Multi-objective optimization of electrochemical machining process parameters using a particle swarm optimization algorithm

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The manuscript was received on 3 March 2008 and was accepted after revision for publication on 1 May 2008.

DOI: 10.1243/09544054JEM1158

Abstract: The selection of optimum values of important process parameters of electrochemical machining processes such as the tool feed rate, electrolyte flow velocity, and applied voltage play a significant role in optimizing the measures of process performance. These performance measures generally include dimensional accuracy, tool life, material removal rate, and machining cost. In this paper, a particle swarm optimization algorithm is presented to find the optimal combination of process parameters for an electrochemical machining process. The objectives considered are dimensional accuracy, tool life, and the material removal rate subjected to the constraints of temperature, choking, and passivity. Both single- and multi-objective optimization aspects are considered. The results of the proposed algorithm are compared with the previously published results obtained by using other optimization techniques.

Keywords: electrochemical machining, multi-objective optimization, particle swarm optimization

1 INTRODUCTION

Traditional machining processes, such as turning, grinding, drilling, milling, etc., remove material by chip formation, abrasion, or microchipping. There are situations, however, where these processes are not satisfactory, economical, or even possible, for the following reasons [**1**]:

1. The hardness and strength of the material is very high (typically above 400 HB) or the material is too brittle.
2. The workpiece is too flexible, slender, or delicate to withstand the cutting or grinding forces, or the parts are too difficult to fix.
3. The shape of the part is complex.
4. Surface finish and dimensional tolerance requirements are more rigorous than those obtained by other processes.
5. Temperature rise and residual stresses in the workpiece are not desirable or acceptable.

These requirements have led to the development of chemical, electrochemical, thermal, electrothermal, mechanical, and other means of material removal. Beginning in the 1940s, these advanced methods are called non-traditional or unconventional machining processes. Over the last four decades, there has been a large increase in the number of non-traditional machining processes (NTMP). Today, non-traditional machining processes with vastly different capabilities and specifications are available for a wide range of applications. These processes are classified according to the nature of energy employed in machining, as discussed below:

(a) chemical and electrochemical processes like chemical milling, electrochemical machining, electrochemical grinding, electrochemical honing, etc.;
(b) thermal and electrothermal processes like electric discharge machining, laser beam machining,

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plasma arc machining, electron beam machining, ion beam machining, etc.;
(c) mechanical processes like ultrasonic machining, abrasive jet machining, water jet machining, etc.;
(d) hybrid processes like electrochemical discharge grinding, abrasive electrical discharge machining, vibration-assisted electrochemical machining, etc.

Among various advanced machining processes mentioned above, the electrochemical machining (ECM) process is one of the most highly developed processes and the present study is mainly focused on this process. The basis of the ECM process is the phenomenon of electrolysis, whose laws were established by Faraday in 1833. The principle and equipment used in the ECM process are illustrated in Fig. 1 [**2**]. The workpiece and tool are the anode and cathode, respectively, of an electrolytic cell, and a constant potential difference (usually about 5–30 V) is applied across them, producing a high current density of 10–200 A/cm^2. A suitable electrolyte (NaCl or $NaNO_3$ aqueous solution) is chosen so that the cathode shape remains unchanged during electrolysis. The electrolyte is pumped at a rate of 3–60 m/s, through the gap between the electrodes, to remove the machining waste (i.e. dissolved material, usually metal hydroxide) and to diminish unwanted effects such as those that arise with cathodic gas generation and electrical heating. The rate at which metal is then removed from the anode is approximately in inverse proportion to the distance between the electrodes. As machining proceeds, and with the simultaneous movement of the cathode at a typical rate, e.g. 0.02 mm/s towards the anode, the gap width along the electrode length will gradually tend to reach a steady state value. Under these conditions, a shape that is approximately a negative mirror image of the cathode will be reproduced on the anode as the cathode does not alter during the ECM process. A typical gap width then can be about 0.4 mm.

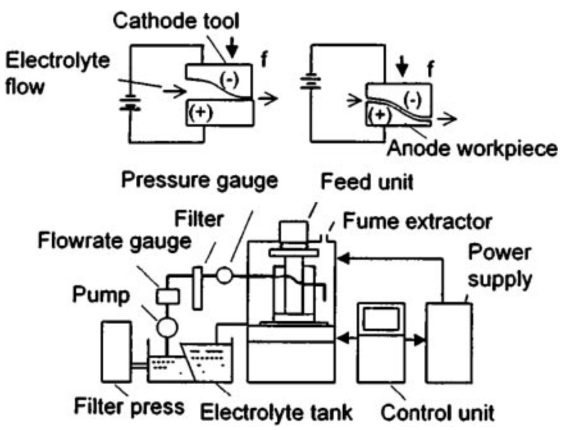

Fig. 1 ECM principle and equipment [**2**]

The ECM process can handle a large variety of materials, limited only by their electrochemical properties and not by their strength. This process is characterized by high metal removal rates for high-strength and difficult-to-machine alloys. Fragile parts that are not easily machinable can be shaped by the ECM process. Certain characteristics of the ECM process, such as the ability to machine three-dimensional curved surfaces without the striation marks, stress-free and burr-free machining, no thermal damage to the workpiece, and ideally no tool wear, etc., make this process widely applicable.

However, the main limitation of the ECM process is the high initial investment along with high power consumption and large floor space requirement. Therefore use of this process is a costly affair. This problem is further compounded by the corrosion, toxicity, and safety-related problems of the electrochemical machining process. Also, electrochemical machining is a complex process and it is difficult to predict the changes that may occur in the interelectrode gap. The electrolyte properties vary due to the emission of a considerable amount of heat and gas bubbles. In addition, hydrodynamic parameters, such as pressure, also vary along the electrolyte flow direction and make the analysis quite complicated. It is therefore essential to make a careful decision during process planning before using electrochemical machining for practical purposes. A human process planner selects proper machining process parameters using his or her own experience or from the handbooks. However, these parameters do not give an optimal result. The selection of optimum process parameters plays a significant role in improving the process performance and process economics by reducing various costs.

The next section presents a brief review of the past research work done on the optimization of ECM process parameters.

2 REVIEW OF PAST RESEARCH WORK

Bhattacharyya *et al.* [**3**] proposed a two-dimensional interelectrode gap model in which maximization of the metal removal rate was considered as the objective function with the tool feed rate and electrolyte flow velocity as the design variables. The three constraints considered were temperature, passivity, and choking. However, the authors had considered only a single-objective optimization problem and solved the same using a graphical solution technique, which, in itself, was less accurate. This model was also based on many simplified assumptions, such as the constant void fraction, electrolyte conductivity as a function of the void fraction only, and constant electrolyte pressure along its flow path. Furthermore, no variable bounds were used.

El-Dardery [**4**] proposed a cost model of the ECM process considering various costs involved in the process. The cost equation was arranged in terms of decision variables, namely feed rate, electrolyte flowrate, and voltage. The optimum values of the decision variables were obtained by partial differentiation of the cost equation with respect to the decision variables. However, as no constraints were considered in this model, the values of decision variables obtained were not practical.

Hewidy *et al.* [**5**] analysed the components of ECM cost (such as costs of power consumption, machining, electrolyte, and labour) with the objective to set out the basic principles for selecting a suitable electrochemical machine to meet the local production requirements of a company. The authors mentioned the impossibility of having a generalized model for this purpose. In another work, Hewidy *et al.* [**6**] modelled the performance of ECM assisted by low-frequency vibrations using an analytical approach.

Acharya *et al.* [**7**] considered the multi-objective optimization model for the ECM process with maximization of the material removal rate, minimization of dimensional inaccuracy, and maximization of tool life as three conflicting objectives. The decision variables were the tool feed rate, electrolyte flow velocity, and applied voltage. The constraints used in this model were temperature constraint, passivity constraint, and choking constraint. The optimization problem was solved by goal programming after linearizing the objective functions and constraint equations by regression analysis. This model overcame the limitations of the model proposed by Bhattacharyya *et al.* [**3**]. However, it did not include the variable bounds for feed rate and differences in the interelectrode gap.

The drawbacks of the model proposed by Acharya *et al.* [**7**] were overcome by Choobineh and Jain [**8**]. The authors had considered only two objective functions, i.e. maximization of the material removal rate and maximization of dimensional accuracy. The third objective to maximize the tool life was eliminated as tool life is overachieved in most practical cases. They used the vertex method to find appropriate distribution of the objective functions. The modified goal-programming problem was then solved in the same manner as in Acharya *et al.* [**7**].

Jain and Jain [**9**] formulated the optimization model based on the analysis given in Acharya *et al.* [**7**] with certain modifications, i.e. expanding the variable bound ranges for the tool feed rate and electrolyte flow velocity but without linearizing the objective functions and constraints. The optimization problem was then solved using a genetic algorithm. However, the authors had considered only a single-objective optimization problem, i.e. to minimize the dimensional inaccuracy. Also the passivity constraint was violated in their approach. Furthermore, the genetic algorithm has its own limitations, such as the risk of replacement of a good parent string with the deteriorated child, less convergence speed, and difficulty in selecting the controlling parameters such as population size, crossover rate, and mutation rate.

It is observed from the review of past work that the graphical solution technique and mathematical programming techniques like goal programming, partial differentiation, etc., had been used to solve the problem of optimization of process parameters of electrochemical machining. However, these traditional methods of optimization do not fare well over a broad spectrum of problem domains. Moreover, traditional techniques are not robust. Due to the complex nature of the optimization problem, these techniques are not ideal for solving these problems, as they tend to obtain a local optimal solution. To overcome the drawbacks of traditional optimization techniques, researchers are now using evolutionary optimization techniques. Evolutionary computation consists of a variety of methods including optimization paradigms that are based on evolution mechanisms such as biological genetics and natural selection. These methods use the fitness information instead of the functional derivatives, making them more robust and effective. The most commonly used non-traditional optimization technique is the genetic algorithm. However, this method provides a near-optimal solution for a complex problem having large numbers of variables and constraints. This is mainly due to difficulty in determining optimum controlling parameters. Therefore efforts are continuing to use more recent optimization algorithms, which are more powerful, robust, and able to provide accurate solution. This paper is intended to apply one of such recently developed optimization algorithms, known as particle swarm optimization, for optimization of process parameters of electrochemical machining.

In the present work, an effort is made to verify whether any improvement in the solution is possible by employing some other recent optimization techniques such as particle swarm optimization to the same optimization model. Particle swarm optimization (PSO) is reported to be the better algorithm for continuous optimization as well as discrete optimization problems [**10–12**]. Hence, PSO is considered in this work for single-objective optimization and multi-objective optimization of electrochemical machining process parameters. The optimization model given in

Acharya *et al.* [**7**] is also considered by expanding the variable bound ranges for the tool feed rate and electrolyte flow velocity.

3 OPTIMIZATION MODEL OF ELECTROCHEMICAL MACHINING

Formulation of the optimization model is the most important task in the optimization process. It involves identifying decision variables to be optimized, expressing the objective functions and constraints as functions of decision variables, setting up the bounds for decision variables, and finally expressing the optimization problem as a mathematical model in a standard format that can be directly solved by the optimization algorithm. The optimization model for the electrochemical machining process is formulated in the present work based on the analysis given by Acharya *et al.* [**7**]. The three decision variables considered for this model are tool feed rate $f\,(\mu\text{m/s})$, electrolyte flow velocity $U(\text{cm/s})$, and applied voltage V.

3.1 Objectives

The various objectives under consideration are described below.

3.1.1 Maximization of the material removal rate

This is the product of projected area and tool feed rate. Maximization of the tool feed rate would maximize the material removal rate (MRR) since the projected area is constant. Thus

$$\text{MRR}_{\max} = f_{\max} \tag{1}$$

3.1.2 Maximization of dimensional accuracy

ECM is perhaps the only machining process that does not allow the workpiece dimensions to be checked in the course of machining. Although few techniques such as ultrasonic measurement of the interelectrode gap can be used [**13**], it is necessary to predetermine the control parameters to ensure the desired dimensional accuracy. Dimensional accuracy depends upon the difference in the interelectrode gap from inlet Y_i to outlet Y_o, which is given by

$$Y_o - Y_i = \left(\frac{K_o}{K_i} - 1\right)\frac{K_i M_w \eta_i V}{\rho_w Z_w F f} \tag{2}$$

$$\frac{Kx}{Ki} = \left(1 - \alpha_x'\right)^n [1 + \alpha(T_x - T_i)] \tag{3}$$

with $K_o = K_x$ at the outlet. The objective of maximizing the dimensional accuracy is attained by minimizing $(Y_o - Y_i)$

3.1.3 Maximization of tool life

Maximization of tool life is ensured by minimizing the number of sparks per cm as given by the equation

$$N_{\min} = a + bE_i \frac{f^2}{VU} + c\frac{f}{V} \tag{4}$$

where a, b, and c are constants and

$$E_i = 1000 \times \frac{A_a}{\text{B}}\left(\frac{\rho_w^2 Z_w F}{K_i M_w \eta_i}\right) \tag{5}$$

3.2 Constraints

The following three constraints are considered in this optimization model.

3.2.1 Temperature constraint

To avoid boiling the electrolyte, the electrolyte temperature at the outlet should be less than the electrolyte boiling temperature. Mathematically this can be expressed as

$$T_i - \frac{1}{\alpha}\left[1 - \left(\frac{1 + S_k f^2}{(1 - \alpha'_{\max})^n U}\right)^{1/2}\right] \leqslant T_b \tag{6}$$

where

$$S_k = \frac{2\alpha\gamma^2 L}{K_i \rho_e C_e J_{cn}} \tag{7}$$

$$\gamma = \frac{Z_w F \rho_w}{M_w \eta_i} \tag{8}$$

3.2.2 Passivity constraint

Oxygen evolved during electrochemical machining forms an oxide film, which is the root cause of passivity. To avoid passivity, the thickness of the oxygen gas bubble layer must be greater than the passive layer thickness. Mathematically, this can be expressed as

$$G_t \frac{f(T_o + 273)}{U\alpha'_{\max}} \geqslant 1 \tag{9}$$

where

$$G_t = \frac{R\rho_f R_f L\gamma}{P_0 t_p i} \tag{10}$$

3.2.3 Choking constraint

Hydrogen evolved at the cathode during the ECM process can choke the electrolyte flow. To avoid choking the electrolyte flow, the maximum thickness of the hydrogen bubble layer should be less than the equilibrium interelectrode gap. Mathematically, it can be expressed as

$$\frac{H_t f^2 (T_o + 273)}{VU\alpha'_{max}(1-\alpha'_{max})^n [1+\alpha(T_o - T_i)]} \leqslant 1 \tag{11}$$

where

$$H_t = \frac{M_h R L \gamma^2}{Z_h P_o F K_i} \tag{12}$$

The next section briefly describes the PSO algorithm.

4 PARTICLE SWARM OPTIMIZATION (PSO)

Particle swarm optimization (PSO) is an evolutionary computation technique developed by Kennedy and Eberhart [**10**]. It exhibits common evolutionary computation attributes, including initialization with a population of random solutions and searching for optima by updating generations. Potential solutions, called particles, are then 'flown' through the problem space by following the current optimum particles. The particle swarm concept was originated as a simulation of a simplified social system. The original intent was to simulate the graceful but unpredictable choreography of a bird flock graphically. Each particle keeps track of its coordinates in the problem space, which are associated with the best solution (fitness) it has achieved so far. This value is called 'pBest'. Another 'best' value that is tracked by the *global* version of the PSO is the overall best value and its location obtained so far by any particle in the population. This location is called 'gBest'. The PSO concept consists of, at each step, changing the velocity (i.e. accelerating) of each particle towards its 'pBest' and 'gBest' locations (global version of PSO). Acceleration is weighted by a random term with separate random numbers being generated for acceleration towards 'pBest' and 'gBest' locations. The updates of the particles are accomplished as per the following equations

$$V_{i+1} = wV_i + c_1 r_1 (\text{pBest}_i - X_i) + c_2 r_2 (\text{gBest}_i - X_i) \tag{13}$$

$$X_{i+1} = X_i + V_{i+1} \tag{14}$$

Equation (13) calculates a new velocity (V_{i+1}) for each particle (potential solution) based on its previous velocity, the best location it has achieved ('pBest') so far, and the global best location ('gBest') the population has achieved. Equation (14) updates an individual particle's position (X_i) in the solution hyperspace. The two random numbers r_1 and r_2 in equation (13) are independently generated in the range [0,1].

The acceleration constants c_1 and c_2 in equation (13) represent the weighting of the stochastic acceleration terms that pull each particle towards 'pBest' and 'gBest' positions, where c_1 represents the confidence the particle has in itself (cognitive parameter) and c_2 represents the confidence the particle has in a swarm (social parameter). Thus, adjustment of these constants changes the amount of tension in the system. Low values of the constants allow particles to roam far from target regions before being tugged back, while high values result in abrupt movement towards, or through target regions [**12**]. The inertia weight w plays an important role in the PSO convergence behaviour since it is employed to control the exploration abilities of the swarm. The large inertia weights allow wide velocity updates to explore the design space globally while small inertia weights concentrate the velocity updates to nearby regions of the design space. The optimum use of the inertia weight w provides improved performance in a number of applications. The effect of w, c_1, and c_2 on convergence for standard numerical benchmark functions is provided by Bergh and Engelbrecht [**14**].

Particle velocities on each dimension are confined to a maximum velocity parameter V_{max}, specified by the user. If the sum of accelerations cause the velocity on that dimension to exceed V_{max}, then the velocity on that dimension is limited to V_{max}.

Although the heuristics are developed to determine the inertia weights and acceleration constants for guaranteed convergent trajectories, it is mainly applicable to single-objective optimization. It is very difficult to obtain the values of inertia weights and acceleration constants for multi-objective optimization problems, due to the inherent conflicting nature of objectives to be optimized. To overcome this problem a time variant PSO was described by Tripathi *et al.* [**15**]. The proposed algorithm was made adaptive in nature by allowing its vital parameters, i.e. inertia weights and acceleration constants, to change with iterations. This adaptiveness helps the algorithm to explore the search space more efficiently. A mutation operator was also included to overcome the premature convergence. The performance of the algorithm was then measured with respect to the main four performance measures, i.e. convergence rate, diversity, purity, and minimal spacing.

Unlike a genetic algorithm, the PSO algorithm does not need a complex encoding and decoding process or a special genetic operator. PSO takes the real number as a particle in the aspect of the representation solution and the particles update themselves with internal velocity. In this algorithm, the evolution looks only for the best solution and all particles tend to converge to the best solution. In the implementation process, particles randomly generated at the beginning or generated by internal velocity during the evolutionary process usually violate the system constraints, resulting in infeasible particles. Therefore, the handling of system constraints, particularly non-linear equation constraints and the measurement and evaluation of infeasible particles, is very important. To cope with constrained problems with evolutionary computation, various approaches such as rejection of infeasible individuals, repair of infeasible individuals, replacement of individuals by their repaired versions, and penalty function methods can be adopted. Among them, the penalty function methods are particularly promising [**12**], as evidenced by recent developments.

The next section presents two examples considering single-objective and multi-objective optimization to demonstrate and validate the proposed PSO algorithm with constant values of inertia weight and acceleration coefficient. Variable inertia weight and acceleration coefficients may be applied to the multi-objective optimization problem [**15**]. However, the multi-objective optimization problem considered in the present work is limited to three objectives subjected to only three constraints; hence the constant values of inertia weight and acceleration coefficients are used. The values of inertia weight and acceleration coefficients, for which the algorithm shows better performance in terms of the convergence rate, are obtained through several trials with an initial guess as given by Bergh and Engelbrecht [**14**].

5 EXAMPLES

To demonstrate and validate the proposed PSO algorithm, two examples are considered for the optimization of electrochemical machining process parameters, based on the model given by Acharya *et al.* [**7**].

5.1 Example 1

This example presents the single-objective optimization case for minimization of dimensional inaccuracy subjected to the constraints of temperature, passivity, and choking. Data for this example are the same as those considered by Acharya *et al.* [**7**] and are given in Table 1. On substituting the values given in Table 1 in equations (1) to (12), the following objective function and constraints are formulated:

Objective function:

$$\text{Minimize } Z = f^{0.381067} U^{-0.372623} V^{3.155414} \mathrm{e}^{-3.128926} \tag{15}$$

where, Z = dimensional inaccuracy (μm)

Constraints:

(a) Temperature constraint:

$$1 - \left(f^{2.133007} U^{-1.088937} V^{-0.351436} \mathrm{e}^{0.321968}\right) \geqslant 0 \tag{16}$$

(b) Passivity constraint:

$$\left(f^{-0.844369} U^{-2.526076} V^{1.546257} \mathrm{e}^{12.57697}\right) - 1 \geqslant 0 \tag{17}$$

(c) Choking constraint:

$$1 - \left(f^{0.075213} U^{-2.488362} V^{0.240542} \mathrm{e}^{11.75651}\right) \geqslant 0 \tag{18}$$

Parameter bounds:

$8 \leqslant f \leqslant 200$ (μm/s)
$300 \leqslant U \leqslant 5000$ (cm/s)
$3 \leqslant \mathrm{V} \leqslant 21$ (V)

Now, the proposed PSO algorithm is applied to solve the above optimization problem. The following parameters of optimization are selected after various trials:

(a) maximum number of iterations: 50;
(b) inertia weight factor (w): 0.65;
(c) acceleration coefficients: $c_1 = 1.65$ and $c_2 = 1.75$.

Table 2 shows the optimum process parameter data for Example 1, along with the previously published results using other methods. The optimum process parameter values are $f = 8$ μm/s, $U = 300$ cm/s, and $V = 9.835$ V. It is observed from the results that the solution obtained by PSO gives a significantly smaller value of dimensional inaccuracy as compared to that of Acharya *et al.* [**7**], Choobineh and Jain [**8**], and Jain and Jain [**9**] when applied to the model of Acharya *et al.*[**7**]. This improvement is mainly due to the use of a better optimization technique, PSO. It is also observed that the results obtained by using a genetic algorithm [**9**] violate the passivity constraint when applied to the model as Acharya *et al.* [**7**]. Jain and Jain [**9**] had used the same model as Acharya *et al.* [**7**] for optimization of ECM process parameters. Even though Jain and Jain [**9**] had mentioned that all constraints were satisfied, when the values obtained by them are substituted in the constraint equations, the passivity constraint becomes violated. The same is the case for the value of dimensional inaccuracy (Z). Jain and Jain [**9**] mentioned that they had obtained an optimum value of Z equal to 7.4633. However, by putting the optimum values of f, U, and V obtained by Jain and Jain [**9**] using the genetic algorithm in the model proposed by Acharya *et al.* [**7**], the value of Z turns out to be 33.62 with violation of the passivity constraint.

Table 1 Values of the constants and decision variables used in the single-objective optimization problem

Notation	Details	Unit	Value
a, b, and c	Constants used in the tool life equation		−2.05, −0.325, and 26.78
A_a	Projected area	cm^2	
B	Width of workpiece	cm	
C_e	Specific heat of electrolyte	cal/g °C	0.997
f	Feed rate	μm/s	
f_{max}	Maximum feed rate	μm/s	
F	Faraday's constant	coulombs	96500
i	Ionic current density	A/cm^2	1.25
J_{cn}	Joule's constant	J/cal	4.186
K_i	Electrical conductivity of electrolyte at inlet	S/cm	0.3333
K_o	Electrical conductivity of electrolyte at outlet	S/cm	
K_x	Electrical conductivity of electrolyte at a distance x from inlet	S/cm	
L	Length of workpiece	cm	3
M_h	Atomic weight of hydrogen	g	1
M_w	Atomic weight of workpiece	g	56
n	Exponent		
N_{min}	Minimum number of sparks per cm		
P	Pressure of electrolyte	MPa	
P_o	Pressure of electrolyte at outlet	MPa	
R	Gas constant	g cm/g K	4.203×10^4
R_f	Roughness factor		1.25
t_p	Time taken for film formation	s	60
T_b	Permitted electrolyte temperature	°C	65
T_i	Room temperature	°C	27
T_o	Electrolyte temperature at outlet	°C	
T_x	Electrolyte temperature at a distance x from inlet	°C	
U	Electrolyte flow velocity	cm/s	
V	Voltage	V	
Y_i	Interelectrode gap at inlet	μm	0.0002
Y_o	Interelectrode gap at outlet	μm	
Z_h	Valency of hydrogen		2
Z_w	Valency of workpiece		2
α	Temperature coefficient of electrolyte conductivity	/°C	0.02
α'	Void fraction		
α'_{max}	Maximum void fraction		0.7
η_i	Current efficiency		0.95
ρ_e	Density of electrolyte	g/cm^3	1
ρ_f	Passive film density	g/cm^3	0.042
ρ_h	Density of hydrogen	g/cm^3	7.86
ρ_w	Density of workpiece	g/cm^3	7.86
σ	Slip ratio between electrolyte and hydrogen gas		1

Table 2 Results of single-objective optimization

Method	Author(s)	f(μm/s)	U(cm/s)	V(V)	TC	PC	CC	Z
GP	Acharya *et al.* [**7**]	18.96	179	15	0.001	2.422	0.204	100
Fuzzy sets	Choobineh and Jain [**8**]	12.75	400	21	0.841	0.0559	0.886	181.07
GA*	Jain and Jain [**9**]	8	2978.45	16.5	0.992	**−0.993**	0.999	33.62
PSO		8	300	9.835	0.895	0.001	0.810	15.452

TC = value of temperature constraint; PC = value of passivity constraint; CC = value of choking constraint; Z = dimensional inaccuracy (single-objective); GP = goal programming.
* For comparison purposes, this result is obtained by putting the optimum values of f, U, and V obtained by Jain and Jain [**9**] using a genetic algorithm (GA) in the model proposed by Acharya *et al.* [**7**].

Optimality of the above-mentioned solution can be confirmed from Figs 2 to 4. As shown in Fig. 2, the dimensional inaccuracy increases with the tool feed rate. Therefore, the smallest possible value of the tool feed rate will minimize the dimensional inaccuracy. Also the passivity constraint will be violated if a higher value of tool feed rate is selected. Hence the tool feed rate at the lower bound ($f = 8$ μm/s) is selected.

The variation of electrolyte flow velocity is shown in Fig. 3. As the dimensional inaccuracy decreases with an increase in the electrolyte flow velocity, selection of a higher value of electrolyte flow

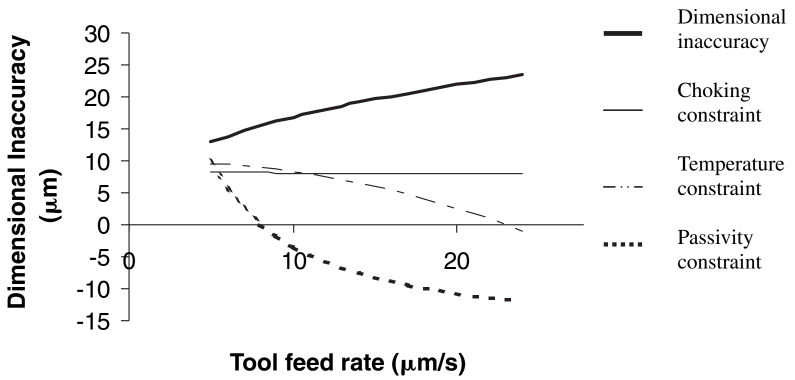

Fig. 2 Variation of the objective function and the constraints with the tool feed rate

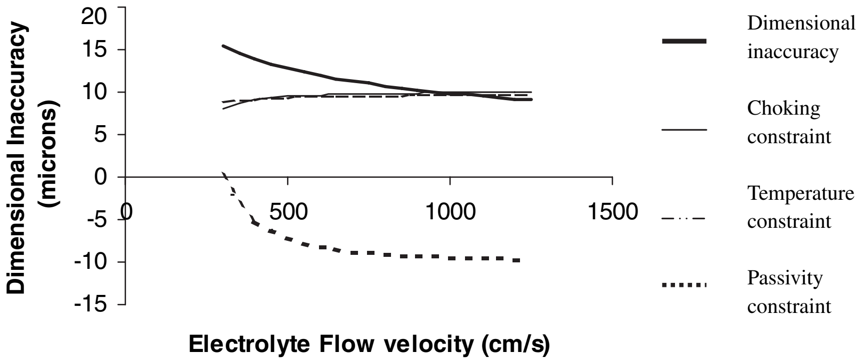

Fig. 3 Variation of the objective function and the constraints with the electrolyte flowrate

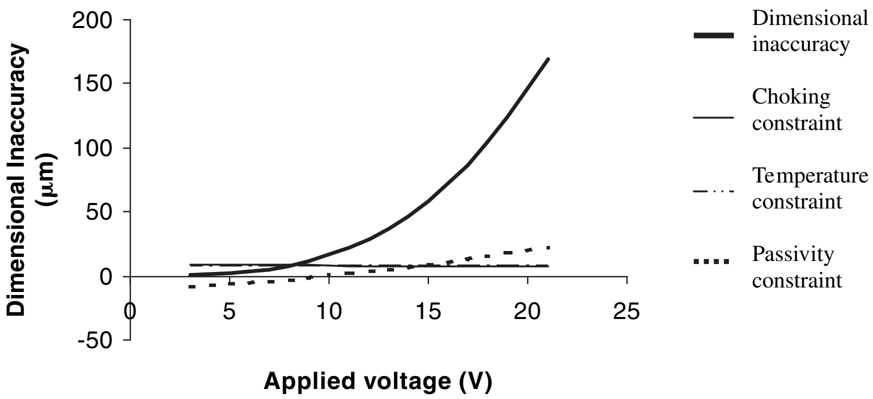

Fig. 4 Variation of the objective function and the constraints with the applied voltage

velocity is desirable. However, the value of electrolyte flow velocity at the lower bound ($U = 300$ cm/s) is obtained, for at any higher value than this the passivity constraint is violated. This may be the reason for violation of the passivity constraint for the solution obtained by using the genetic algorithm [**9**] when used in the optimization model of Acharya *et al.* [**7**].

Figure 4 shows that the dimensional inaccuracy increases with an increase in voltage. Hence, selection of a lower value of applied voltage is desirable. However, to ensure the non-negativity of the passivity constraint, the value of voltage (V) equal to 9.835 V is selected.

5.2 Example 2

This example presents a multi-objective optimization case considering all three objectives, namely material removal rate, tool life, and dimensional inaccuracy

Table 3 Results of multi-objective optimization

Method	f(μm/s)	U(cm/s)	V(V)	TC	PC	CC	Z_1	Z_2	Z_3	Z
GP	18.96	179	15	0.001	2.422	0.204	100	51.79	18.96	18.22
Fuzzy set	12.75	400	21	0.841	0.0559	0.886	181.1	5.47	12.75	5.47
GA*	8	2978.45	16.5	0.992	**−0.993**	0.999	33.62	1.94	8	1.23
PSO	8	300	13.225	0.905	0.583	0.799	39.34	3.39	8	1.811

Z = normalized combined objective function.
* For comparison purposes, this result is obtained by putting the optimum values of *f, U,* and *V* obtained by Jain and Jain [**9**] using a genetic algorithm (GA) in the model proposed by Acharya *et al.* [**7**].

subjected to the constraints of temperature, passivity, and choking.

The first objective is to minimize the dimensional inaccuracy as given by the following expression

$$Z_1 = f^{0.381067} U^{-0.372623} V^{3.155414} \mathrm{e}^{-3.128926} \tag{19}$$

where Z_1 = dimensional inaccuracy (μm).

The second objective is to maximize the tool life by minimizing the number of sparks per millimetre, which is given by the following expression

$$Z_2 = f^{3.528345} U^{0.000742} V^{-2.52255} \mathrm{e}^{0.391436} \tag{20}$$

where Z_2 = number of sparks per millimetre.

The third objective is to maximize the material removal rate, which is given by the following expression

$$Z_3 = f \tag{21}$$

where Z_3 = material removal rate (μm/s)

Decision variables, variable bounds, and constraints are the same as specified in Example 1. The normalized combined objective function (Z) is formulated by considering different weightages to all objectives and is given by the following equation

$$Z = (\mathrm{w}_1 Z_1/Z_{1\min}) + (w_2 Z_2/Z_{2\min}) - (w_3 Z_3/Z_{3\max}) \tag{22}$$

where

$Z_{1\min}$ = minimum value of dimensional inaccuracy obtained when the single-objective optimization problem considering only dimensional inaccuracy as an objective was solved for the given three constraints = 15.452 μm

$Z_{2\min}$ = minimum value of number of sparks per millimetre obtained when the single-objective optimization problem considering only the tool life (in terms of number of the sparks) as an objective was solved for the given three constraints = 1.055

$Z_{3\max}$ = maximum value of feed rate obtained when the single-objective optimization problem considering only the material removal rate (in terms of the feed rate) as an objective was solved for the given three constraints = 25 μm/s.

w_1, w_2, and w_3 = weightages assigned to the objective functions Z_1, Z_2, and Z_3 respectively.

The values of weightages can be calculated by using the analytic hierarchy process [**16**]. However, in the present example, equal weightages are assumed.

The following parameters of optimization were selected after various trials:

(a) maximum number of iterations: 50;
(b) inertia weight factor (w): 0.65;
(c) acceleration coefficients: c_1 = 1.65 and c_2 = 1.75.

The results of multi-objective optimizations along with the previously published results using other methods are as given in Table 3. It is observed from the results that the combined objective function (Z) obtained by PSO shows substantial improvement over Acharya *et al.* [**7**] and Choobineh and Jain [**8**]. The results obtained by putting the optimum values of the genetic algorithm [**9**] in the model proposed by Acharya *et al.* [**7**] are also shown for comparison purposes for the three objectives and the combined objective function. Although the results obtained by the genetic algorithm seem to be better than those obtained by PSO, they violate the passivity constraint and hence are not valid.

In the present work, both single objective and multi-objective aspects of optimization are considered using the PSO algorithm. The comparative performance of single-objective and multi-objective optimizations is shown in Table 4. As shown in Table 4, the solution converges to a higher value of voltage using multi-objective optimization as compared to the results obtained for single-objective optimization. As the voltage increases, the dimensional inaccuracy (Z_1) increases, but the number of spark (Z_2) decreases (and hence increasing the tool life) substantially. The value of the material removal rate (Z_3) is the same for both single-objective optimization and multi-objective optimization. The combined objective function (Z) therefore seems to be better.

6 CONCLUSIONS

The selection of proper values for the parameters of an electrochemical machining process is crucial to the efficiency and high quality of the outcome of

Table 4 Comparison of results of single-objective optimization and multi-objective optimization using PSO algorithm

Optimization using PSO	f(μm/s)	U(cm/s)	V(V)	Z_1	Z_2	Z_3	Z
Single-objective	8	300	9.835	15.45			15.45
Multi-objectives	8	300	13.225	39.34	3.39	8	1.811

that process. The efficiency and quality of the outcome may be measured based on one or more objectives. These objectives, generally, are functions of the operating parameters and often their functional forms are defined through careful deduction of physical laws of nature.

In the present work, both single-objective optimization and multi-objective optimization aspects of electrochemical machining process parameters are considered using a PSO algorithm. The three objectives considered are minimization of dimensional inaccuracy, maximization of tool life by minimizing the number of sparks per millimetre, and maximization of the material removal rate subjected to the constraints of temperature, passivity, and choking. It is observed that the results obtained by using the PSO algorithm show significant improvement over other optimization techniques such as goal programming, fuzzy set theory, and genetic algorithms. When the results of single-objective and multi-objective optimizations obtained by PSO are compared, the combined objective function seems to be better. However, it can be observed that in both cases, the tool life is overachieved, as indicated by very low values of the number of sparks. Therefore, the solution obtained using single-objective optimization is preferred as it gives maximum dimensional accuracy along with the necessary tool life.

In this paper, the performance of PSO in terms of convergence rate and accuracy of the solution is studied. Compared to other non-conventional optimization methods, few trials are required to predict the best and worst operating parameters of the PSO algorithm. The proposed algorithm requires only 30 to 40 iterations for convergence to the optimal solution. The algorithm can also be easily modified to suit optimization of process parameters of other advanced machining processes such as electrical discharge machining, ultrasonic machining, abrasive jet machining, water jet machining, etc. Also, the proposed algorithm can handle the multi-objective optimization models efficiently.

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