Optimisation of process parameters of mechanical type advanced machining processes using a simulated annealing algorithm

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Abstract: The optimum selection of process parameters is essential for advanced machining processes as these processes incur high initial investment, tooling cost, operating and maintenance cost. This paper presents the results of optimisation of process parameters of mechanical type advanced machining processes using a simulated annealing algorithm. The results obtained are then compared with those obtained using a genetic algorithm. It is observed that simulated annealing algorithm has outperformed the genetic algorithm in the present work.

Keywords: advanced machining processes; optimisation; simulated annealing.

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1 Introduction

The use of new materials like carbides, ceramics, nimonics, diamonds, etc., is increasing in industries like aerospace, nuclear engineering and many others owing to their high strength to weight ratio, hardness and heat resistant qualities (Benedict, 1987). The conventional machining processes in spite of recent technological advancements are inadequate to machine these materials from standpoint of economic production. Besides, machining of these materials into complex shapes is difficult, time consuming and sometimes impossible. Advanced machining processes have emerged to overcome these difficulties. According to the nature of energy employed in machining, these processes are classified as below:

1. Mechanical processes like Ultrasonic Machining (USM), Abrasive Jet Machining (AJM), Water Jet Machining (WJM) and Abrasive Water Jet Machining (AWJM).
2. Chemical and electrochemical processes like Electro Chemical Machining (ECM), Electro Chemical Grinding (ECG) and Electro Chemical Honing (ECH)
3. Thermal and electro thermal processes like Electric Discharge Machining (EDM), Laser Beam Machining (LBM), Plasma Arc Machining (PAM) and Ion Beam Machining (IBM).

The present study is mainly focused on mechanical type advanced machining processes. Most of these processes (except WJM) use abrasive particles as tool, and the work material is invariably removed by the phenomenon of erosion. Erosion or erosive cutting is defined as removal of material from the work surface by a stream of impinging solid abrasive particles carried in a suitable carrier. Basically, the process of material removal by erosion is similar to that of grinding and single point tool cutting in the sense that material is removed by an individual particle (or tool) by displacing (in ductile materials) or fracturing (in brittle materials). Erosion is difficult to describe due to uncertainties involved in determining number, shape, velocity and direction of the striking particles. Material removal through erosion may take place either due to cutting wear or deformation wear or both depending on the work material characteristics and conditions of striking.

Comprehensive qualitative and quantitative analysis of the material removal mechanism and subsequently the development of analytical models of material removal are necessary for a better understanding and to achieve the optimum process performance. Analytical material models are also necessary for simulation, optimisation and planning of the process, prediction of process performance indicators, verification and improvements of experimental results, selection of appropriate models for specific type of work material and machining conditions. Since the inception of different advanced machining processes, various investigators have proposed different analytical models of material removal as functions of controllable process variables.

An analytical model for erosive cutting of brittle materials by normal impact of abrasives was developed by Sheldon and Finnie (1966a). Sarkar and Pandey (1976) proposed a model for AJM of the brittle materials considering the effect of nozzle pressure on the material removal rate.

Moore and King (1980) studied abrasive wear and indentation properties, hardness and fracture of a wide range of engineering ceramics and brittle solids and concluded that both plastic deformation mechanisms and fracture mechanisms cause material removal during the abrasive wear of brittle solids. The effects of various input parameters such as stand-off distance, mixture ratio, carrier fluid pressure, grain size, etc., on the material removal rate as well as penetration rate in the AJM were presented by Verma and Lal (1984). Experimental results indicated that both material removal rate as well as penetration rate depends on stand off distance, mixture ratio, and pressure and grain size.

Sundarajan (1984) proposed an empirical equation relating the volume of the crater formed during high velocity oblique impact tests to the velocity and angle of impact and to the target material hardness, which can predict the volume of the crater formed quite accurately over a wide range of impact velocities. Chen et al. (1996) developed experimental techniques based on statistical experimental design principles and theoretical investigations were conducted to study AWJM cutting of alumina-based ceramics. Semi-empirical cutting depth equations were determined for the prediction and optimisation of the abrasive WJM cutting performance.

Wang and Rajurkar (1996) suggested more realistic model considering the stochastic and dynamic nature of the ultrasonic process. However, it is applicable to perfectly brittle materials only. Choi and Choi (1997) developed an analytical model for material removal in AWJM of brittle material. The proposed model was experimentally evaluated. The experimental results suggested that the abrasion mechanisms for ductile and brittle materials are different. For ductile materials, material removal is mainly due to plastic deformation, while crack propagation plays a major role for brittle materials, due to which the material removal rate is high for brittle materials in case of AWJM.

Lee and Chan (1997) measured the effects of amplitude of the tool tip, the static load applied and the size of the abrasive on the material removal rate and the surface roughness and concluded that any increase in the amount of energy imparted to the engineering ceramics in terms of the amplitude of the tool vibration, the static load applied and the grit size of the abrasive, would result in an increase in the material removal rate and roughening of the machined surfaces.

Chen et al. (1998) investigated geometry characteristics, especially of the uncut-through kerf of ceramics. The experimental study involving multi-directional cutting and the application of cutting head oscillation techniques in the AWJM was conducted for enhancing AWJM cutting quality. Paul et al. (1998) developed a mathematical model to predict the total depth of cut of brittle polycrystalline material by AWJM taking into account the variation in the shape and size of the abrasive particles. Process parameter optimisation for AWJM using fuzzy rules was carried out by Chakravathy and Babu (1999) with objectives to maximise the production rate and to minimise abrasive consumption. Jain and Jain (2001) reviewed various analytical and some semi-empirical/empirical material removal models for different mechanical type advanced machining processes comprehensively and exhaustively.

Khan and Haque (2007) presented a comparative analysis of the performance of garnet, aluminium oxide and silicon oxide during AWJM of glass. The study showed that width of cut increases as the stand-off distance of the nozzle from the work is increased which is due to divergence shape of the abrasive water jet. Jain et al. (2007) used genetic algorithms for optimisation of process parameters of mechanical type advanced machining processes. The authors had considered optimisation of four processes namely, USM, AJM, WJM and AWJM. The formulated optimisation models were multi variable, non-linearly constrained single objective optimisation problems. To ensure that the obtained optimum solution was global optimum or near global optimum, concept of statistical DOEs was used to optimise the three most influential and important parameters of real–coded GA namely population size, SBX parameter, and polynomial mutation parameter. The optimisation model proposed by the authors was based on the material removal rate only. Further, different crossover and mutation probabilities were not tried to check the accuracy of results.

Advanced machining processes are having relatively higher initial investment cost, tooling cost, power consumption and maintenance. It is therefore very essential to optimise the various parameters of these processes to get higher material removal rate and low specific energy in order to make them cost effective. The traditional methods of optimisation and search do not fare well over a broad spectrum of problem domains. Traditional techniques are not efficient when practical search space is too large. Numerous constraints and number of passes make the machining process optimisation problem more complicated. Traditional techniques such as geometric programming, dynamic programming, branch and bound techniques and quadratic programming may not be suitable to solve these problems and they are inclined to obtain a local optimum solution.

It is with this spirit, a simulated annealing algorithm for optimisation of process parameters of advance machining processes is proposed in this paper. This paper also compares the results obtained using genetic algorithm by Jain et al. (2007).

The next section describes the simulated annealing algorithm.

2 Simulated annealing algorithm

The simulated annealing algorithm simulates this process of slow cooling of molten metal through annealing to achieve the minimum function value in the minimisation problem. The cooling phenomenon is simulated by controlling a temperature like parameter introduced with the concept of Boltzman probability distribution. According to Boltzman probability distribution, a system in a thermal equilibrium at a temperature '*T*' has its energy distributed probabilistically according to the following expression:

$$P(E) = \exp(-E/KT) \quad (1)$$

where '*K*' is Boltzman constant. This expression suggests that a system at high a temperature has almost uniform probability of being at any energy state. Therefore by controlling the temperature '*T*' and assuming that the search process follows Boltzman probability distribution, the convergence of an algorithm can be controlled. At any current point $X(t)$, the new value of the variables for the successive iterations is calculated using the formula,

$$X(t+1) = X(t) + \sigma\left(\sum_{i=1}^{N} R_i - \frac{N}{2}\right) \quad (2)$$

where $\sigma = (X_{\max} - X_{\min})/6$; R = Random number; and N = Number of random numbers used.

In this work six random numbers are used. While starting the process, the initial values for the variables are taken as the average of the respective variable limits.

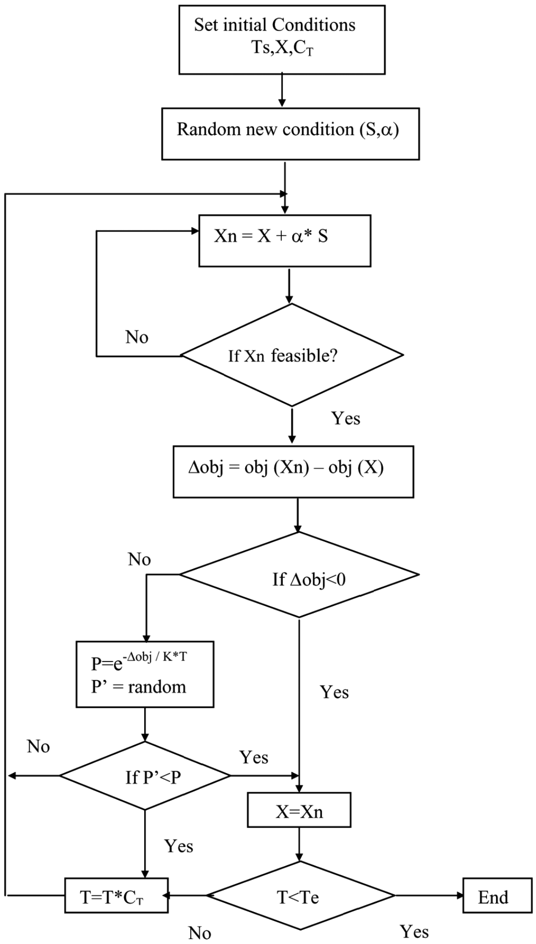
Using the Metropolis algorithm (Metropolis et al., 1953), we can say that the probability of the next point being accepted at $X(t + 1)$ depends on the difference in the function value at these two points or on $\Delta E = E(t + 1) - E(t)$ and is calculated using the Boltzman probability distribution:

$$P(E(t + 1)) = \min (1, \exp(-\Delta E/KT). \quad (3)$$

If $\Delta E \leq 0$, this probability is one and the point $X(t + 1)$ is always accepted. In the function minimisation context, this makes sense because if the functional value at $X(t + 1)$ is better than $X(t)$, the point $X(t + 1)$ must be accepted. The interesting situation happens when ΔE is bigger than zero, which implies that the function value at $X(t + 1)$ is worst than at $X(t)$. According to many traditional algorithms, the point should not be chosen. According to the Metropolis algorithm, there is some finite probability of selecting the point $X(t + 1)$ even though it is worst than point $X(t)$. However the probability is not same in all situations. This probability depends on the magnitude of ΔT and T values.

Simulated annealing algorithm begins with an initial point and a high temperature T. A second point is created at random in the vicinity of .the initial point and the difference in the function values [ΔE] at these two points is calculated. If the second point has a smaller function value, the point is accepted, otherwise the point is accepted with the probability of exp [$-\Delta E/T$]. This completes an iteration of this simulated annealing procedure. In the next generation, another point is created at random in the neighbourhood of the current point and the Metropolis algorithm is used to accept or reject the point in order to simulate the thermal equilibrium at every temperature the number of points '*n*' is usually tested at a particular temperature, before reducing the temperature. The algorithm is terminated when a sufficiently small temperature is obtained or a small enough change in function value is found. The flowchart for simulated annealing algorithm is shown in Figure 1.

Figure 1 Flow chart for simulated annealing algorithm



3 Examples

Now to demonstrate and validate the proposed simulated annealing algorithm, four examples are considered. Jain et al. (2007) had presented a genetic algorithm approach for optimisation of the process parameters of four mechanical type advanced machining processes namely, USM, AJM, WJM and AWJM.

Now, considering the same objective functions and constraints for the four mechanical type advanced machining processes, a simulated annealing algorithm is applied for optimisation and discussed in the following subsections.

3.1 Ultrasonic Machining (USM)

The term ultrasonic is used to describe the vibratory wave of frequency above that of upper frequency limit of the human ear. USM is a process in which material is removed due to action of abrasive grains. The abrasive particles are driven into the work surface by a tool oscillating normal to the work surface at a high frequency. For this process, the material removal rate is not a function of a load or imposed stress on the tool. There appears no simple relationship between the material removal rate and the physical characteristic of the process such as tensile strength, percent elongation, microhardness or impact strength (Pandey and Shan, 1980). Various researchers had reported the different relationships between the amplitude of vibration and material removal rate. The results presented by Shaw (1956) indicating the variation of the material removal rate to be proportional to $(\text{amplitude})^{3/4}$ are used in the present study, as it can be applicable to all type of materials. The frequency of vibration used for the machining process must be the resonant frequency of the acoustic system in order to obtain the greatest amplitude at the tool tip and thus achieve the maximum utilisation of the acoustic system.

Material removal rate increases linearly with the grain size. However, the optimum grain size is governed by the amplitude of the tool vibration and as the grain size become comparable with the amplitude the optimum condition is achieved. Surface finish is also greatly affected by the grain size. The machining load reaches the maximum as the static load on the tool is increased. The optimum value static load is governed by the amplitude of vibration and the cross sectional area of the tool. The rise in material removal rate can be achieved with an increase in the slurry concentration. However, the saturation occurs when a volume of slurry is 30–40% of the abrasive water mixture (Neppiras and Foskett, 1957).

The same decision variables, objective function, surface roughness constraint and variable bounds as considered by Jain et al. (2007) are considered in the present work and are given below.

Decision variables: Five, i.e., amplitude of vibration 'A_v' (mm); frequency of vibration 'f_v' (Hz); Mean diameter of abrasive grain 'd_m' (mm); Volumetric concentration of abrasive particles in slurry 'C_{av}' and static feed force 'F_s' (N).

Objective function: Maximise material removal rate (Z_1):

$$Z_1 = \frac{4.963\, A_t^{0.25} K_u^{\;0.75}}{[\sigma_{fw}(1+\lambda)]^{0.75}} F_s^{0.75} A_v^{0.75} C_{av}^{0.25} d_m f_v \tag{4}$$

where K_u is a constant of proportionality (mm^{-1}) relating mean diameter of abrasive grains, and diameter of projections on an abrasive grain ($= K_u d_m^{\,2}$).

Surface roughness constraint (Z_2):

$$Z_2 = 1.0 - \frac{1154.7}{[A_t \sigma_{fw}(1+\lambda)]^{0.5} (\text{Ra})_{\max}} \left(\frac{F_s A_v d_m}{C_{av}} \right)^{0.5} \geq 0. \tag{5}$$

Variable bounds:

$0.005 \leq A_v \leq 0.1$ (mm); $10000 \leq f_v \leq 40000$; $0.007 \leq d_m \leq 0.15$ (mm);
$0.05 \leq C_{av} \leq 0.5$; $4.5 \leq F_s \leq 45$ (N).

Now using the simulated annealing algorithm, the objective function is written as:

Minimise $Z = -Z_1 - \text{Penalty} \times Z_2$ (penalty = 21 if $Z_2 < 0$, else penalty = 0).

Penalty is defined in such a way that a point having higher value of Z_1 but with small negative value of Z_2 should be accepted at higher temperature to search another point in the vicinity.

Value of Z_2 must be positive to satisfy the constraint. However, at some iteration a solution may exist with significant improvement in Z_1 but slightly violating the constraint Z_2. As this solution has a potential to search global optimum solution, it must be accepted at high temperatures even though the constraints are violated. However, care should be taken that such a point should never appear in the final solution. This task is achieved by assigning suitable penalty value to the constraint function. The penalty value is assigned only to the solutions which violates the constraint function. For all other solutions the penalty value assigned is zero.

The procedure used in this work for selecting the penalty function is summarised as below:

i Calculate values of Z_1 and Z_2 at all boundary points.

ii Select the points which violate the constraint i.e., having negative values of sZ_2.

iii Calculate the ratio $(r) = Z_1 / Z_2$ at all points selected in step (ii).

iv Select the maximum value (r_{max}) among the ratio 'r' calculated in step (iii). Assign a penalty as a value slightly higher than $r_{max.}$

In the case of USM process described above, a boundary point having process parameter values as, $F_s = 4.5$, $A_v = 0.1$, $C_{av} = 0.5$, $d_m = 0.007$, $F_v = 40000$, has objective function value $Z_1 = 5.79$ with surface roughness constraint value $Z_2 = 0.303$. Thus $r_{max} = (5.79/0.303) = 19.10$. Hence a penalty value of 21 is selected.

The same procedure is followed to assign the penalty values for the optimisation of process parameters of all other processes discussed ahead in this paper such as AJM, WJM and AWJM.

The constants used in the process parameter optimisation of USM and their values are given in Table 1. The results of optimisation of process parameters of USM are presented in Table 2. From the results shown in Table 2, it is observed that for USM the material removal rate obtained using simulated annealing is 3.660 mm^3/s which is better (by about 3%) than the final results obtained by Jain et al. (2007) using genetic algorithm. It can be observed from the model that the effect of two parameters i.e., static feed force (F_s) and amplitude of vibration (A_v) on the objective function is almost same. However 'A_v' is more sensitive to the objective function as compared to 'F_s' and hence higher value of 'A_v' up to the limit of specified constraint leads to the improved solution.

Table 1 Values of the constants used in process parameter optimisation of USM

Notation	*Details*	*Units*	*Values*
A_t	Cross section area of cutting tool	mm^2	20
σ_{fw}	Flow stress of work material	MPa	6900
K_u	Constant of proportionality	mm^{-1}	0.1
$(Ra)_{max}$	Allowable surface roughness	μ	0.8
λ	Indentation ratio		0.246

Table 2 Results of optimisation for USM process

Variables	*Results of S.A.*	*Results of G.A.*
Static feed force (N)	4.53	10.8
Amplitude of vibration (mm)	0.077	0.0263
Volumetric concentration of particles in slurry	0.5	0.479
Mean diameter of abrasive grain (mm)	0.114	0.1336
Frequency of vibration (Hz)	40,000	39333.9
Material removal rate (mm^3/s)	*3.660*	*3.553*

3.2 Abrasive Jet Machining (AJM)

AJM uses a stream of fine-grained abrasives mixed with air or some other carrier gas at high pressure. This stream is directed by means of suitably designed nozzle on to the work surface. Metal removal occurs due to erosion caused by the abrasive particles impacting the work surface at high speed (Pandey and Shan, 1980) Metal removal rate increases with increase in mass flow rates of the abrasive particles. However, large value of mass flow rate has been found to adversely influence jet velocity and hence optimum value of mass flow rate is required to found out. The rate of metal removal also depends on the grain size. Finer grains are less irregular in shape and hence possess lesser cutting ability. The kinetic energy of the abrasive jet is utilised for metal removal by erosion. For erosion to occur the jet must impinge the work surface with a certain minimum velocity (Sheldon and Finnie, 1966b).

The decision variables, objective function, surface roughness constraint and variable bounds as considered by Jain et al. (2007) are considered in the present work also and are given below:

3.2.1 Optimisation model for brittle materials at normal impingement of abrasive particles

The decision variables, objective function, surface finish constraint and the variable bounds are given below:

Decision variables: Three, namely Mass flow rate of abrasives 'M_a' (kg/s); Mean radius of abrasives 'r_m' (mm); and Velocity of abrasive particles 'v_a' (mm/s).

Objective function: Maximise material removal rate (Z_1)

$$Z_1 = \frac{0.0035\, n_a\, M_a\, v_a^{1.5}}{\sigma_{fw}^{0.75}\, \rho_a^{0.25}}. \tag{6}$$

Surface finish constraint (Z_2):

$$Z_2 = 1.0 - \frac{18.26}{(\mathrm{Ra})_{\max}} \left(\frac{\rho_a}{\sigma_{fw}} \right)^{0.5} r_m v_a \geq 0. \tag{7}$$

Variable bounds:

$$0.0000167 \leq M_a \leq 0.0005 \text{ (kg/s)}; \quad 0.005 \leq r_m \leq 0.075; \quad 150000 \leq v_a \leq 400000.$$

Now using the simulated annealing algorithm, the objective function is written as:

Min. $Z = -Z_1 - \text{Penalty} \times Z_2$ (penalty = 44.21 if $Z_2 < 0$, else penalty = 0).

Values of the constants used in the process parameter optimisation of AJM for brittle materials are given in Table 3. The results of optimisation of process parameter of AJM for brittle materials are presented in Table 4. From the results shown in Table 4, it is observed that for AJM of brittle materials, the material removal rate obtained using simulated annealing is 8.257 mm^3/s which is slightly more (by about 0.254%) than the final results obtained by Jain et al. (2007) using genetic algorithm. This is because of higher value of velocity of abrasive particle is obtained using simulated annealing than that obtained using genetic algorithm.

Table 3 Values of the constants used in process parameter optimisation of Abrasive Jet Machining of brittle materials

Notation	*Details*	*Units*	*Values*
ρ_a	Density of abrasive particle	kg/mm^3	3.85×10^{-6} (Al_2O_3)
n_a	Proportion of abrasive particles effectively participating in the machining process		0.7
σ_{fw}	Flow stress of work material	MPa	5000 (Glass)
$(Ra)_{max}$	Allowable surface roughness	μm	0.8

Table 4 Results of optimisation for Abrasive Jet Machining of brittle materials

Variables	*Results of S.A.*	*Results of G.A.*
Mass flow rate of abrasives (kg/s)	0.0005	0.0005
Mean radius of abrasives (mm)	0.005	0.005
Velocity of abrasive particles (mm/s)	315764.8	315504.3
Material removal rate (mm^3/s)	*8.257*	*8.236*

3.2.2 *Optimisation model for ductile materials at normal impingement of abrasive particles*

The decision variables, objective function, surface finish constraint and the variable bounds are given below:

Decision variables: Three, namely Mass flow rate of abrasives 'M_a' (kg/s); Mean radius of abrasives 'r_m' (mm); and Velocity of abrasive particles 'v_a' (mm/s).

Objective function: Maximise material removal rate (Z_1)

$$Z_1 = 1.0436 \times 10^{-6} \xi \frac{\rho_w}{\delta^2 H^{1.5} \rho_a^{0.5}} M_a V_a^3 \ (\text{mm}^3/\text{s}). \tag{8}$$

Surface finish constraint (Z_2):

$$Z_2 = 1 - \frac{25.82}{Ra} \left(\frac{\rho_a}{H} \right)^{0.5} r_m v_a \geq 0. \tag{9}$$

Variable bounds:

$0.0000167 \le M_a \le 0.0005$ (kg/s); $0.005 \le r_m \le 0.075$; $150000 \le v_a \le 400000$.

Now using the simulated annealing algorithm, the objective function is written as:

Min. $Z = -Z_1 - \text{Penalty} \times Z_2$.

Values of the constants used in the process parameter optimisation of AJM for ductile materials are given in Table 5. The results of optimisation of process parameter of AJM of ductile material are presented in Table 6. From the results shown in Table 6, it is observed that for AJM of ductile material, the material removal rate obtained using simulated annealing is 0.6053 mm^3/s which is slightly higher (by about 0.46%) than the final results obtained by Jain et al. (2007) using genetic algorithm. This is because of higher value of velocity of abrasive particle is obtained using simulated annealing than that obtained using genetic algorithm.

Table 5 Values of the constants used in the process parameter optimisation of Abrasive Jet Machining for ductile materials

Notation	*Details*	*Units*	*Values*
ρ_a	Density of abrasive particle	kg/mm^3	2.48×10^{-6} (Glass bead)
δ	Critical plastic strain of work material		1.5
H	Dynamic hardness of work material	MPa	1150 (Al–6061–T6)
ξ	Indentation volume plastically deformed		1.6
$(Ra)_{max}$	Allowable surface roughness	µm	2.0

Table 6 Results of optimisation for Abrasive Jet Machining of ductile materials

Variables	*Results of S.A.*	*Results of G.A.*
Mass flow rate of abrasives (kg/s)	0.0005	0.0005
Mean radius of abrasives (mm)	0.005	0.005
Velocity of abrasive particles (mm/s)	333549.08	333214.7
Material Removal Rate (mm^3/s)	*0.6053*	*0.6025*

3.3 Water Jet Machining (WJM)

Important process parameters of WJM are stand-off-distance, water pressure, travel speed of jet, and nozzle diameter. An optimum value of stand-off-distance exists for maximum material removal rate and it has been confirmed experimentally by Hashish and duPlessis (1979). Material removal rate increases with increasing stand-off-distance, but beyond a certain value of stand-off-distance, material removal rate starts decreasing due to a reduction in jet velocity with increasing distance from the nozzle tip. Stand-off-distance also affects accuracy and quality of cut. A divergent jet cuts less effectively and less accurately. Machined depth increases with an increase in water pressure for both metals and non-metals. Also, a threshold pressure exists below which no cutting takes place. The rate of cutting increases with an increase in nozzle diameter (Meng et al., 1998) an optimum value of feed rate of jet also exists for maximum material removal. Cutting and piercing ability of the jet is most effective when it is directed normal to the work surface. Depth of groove also decreases with increasing feed rate. Geometry and finish of cut depend on nozzle design, jet velocity, feed rate, depth of cut and properties of target material. As with the AJM process, nozzle wear depends on nozzle material, its hardness, nozzle design and pressure of the water jet (Mishra, 1997).

For WJM, the multi-objective optimisation model is formulated with maximisation of material removal rate and minimisation of specific energy. However, Jain et al. (2007) had solved a single objective optimisation problem considering maximisation of the material removal rate only. In this work, this problem is solved initially as a single objective optimisation problem (for comparison purpose) and is subsequently solved as multi-objective optimisation problem.

The same decision variables, objective function, power consumption constraint and variable bounds as considered by Jain et al. (2007) are considered in the present work and are given below.

Decision variable: Water jet pressure at nozzle exit 'P_w' (MPa); diameter of water jet nozzle 'd' (mm); transverse or feed rate of the nozzle 'f_n' (mm/s); stand off distance 'X' (mm).

Objective function:

1 Maximise material removal rate (Z_1):

$$Z_1 = \frac{0.297\ d^{15} f_n X^{0.5} \psi^{2/3}}{C_{fw}} \left[1 - \frac{\sigma_{yw}}{2P_w\phi}\right]\left[1 - \mathrm{e}^{-2256.76 C_{fw} P_w \phi / (n_w f_n)}\right] \tag{10}$$

2 Minimise specific energy (Z_3):

$$Z_3 = \frac{3.74\times 10^{15} C_{fw} d^{0.5} P_w^{1.5}}{\psi^{2/3}[1-(\sigma_{yw}/2P_w\phi]\left[1-\mathrm{e}^{-2256.76 C_{fw} P_w \phi/(n_w f_n)}\right] f_n X^{0.5}} \tag{11}$$

where

$$\phi = \frac{2}{K_1}[0.5 - 0.57\psi + 0.2\psi^2]$$

$$\psi = 1 - \sqrt{\frac{1}{P_w}\frac{\sigma_{cw} K_1}{2}}$$

$K_1 = X/X_i$; and $C_D \approx 0.7$.

Power consumption constraint (Z_2):

$$Z_2 = 1 - \frac{1.11\times 10^{-1.5} C_D d^2 P_w^{1.5}}{P_{\max}} \geq 0. \tag{12}$$

Variable bounds:

$1.0 \leq P_w \leq 400.0$ (MPa); $0.05 \leq d \leq 0.5$ (mm); $1.0 \leq f_n \leq 300.0$ (mm/s); $2.5 \leq X \leq 50.0$ (mm).

Now using the simulated annealing algorithm, the objective function considering maximisation of material removal rate only is written as:

Min. $Z = -Z_1 - \text{Penalty} \times Z_2$ (penalty = 8540 if $Z_2 < 0$, else penalty = 0).

Values of the constants used in the process parameter optimisation of WJM are given in Table 7. The results of optimisation of process parameter of WJM are presented in Table 8. From the results shown in Table 8, it is observed that for WJM (considering single objective function) the material removal rate obtained using simulated annealing is almost same as that obtained using genetic algorithm. However, using simulated annealing, the solution converges to slightly higher values of water jet pressure and transverse rate of nozzle and lower value of stand off distance which indicates that simulated annealing algorithm provides better search than genetic algorithm.

Table 7 Values of the constants used in the process parameter optimisation of Water Jet Machining

Notation	*Details*	*Units*	*Values*
C_{fw}	Skin friction coefficient of work material		0.005
n_w	Damping coefficient of work material	kg mm^{-1}s^{-1}	2357.3
σ_{cw}	Compressive yield strength of work material	MPa	26.2
σ_{yw}	Tensile yield strength of work material	MPa	3.9
X_i	Length of initial region of water jet	mm	20
$P_{\max}$	Allowable power consumption value	kW	50

Table 8 Results of optimisation for Water Jet Machining considering only material removal rate as an objective function

Variables	*Results of S.A.*	*Results of G.A.*
Water jet pressure at nozzle exit (MPa)	398.12	397
Diameter of water jet nozzle (mm)	0.5	0.5
Traverse rate of nozzle (mm/s)	215.63	214.41
Stand off distance (mm)	2.5	2.54
Material removal rate (mm^3/s)	*140.25*	*139.79*

In addition to the material removal rate, specific energy can also be considered as another objective and the problem is solved as multi-objective optimisation problem. The methodology for multi-objective optimisation problem in brief can be evolved as follows.

Step 1: Defining the combined objective function (Z):

$$Z = w_1 \times (-Z_1) + w_2 \times Z_3$$

where w_1 and w_2 are the weights assigned to given objectives Z_1 and Z_3.

Step 2: Determining the values of weightages w_1 and w_2:

The weightages assigned to the objectives in this example are given below:

w_1 = weight factor for material removal rate = 0.555.

w_2 = weight factor for specific energy = 0.444.

However, it may be added that, in actual practice, these values of weightages can he judiciously decided by the decision-maker depending upon the policies of the company. Analytical Hierarchy Process (AHP) method proposed by Saaty (2000) may be used to assign the weights of importance to the objectives. The assigned values in this paper are for demonstration purpose only.

Step 3: Normalising the objective function:

In order to achieve the proportionate contribution of each objective in the combined objective function, each objective at any point is divided by its maximum or minimum value (considering the optimisation of individual objective function for the given constraints) to get the normalised combined objective function as given below.

Min. $Z_N = w_1 \times (-Z_1)/Z_{1\max} + w_2 \times Z_3/Z_{3\min}$

where, $Z_{1\max}$ = Maximum value of Z_1

$Z_{3\min}$ = Minimum value of Z_3

Z_N = Normalised combined objective function.

Step 4: Assigning penalty:

To take into account the effect of power consumption constraint, the penalty is assigned as below.

Penalty = $Z_{\min}/Z_{2\max}$

where, $Z_{\min}$ = Minimum value of combined objective function without considering penalty.

$Z_{2\max}$ = Maximum of negative values of power consumption constraint.

In present case, penalty = 28.56 if $Z_2 < 0$; else penalty = 0.

Step 5: Determining the initial temperature and initial solution:

The initial temperature is obtained by calculating the average of the function values at a boundary points.

Initial temperature $T_0 = \Sigma Z_{Nb}/n$

where, Z_{Nb} = Value of normalised objective function at each boundary point and n = Number of boundary points.

The initial solution is taken as: $P_w = 250$; $d = 0.3$; $f_n = 200$; $X = 20$.

Step 6: Optimising the process parameters using simulated annealing algorithm. The results of optimisation of process parameter of WJM (considering multi-objective optimisation) are presented in Table 9.

Table 9 Results of optimisation for Water Jet Machining considering material removal rate as well specific energy

Variables	*Results of S.A.*
Water jet pressure at nozzle exit (MPa)	398.12
Diameter of water jet nozzle (mm)	000.5
Traverse rate of nozzle (mm/s)	215.63
Stand off distance (mm)	002.5
Material removal rate (mm^3/s)	140.25
Specific energy (J/mm^3)	514.316
Value of normalised objective function	000.441

3.4 Abrasive Water Jet Machining (AWJM)

The AWJM process uses a high velocity water jet in combination with abrasive particles for cutting different types of materials. A stream of small abrasive particles is introduced and entrained in the water jet in such a manner that water jet's momentum is partly transferred to the abrasive particles. The role of carrier water is primarily to accelerate large quantities of abrasive particles to a high velocity and to produce a highly coherent jet. Important process parameters of AWJM can be categorised as hydraulic parameters: water pressure, and water flow rate, abrasive parameters: type, size, shape, and flow rate of abrasive particles; cutting parameters: traverse rate and stand-off-distance. The relationship between water pressure and for different abrasive flow rates and nozzle diameters is almost linear. Depth of cut increases with an increase in water flow rate with decreasing slope as the saturation point is reached, while it varies linearly with water jet nozzle diameter for a given pressure (Hashish, 1989). The relationship between abrasive flow rate and depth of cut is linear up to a point. This linearity terminates at higher abrasive flow rates because particle velocity decreases more rapidly than the increase in number of impacts (Hashish, 1984). Depth of cut decreases with increasing traverse rate while an optimum traverse rate exists for maximum kerf area generation (= traverse rate times depth of cut). A minimum (or critical) traverse rate exists below which no further increase in depth of cut can be obtained. Cutting wear may often occur at low traverse rates and deformations wear at higher traverse rates. An increase in stand-of-distance rapidly decreases machined depth, because with an increase the jet breaks into droplets resulting in free abrasive particles hence shallow penetration. A maximum value of stand off distance exists beyond which no cutting will take place (Jain, 2001).

The decision variables, objective function, power consumption constraint and variable bounds as considered by Jain et al. (2007) are considered in the present work also and are given below.

Decision variables: Five, namely water jet pressure at the nozzle exit 'P_w' (MPa); diameter of abrasive water jet nozzle 'd_n' (mm); traverse rate of nozzle 'f' (mm/s); Mass flow rate of water 'M_w' (kg/s); Mass flow rate of abrasives 'M_a' (kg/s).

Objective function: Maximise material removal rate (Z_1)

$$Z_1 = d_n f (h_c + h_d). \tag{13}$$

The indentation depth of cutting wear 'h_c' can be calculated using the following equation.

$$h_c = (1.028 \times 10^{4.5}\ \xi/Ck\ \rho^{0.4})(d_n^{0.2}Ma^{0.4}/f^{0.4})[M_wP_w^{0.5}/(M_a + M_w)] - [(18.48\ K^{2/3}\xi^{1/3})/(C_k^{1/3}f_r^{0.4})][M_wP_w^{0.5}/(M_a + M_w)]^{1/3};\quad \text{if } \alpha_t \leq \alpha_0$$

$$h_c = 0,\quad \text{otherwise.} \tag{14}$$

Here α_0 is the angle of impingement at which the maximum erosion occurs and is given by

$$\alpha_0 = [(0.02164\ Ck^{1/3}f_r^{0.4})/(K^{2/3}\xi^{1/3})][(M_a + M_w)/(M_wP_w^{0.5})]^{1/3}. \tag{15}$$

And α_t is the angle of impingement at the top of the machined surface, which is approximately given by

$$\alpha_t = (0.389 \times 10^{-4.5}\rho_a^{0.4}C_k/\xi)[d_n^{0.8}f^{0.4}(M_a + M_w)/M_a^{0.4}M_wP_w^{0.5}] \tag{16}$$

$$C_k = (3000\ \sigma_{fw}f_r^{0.6}/\ \rho_a)^{0.5}\ \text{(mm/s)}. \tag{17}$$

Indentation depth due to the deformation wear 'h_d' can be found using

$$h_d = \frac{\eta_a dM_a[K_1M_wP_w^{0.5} - (M_a + M_w)v_{ac}]^2}{(1570.8\ \sigma_{fw})d_n^2 f(M_a + M_w) + (K_1C_{fw}\eta_a)[K_1M_wP_w^{0.5} - (M_a + M_w)v_{ac}]^2 M_aM_wP_w^{0.5}} \tag{18}$$

where v_{ac} is a critical velocity of the abrasive particles,

$$v_{ac} = 5\pi^2 \frac{\sigma_{ew}^{2.5}}{\rho_a^{0.5}}\left(\frac{1-v_a^2}{E_{Ya}} + \frac{1-v_w^2}{E_{Yw}}\right)^2 \tag{19}$$

and $K_1 = 1.4142 \times 10^{4.5}\ \xi$.

Power consumption constraint: (Z_2)

$$Z_2 = 1 - \frac{P_wM_w}{P_{\max}} \geq 0. \tag{20}$$

Variable bounds:

$50.0 \leq P_w \leq 400.0$ (MPa); $0.5 \leq d_n \leq 5.0$ (mm); $0.2 \leq f \leq 25.0$ (mm/s);
$0.02 \leq M_w \leq 0.2$ (kg/s); $0.0003 \leq M_a \leq 0.08$ (kg/s).

Now using the simulated annealing algorithm, the objective function is written as:

Min. $Z = -Z_1 - \text{Penalty} \times Z_2$ (penalty = 270 if $Z_2 < 0$, else penalty = 0).

Values of the constants used in the process parameter optimisation of AWJM are given in Table 10. The results of optimisation of process parameters of AWJM are presented in Table 11.

For AWJM, it can be seen from the model that, as the angle 'α_t' increases, indentation depth of cutting wear (h_c) and hence the material removal rate increases. However, Paul et al. (1998) had reported that if angle 'α_t' exceeds the critical impact angle 'α_0' then no material removal is assumed to occur by cutting wear ($h_c = 0$) and the material removal takes place only due to the deformation wear, which causes the material removal rate to reduce drastically.

Table 10 Values of constants used in process parameter optimisation of Abrasive Water Jet Machining

Notation	*Details*	*Units*	*Values*
ρ_a	Density of abrasive particles	kg/mm^3	3.95×10^{-6} for Al_2O_3
ν_a	Poisson's ratio of abrasive particles		0.25
E_{Ya}	Young's modulus of elasticity of abrasive	MPa	350000
f_r	Roundness factor of abrasive particle		0.35
f_s	Sphericity factor of abrasive particle		0.78
η_a	Proportion of abrasive grains effectively participating in the machining		0.7
ν_w	Poisson's ratio of the work material		0.2 (for Ti)
E_{Yw}	Young's modulus of elasticity of work material	MPa	114000
σ_{ew}	Elastic limit of work material	MPa	883
σ_{fw}	Flow stress of the work material	MPa	8142
C_{fw}	Skin friction of the work material		0.002
ξ	Mixing efficiency between water and abrasives		0.8
$P_{\max}$	Allowable power consumption value	kW	56

Table 11 Results of optimisation for Abrasive Water Jet Machining process

Variables	*Results of S.A.*	*Results of G.A.*
Water jet pressure at nozzle exit (MPa)	400	398.3
Diameter of abrasive water jet nozzle (mm)	2.9	3.726
Traverse rate of nozzle (mm/s)	15	23.17
Mass flow rate of water (kg/s)	0.138	0.141
Mass flow rate of abrasives (kg/s)	0.08	0.079
Material removal rate (mm^3/s)	*218.19*	*90.28*

For the solution obtained by Jain et al. (2007) using genetic algorithm as 'α_t' exceeds 'α_0', indentation depth of cutting wear (h_c) becomes zero and hence results in less material removal rate as compared to the solution obtained by simulated annealing algorithm (for which 'α_t' < 'α_0'). The solution obtained by simulated annealing algorithm also results in higher value of depth of deformation wear (h_d) than that obtained with genetic algorithm, which further increases the value of material removal rate. The combined effect thus leads to the improvement in objective function by 141.68%.

The simulated annealing algorithm used in this paper needs to store a single value of objective function, which is then compared with the value obtained in the next iteration; hence the memory requirement is less. Also as most of the operations performed by simulated annealing are comparisons only, the computational efforts and time required is less which may lead to reduced computational cost as compared to genetic algorithm.

4 Conclusion

The simulated annealing algorithm proposed in the present work has outperformed the genetic algorithm for optimisation of process parameters of mechanical type advanced machining processes. About 3% improvement is obtained for optimisation of process parameters of USM and the improvement is up to 141.68% in the case of optimisation of process parameters of AWJM. The proposed algorithm can be easily modified to suit advance machining processes of other types like EDM, PAM, etc. The proposed algorithm is easy to use, simple to implement and can efficiently handle the multi-objective optimisation models.

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