

# Composition Method

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**Abstract** -Composition method provides easy solution to some common life situation . It also describes some mathematical situations like set concept as well as applications of sequences for solution . In this paper we provide two form of this method, in the form of three inter-dependent formulas depending upon their area of application. It focuses on grouping, adding and placing n object ,which is similar to making different groups of employees and giving them bonus or facilities etc.

**Keywords:** Sequences, Cycloid, Power set, statistics.

## I. INTRODUCTION

Invention of numbers is major initiative in the field of mathematics as it sets abasic building blocks for foundation of mathematics. Basic operation like addition, subtraction ,multiplication and division which are used and formulated in most of the part of mathematics.

We deal with situation in which we collect n object they are grouped ,placed, added or expressed as a relation between them. Most of the time we assume placing and collection of object while doing addition or making calculations through practice but if we deal with this two constraint it allow us to perform calculations more systematically as we can deal more complex situation through formulas.

We give formulation of method ,expressed by some rules. Basis for this is pigeonhole principle and situation like adding colors which seems to be practical and easy but still admits new formulations. Pigeonhole principle has many forms out of which equivalent form has used here .

## II. DEFINITIONS:

**A.Unit:** It is number ,measurable quantity or object considered for counting or addition which forms the basis for composition.

**Pigeonhole principle:** it says n objects occupy exactly n distinct places such that each place receives exactly one object.

**B.Composition:** It is a operation defined on n units to count or add them such that it has following formulation.

$$C_n = 1 \quad \forall \text{ given } n \quad [1]$$

$$C_n = n \quad \forall \text{ given } n \quad [2]$$

$$S_n = X_n \text{ or } \sum_n X_n \quad \forall \text{ given } n \quad [3]$$

Note that in each case

[1] is *unitary* composition , means addition or counting of n units results in single unit.

[2] is *natural* composition, means addition or counting is like natural number addition.

[3] is *relative* composition , this addition is due to inter relation between n units .

**Illustration:** We illustrate composition for given operation by following steps.

1]Decide unit for given composition operation and write it in unitary form.

2]Apply pigeonhole principle of suitable form to get natural composition

3]With definite relation between the object take sum over natural composition to obtain relative composition.

## III. EXAMPLES:

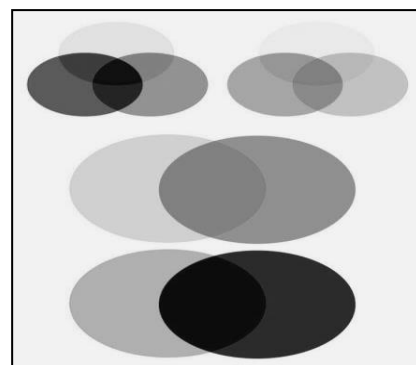
**Example A** illustrate the above composition by example which says that, obtain composition formulas for n distinct coloured places by means of mixing or adding them partially.

**Solution:** i]we have k distinct coloured places ,we observed that by taking unit circle of each colour we add them at a place we get single distinct colour from k colours

Thus

$$P_k = 1$$

$$\forall \text{ given } k$$



ii] Now take the k colours attached or added at single point so according to pigeonhole principle, which says that n objects occupy n distinct places, we get k colours in this addition

i.e.  $1+1+1 \dots \dots +1$  k times

Thus

$$P_k = k \quad \forall \text{ given } k$$

iii] To get relative composition, consider the case of adding colours partially pairwise. Note that  $1+1=3$   $3+1=5$  ..so on

$$\text{i.e. } S_k = 2k - 1 \quad \forall \text{ given } k$$

So we see that *unitary*, *natural* and *relative* composition exist in case of colour mixing by means of variation of places.

Example B Illustrate the composition by means of place variation where you travel  $9n$  km with unit / trip of  $9$  km.

Solution: i] consider a tourist place where you take circular trip of radius  $a$  and circumference  $C = 9$  km. also to complete a journey of  $9n$  km,  $n$  turn are taken along  $C$  of a circular path. but we see only 1 trip

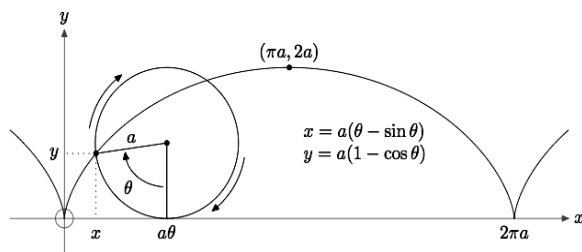
$$\text{i.e. } C_n = 1 \quad \forall \text{ given } n$$

ii] Now if we straighten the way along a line. In tourist place in which at every  $C = 2\pi a$  i.e.  $C = 9$  km we visit to different place [*pigeonhole principle*]. so during journey of  $9n$  km, along straight path  $n$  places with different scenario are seen, that can be expressed as

$$\text{i.e. } C_n = n \quad \forall \text{ given } n$$

So  $n$  trip can be taken by selecting proper tourist track in the journey, here we take straight path.

iii] Now make journey along a cycloid of circle of circumference  $C = 2\pi a$ , which lie along previous linear track with trip  $C = 9$  km with respect to [ii] path.



Since each  $C$  reached by path length  $8a$ , which is a length of arc in one revolution, number of trips attend along cycloid path is given as

$$S_n = \frac{2\pi a}{8a} n$$

$$S_n = \frac{\pi}{4} n$$

Since trip is not done in fraction, so to stop at specific station we take greatest integer part of above formula. In above formula fraction part comes because journey is fixed to  $9n$  km. so trips are reduced from  $n$  to  $(\pi/4)n$

Example C. Take unit as a finite length, write infinite set in terms of finite set using composition.

Solution: Take operation locus of a point which includes infinite points. Now locus of a point equidistance from a single point is circle of finite length  $C$  { = circumference }

$$\text{i.e. } C_n = 1 \text{ finite length } [C]$$

Now take locus of a point along a line [linearly] on which we can make  $n$  partition of length  $C$

$$\text{i.e. } C_n = n \text{ partitions}$$

Again take relative composition as sum over  $n$  units which gives  $S_n = \sum_n 1$

Which is infinite length since  $n$  is infinite. So we expressed infinite length in terms of finite length

$$\text{i.e. } C \cdot S_n = \sum_n 1 \cdot C = nC \dots \text{infinite length}$$

Example D: Illustrate the composition for collection of  $n$  objects.

Solution : i] make single collection of  $n$  object.

$$C_n = 1$$

ii] Now arranged then according to pigeonhole principle in  $n$  boxes such that every object has exactly two neighbours. So we get  $n$  collection which contains single object.

$$\text{i.e. } C_n = n.$$

iii] from above we notice that  $n$  objects are arranged in a circular way any object can take any position then number of arrangement using permutation formula  $P_n^n = n!$

$$\text{i.e. } S_n = n!$$

So for collection of  $n$  similar objects composition formulas can be derived.

Remark: We see that taking collection is analogous with definition of *set* which is defined over collection of  $n$  objects. more specifically similar  $n$  objects count to be 1

$$\text{i.e. } C_n = 1$$

and distinct  $n$  element count to be  $n$

$$\text{i.e. } C_n = n.$$

Further we define set of *power set* of  $n$  element which includes  $S_n = 2^n$  elements

So 'number of element in set' is composable under the operation objects can be similar or distinct.

#### IV. SIGNIFICANCE:

In pigeonhole principle two things are considered one place and second is objects. Both of these things can be categories as similar or distinct.

Composition mainly lies on these facts:  $n$  objects occupy same or distinct places as well as  $n$  objects can be similar or distinct in a collection.

Mixing or adding colours, rotation of body around the axis or along the line, similar or distinct collection of objects, measuring length or area of a circle etc. are the operations over which composition can apply.

In further note composition method has boiled down to some practical formulas that can be readily used in problem solving, thus it becomes applicable in word problems. Second form of this method seems to touch statistics of discrete frequency distribution. Means we

involve numbers and there specifications to find desire results.

#### V. PROPERTIES:

A. If  $C_n=1$  is unitary composition and  $C_n=n$  is natural composition for collection of  $n$  objects then

$$C_n = C_1 + C_1 + \dots + C_1 \quad n \text{ times}$$

Proof:

Consider

$$C_n = n$$

$$= 1+1+ \dots +1 \quad n \text{ times}$$

Now since  $C_n = 1$  we have  $C_1 = 1$  by applying pigeonhole principle.

Substituting this for 1 we get

$$C_n = C_1 + C_1 + \dots + C_1 \quad n \text{ times}$$

Thus

$$C_n = nC_1$$

Here  $C_1$

Represent distinct object appearing once in collection

B. In collection of  $n$  units, subgroups are formed like  $k$  then we have

$$C_k = 1$$

$$C_n = \sum_{k=x,y,z} C_k \quad \text{where } k = x, y, z$$

$$\text{And } n = \sum_{k=x,y,z} k$$

Proof: Consider grouping in form of sub group. Thus total number of sub group as partition of  $n$

Thus from [I]

$$C_n = C_1 + C_1 + \dots + C_1 \quad n \text{ times}$$

For subgrouping or partitions we have

$$C_n = C_x + C_y + C_z \quad i.e$$

$$C_n = \sum_{k=x,y,z} C_k \quad \text{where } k = x, y, z$$

Since in each subgroups there are  $k$  units they all add up to total unit  $n$

Thus

$$n = \sum_{k=x,y,z} k$$

Remark: Now sequentially for  $r$  partitions we have

$$C_n = \sum_r Cx_k \quad \text{where } k = 1, 2, \dots, r$$

$$C_n = r$$

$$n = \sum_r \text{partitions } x_k$$

#### VI. SECOND FORM:

A. Composition for making subgroups of  $k$  units from  $n$  units / objects is given by

$$1) C_k = 1$$

$$2) C_n = \sum_{k=x,y,z} C_k \quad \text{where } k = x, y, z$$

$$\text{and } n = \sum_{k=x,y,z} k$$

$$3) S_{x,y,z} = \sum_{k=xyz} k \cdot t$$

Where  $t$  is positive real number.

B. Composition for making  $r$  subgroups containing  $x_k$  units is given by

$$1) Cx_k = 1 \quad \text{where } k = 1, 2, \dots, r$$

$$2) C_n = \sum_k Cx_k \quad \text{where } k = 1, 2, \dots, r$$

$$\text{and } n = \sum_k x_k$$

$$3) S_r = \sum_k x_k \cdot t \quad \text{where } k = 1, 2, \dots, r$$

Where  $t$  is positive real number.

#### VII. APPLICATION:

A. it is applicable over  $n$  object to count or add them.

Where object specification are rough

B. Some mathematical concepts or situations can be put in this formulations

C. It allow us to do simple word problem as well as basic statistical problem

D. sequential criterion predicts the solution as well as we can set our problems accordingly.

Example A:

There are 9 mathematician, 6 computer scientist and 3 statistician in the university.  $1/3$  are selected from each stream for scholarship, using II<sup>nd</sup> form calculate how many will get selected for scholarship?

Solution:

$$\text{We have } C_k = 1 \quad \text{Where } k=3,6,9$$

$$\text{Thus } C_n = \sum_k C_k$$

$$= \sum_k 1$$

$$= 3$$

$$i.e. C_{18} = 3$$

since

$$n = \sum_k k \quad n=18$$

$$\text{Now } S_3 = \sum_{k=x,y,z} k \cdot t \quad \text{where } t=1/3$$

$$= 1/3(18)$$

$$S_3=6 \quad \text{will get selected for scholarship}$$

Hence the solution.

Example B: Consider a dice which is thrown  $n$  times so  $n$  trials are made. If  $X_k$  denote frequency of  $k$  then express the probability condition using second form.

Solution: We have

$$Cx_k = 1 \quad \text{where } k = 1, 2, \dots, 6$$

Where 1 represent a partition of  $k$  and  $X_k$

Represents occurrence. Further

$$C_n = \sum_k Cx_k \quad \text{where } k = 1, 2, \dots, 6$$

and  $n = \sum_k x_k$

i.e  $C_n = 6$

Now probability of each number is given by  $\frac{x_k}{n}$

Thus probability condition can be expressed as

$$S_6 = \sum_k x_k/n \quad \text{where } k = 1, 2, \dots, 6$$

=1

Hence the solution

#### VIII. CONCLUSION:

It is an attempt to treat basic addition in terms of formulas for objects which are given in rough format and extend same formulation to simple word problem. We may assign weight to relative composition to solve more complex situation.

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