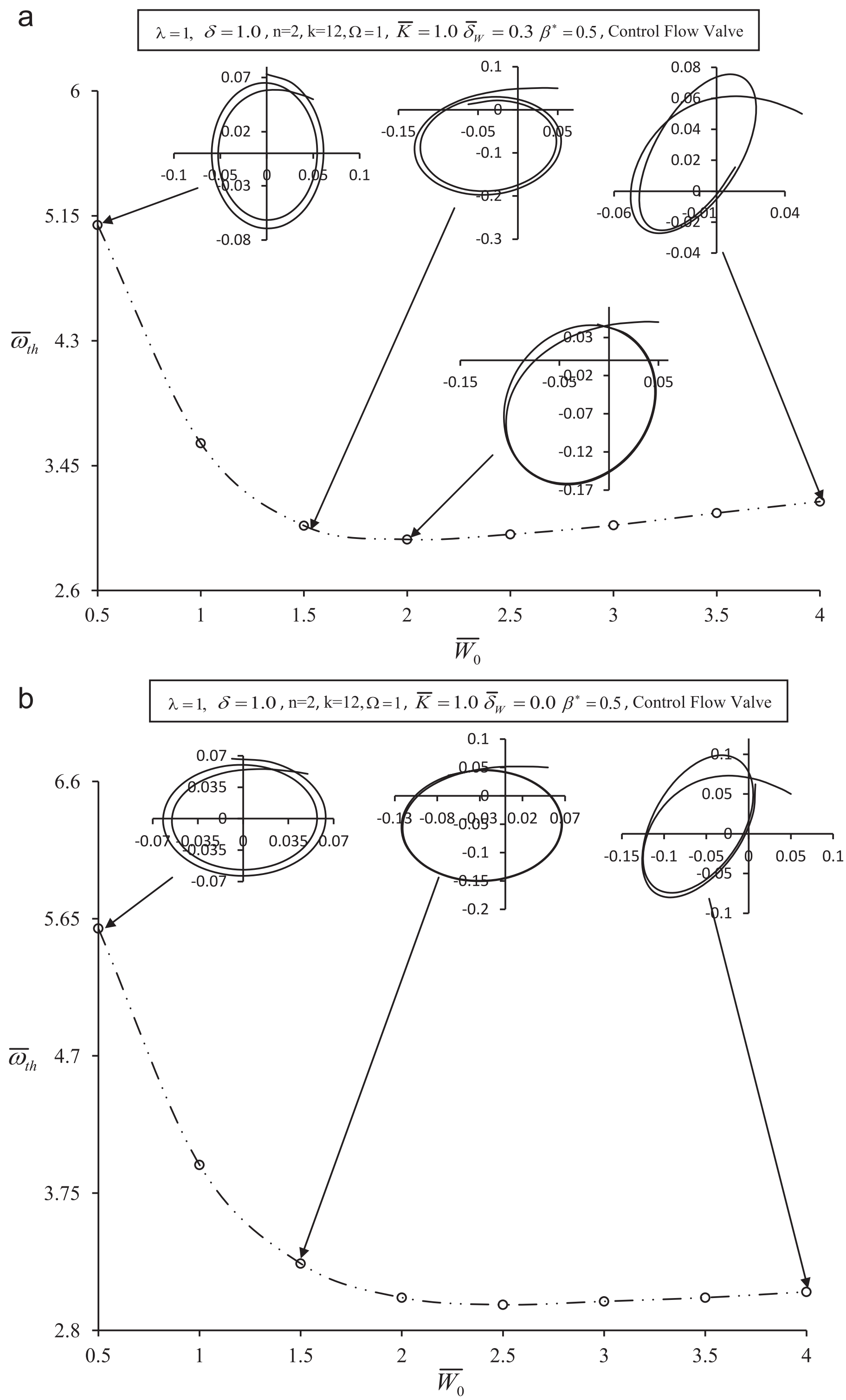

Fig. 15. (a) Variation of $\overline{\omega}_{th}$ with respect to $\overline{W}_0$ for $\overline{K}=1.0$ and $\overline{\delta}_W=0.3$ and (b) variation of $\overline{\omega}_{th}$ with respect to $\overline{W}_0$ for $\overline{K}=1.0$ and $\overline{\delta}_W=0.0$.

reduced by proper selection of an offset factor (δ). The journal motion nonlinear stability viewpoint, The designer can choose a proper bearing geometry from the following criterion:

$\overline{\omega}_{th}|_{\delta=1.25} > \overline{\omega}_{th}|_{\delta=1.0} > \overline{\omega}_{th}|_{\delta=0.75}$.

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