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ORIGINAL ARTICLE

Drag reduction in a gravity-driven flow system using polyethylene oxide solutions

Yennam Rajesh[1] · **Lakshmana Rao Jeeru**[2]

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Abstract
The current study describes the use of a mathematical formula based on macro-scale balances to calculate the efflux time for gravity draining a Newtonian liquid from a large conical tank through an exit pipe at the bottom of the tank when the flow in the pipe line is turbulent. The least amount of time required to drain the tank will be calculated using the efflux time equation, which has been modified using experimental data. When the flow is mixed, that is, partially laminar and partially turbulent, gravity-driven, and once through the system, the percentage reduction in efflux time that the addition of water-soluble polyethylene oxide polymer has on drag reduction is displayed. Therefore, the efflux time equation provides the shortest amount of time needed to acquire the liquid draining from the tank.

Keywords Drag reducing agent · Efflux time · Newtonian liquid · Friction factor · Conical tank

Introduction

The chemical industry uses storage tanks with various shapes. Numerous factors, such as the necessity for insulation, floor space, corrosion requirements, material costs, etc., might affect the choice of a specific tank form. The efflux time, very important from a production standpoint and in emergency situations, is the length of time required to remove the liquid from the storage container (Hart and Sommerfeld 1995). According to Joye and Barret (2003), when a vessel with a large diameter is slowly drained through an exit piping system made up of one or more pipes, the flow is predominantly laminar in the vessel and is likely to be turbulent in the pipe (Peet and Sagaut 2008; Xi 2019; Choi 1997).

From a practical perspective, the drag reduction impact is very intriguing (Subbarao et al. 2011). Adding a small amount of polymers to pipes to reduce drag can result in considerable cost savings and improved transit efficiency for the majority of liquids. In addition to reducing drag, the polymer also inhibits heat transfer, which helps to maintain low oil viscosity (Vov et al.2005). The addition of polymers to oil that is piped from offshore platforms to onshore facilities is an equivalent application (Henoch 2006). In addition, polymers have been used in sewage pipelines and storm-water drains can increase flow rates to prevent overflowing at peak loads; If only relatively rarely use is necessary, this might be much less expensive than having new pipes installed (Agulilar 2006). Another application is to increase the range and coherence of water jets from firefighting hoses, but this idea has not been applied very often (David and Roche 1999). Torpedoes' drag can be reduced in a military use, according to a patent. It has been patented to discharge a seawater-polymer solution from the torpedo nose (Wang et al. 2011). By improving blood flow via stenotic arteries without influencing blood flow through healthy vessels, low quantities of polymers may have a medical benefit (Fortuna, and Hanratty 1972). Additionally, corrosion is decreased through the use of polymers that reduce drag.

In addition to these practical considerations, fundamental fluid dynamics researchers find the phenomenon of drag reduction by polymer additives to be incredibly fascinating (Jurban et al. 2006). The fact that such minute modifications to the fluid can have such a significant impact on the flow characteristics strongly suggests the presence of a crucial momentum transmission mechanism that the polymer interferes with (Drappie et al. 2006). Therefore, research on

✉ Yennam Rajesh
rajeshiitg09@gmail.com; yennamrajesh@kkwagh.edu.in

[1] Department of Chemical Engineering, K. K. Wagh I. E. E. and R Nasik (MS), Affiliated to S. P. Pune University, Pune 422003, India

[2] School of Petroleum Technology, Pandit Deendayal Energy University, Gandhinagar 382426, Gujarat, India

polymeric drag reduction can contribute to a better understanding of the phenomenon.

Experimental research on the phenomenon of polymer-based drag reduction has been focused on horizontal channels and pipelines. Joye and Barret (2003) The majority of the described effort focuses on reducing turbulent drag, with only a small amount of work reducing laminar drag (Subbarao et al. 2013). Some drag reduction is achievable when the flow is mixed, that is, partially laminar, partially turbulent, and gravity-driven (Mohamed et al. 2021).

A conical tank, Newtonian liquid (PEO solution), and an exit pipe are all part of the experimental setup used for this work. The conical tank's base is where the exit pipe is attached. The height of the liquid in the tank, the concentration of PEO solution, and the length of the exit pipe are all operational considerations, have an impact on the efflux time and % Drag reduction. The efflux time for various polymer concentrations has been calculated using a mathematical equation for conical tanks has been developed (Eq. 14).

Materials and methodology

Experimental apparatus description

In Fig. 1a conical tank with a known diameter is placed atop a stainless steel structure. A mild steel pipe with a known diameter (d) was welded to the structure and employed as an escape pipe in the middle of the tank's bottom (Li et al. 2008). The liquid was drained from the tank using a gate valve (GV), which was situated alongside the control valve at the base of the exit pipe. To function as a level indication while the tank was being drained, a clear plastic tube (L) was put within. Four exit pipe lengths of 1 m, 0.75 m, 0.5 m, and 0.25 m were employed in the tests, and two tanks with diameters of 0.28 m and 0.25 m. Experiments were carried out using pipes with diameters of 0.004 m and 0.006 m, respectively. To determine the draining pattern and efflux periods, additives containing polyethylene oxide (PEO) in varying quantities were used. The efflux times have been measured with a stopwatch with a 1 s precision.

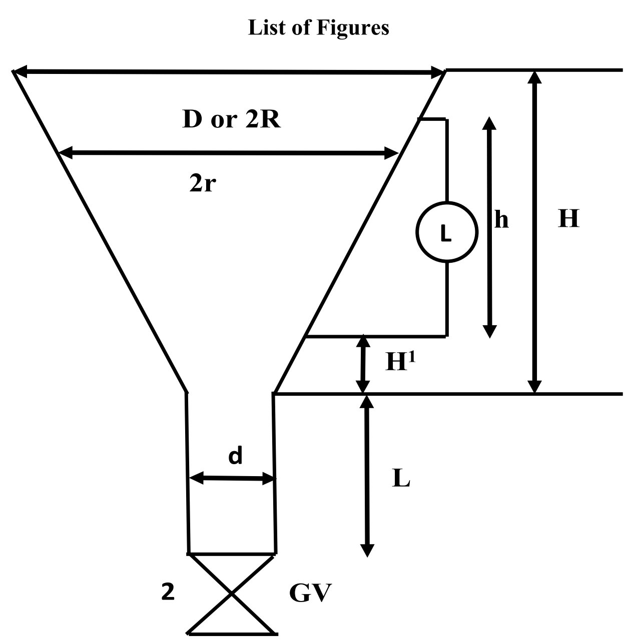

Fig. 1 Conical tank with exit pipe

Synthesis of drag reducing agent

Polyethylene oxide (PEO) having a molecular weight of 1,00,000 was supplied by Shiv Shakti Group, Gujarat. The PEO stock solution was created using the technique outlined by Subbarao et al. (2011). 10, 20, 30, 60, 65, 70, and 75 PPM solutions were created by diluting the prepared stock solution. The conical tank was filled with the pre-blended solutions and exit pipe assembly, and the processes listed below were employed to calculate the efflux timings.

Efflux time measurement for a single exit pipe both with and without polymer

The gating valve at the exit pipe's base was closed, allowing the exit pipe to fill the tank with water/polymer solution up to the level of the tank and at a specific concentration. The author must immediately start a stopwatch after opening the bottom Gate valve to gage the draining time. The tank liquid level indicator displays the drop in liquid level inside the tank. The measuring time was reflecting the liquid level in the tank's known decline. The reading was taken up until the water level, which was 0.02 m just above the tank bottom, reached the intended value. The experiment's efflux time was marked as t_{act}, repeated the experiment several times to ensure uniformity and reproducibility. The aforementioned experimental process was performed with various tank dimensions and outlet pipe lengths.

Tables 1 and 2 provide a summary of the equipment information, the variables covered with and without polymer solutions for various tank diameters, and the specifics of tests utilizing single exit pipes with 0.004 m and 0.006 m diameters.

Development of mathematical model

The mathematical equation of efflux time for conical tank has been developed using the below Fig. 1.

The above illustration demonstrates how a conical tank is connected to an exit pipe to drain the solution. It

Table 1 List of experiments performed in the absence and presence of polymer additives for 0.255 m and 0.28 m dia. tank and 0.004 m dia. single exit pipe

S. no.	*H, m*	*L, m*	*A1/Ap*
1	0.30 0.28 0.26 0.24	0.25, 0.50, 0.75 and 1	0.0043
2	0.30 0.28 0.26 0.24	0.25, 0.50, 0.75 and 1	0.006

Table 2 List of experiments performed in the absence and presence of polymer additives for 0.255 m and 0.28 m dia. tank and 0.006 m dia. Single exit pipe

S. no.	*H, m*	*L, m*	A_1/A_p
1	0.30 0.28 0.26 0.24	0.25, 0.50, 0.75 and 1	0.000017
2	0.30 0.28 0.26 0.24	0.25, 0.50, 0.75 and 1	0.000024

also specifies the length and diameter of the exit pipe, as well as the tank's liquid at its highest point, among other details (Paschkewitz et al. 2004). A Newtonian liquid was poured into the tank, and a quick-opening gate valve was set up at the bottom of the pipe to allow the liquid to be drained from point 2.

Calculating the efflux duration and the pattern of liquid drainage from the tank, excluding the pipe, is necessary.

From mass balance,

Rate of mass inflow (1)—rate of mass outflow at (2)—rate of mass accumulated

$$\frac{d}{dt}\left(M_{\mathrm{tot}}\right) = -W_1 - W_2$$

Here $W_1 = 0$, then

$$\frac{d}{dt}\left(M_{\mathrm{tot}}\right) = -W_2$$

M_{tot} Total mass of liquid in the tank.

Writing the Bernoulli theorem between points i and ii

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gZ_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gZ_2 + \frac{4fLV_2^2}{2d} \tag{1}$$

Since the tank top and bottom are open to the atmosphere, $P_1 = P_2$.

$V_1 = 1$ (The liquid is draining with lower speed; therefore, it is steady state).

At any height h, $Z_1 = Z_2 + h + L$.

These values are substituting in Eq. (1)

$$g(h+L) = \frac{V_2^2}{2} + \frac{4fLV_2^2}{2d} \tag{2}$$

The above equation can be written by the assumption of constant friction factor:

$$g(h+L) = \frac{V_2^2}{2} \times (1 + 4fL/d). \tag{3}$$

$$V_2^2 = \frac{2 \times g \times (h+L)}{1 + 4fL/d} \tag{4}$$

$$V_2^2 = \sqrt{\frac{2 \times g \times (h+L)}{1 + 4fL/d}}. \tag{5}$$

Applying mass balance equation

$$\frac{d}{dt}\left(\frac{1}{3}\pi r^2 h\right) = \sqrt{\frac{2g(h+L)}{1+4f(L/d)}} \times \rho \frac{\pi}{4} d^2. \tag{6}$$

In the above equation, density (ρ) is constant.

From Fig. 1,

$$\frac{r}{h} = \frac{R}{H},$$

$$r = \frac{h \times R}{H}. \tag{7}$$

The above "r" value insert in Eq. (6)

$$\frac{d}{dt}\left(\frac{1}{3}\left(\frac{h \times R}{H}\right)^2 h\right) = -\sqrt{\frac{2g(h+L)}{1+4f(L/d)}} \times \frac{d^2}{4}, \tag{8}$$

$$h^2 \times \frac{dh}{dt}\frac{R^2}{H^2} = -\sqrt{\frac{2g(h+L)}{1+4f(L/d)}} \times \frac{d^2}{4},$$

$$h^2 \times \frac{dh}{\sqrt{h+L}} = -\frac{d^2}{4R^2} \times H^2 \sqrt{\frac{2g}{1+4f(L/d)}} dt. \tag{9}$$

Conduct the integration on L.H.S and R.H.S

$$\int \frac{h^2}{\sqrt{h+L}} dh = -\int \frac{d^2}{4R^2} \times H^2 \sqrt{\frac{2g}{1+4f(L/d)}} dt. \tag{10}$$

At $t=0$, $H=H$ and at $t=t$, $H=H^1$.

$$\int_{H^1}^{H} \frac{h^2}{\sqrt{h+L}} \mathrm{d}h = -\int_0^t \frac{d^2}{4R^2} \times H^2 \sqrt{\frac{2g}{1+4f(L/d)}} \mathrm{d}t. \tag{11}$$

Using integration to the equation's LHS (11),

$$h + L = X$$

$$h = X - L$$

$$\mathrm{d}h = \mathrm{d}X$$

Substituting these values in the above Eq. (12)

$$\int_{H^1}^{H} \frac{h^2}{\sqrt{h+L}} \mathrm{d}h = -\int_{H^1}^{H} \frac{(X-L)^2}{\sqrt{X-L+L}} \mathrm{d}X, \tag{12}$$

$$\int_{H^1}^{H} \frac{h^2}{\sqrt{h+L}} dh = \int_{H^1}^{H} \frac{(X^2+L^2-2XL)}{\sqrt{X}} dX$$

$$\int_{H^1}^{H} \frac{h^2}{\sqrt{h+L}} dh = \int_{H^1}^{H} (\frac{X^2}{\sqrt{X}} + \frac{L^2}{\sqrt{X}} - \frac{2XL}{\sqrt{X}}) dX$$

$$\int_{H^1}^{H} \frac{h^2}{\sqrt{h+L}} dh = \int_{H^1}^{H} (\frac{X^2}{X^{1/2}} + \frac{L^2}{X^{1/2}} - \frac{2XL}{X^{1/2}}) dX$$

$$\int_{H^1}^{H} \frac{h^2}{\sqrt{h+L}} dh = \left[\frac{X^{2.5}}{2.5} + 2\sqrt{X} L^2 - \frac{2X^{3/2} L}{\frac{3}{2}} \right]_{H^1}^{H}$$

$$\int_{H^1}^{H} \frac{h^2}{\sqrt{h+L}} dh = \left[\frac{X^{2.5}}{2.5} + 2L^2\sqrt{h+L} - \frac{2X^{3/2} L}{\frac{3}{2}} \right]_{H^1}^{H}$$

$$\int_{H^1}^{H} \frac{h^2}{\sqrt{h+L}} dh = \left[\frac{(h+L)^{2.5}}{2.5} + 2L^2\sqrt{h+L} - \frac{4}{3}(h+L)^{3/2} L \right]_{H^1}^{H}$$

$$\int_{H^1}^{H} \frac{h^2}{\sqrt{h+L}} \mathrm{d}h = \left[\begin{array}{l} \frac{(H+L)^{2.5} - (H'+L)^{2.5}}{2.5} + 2L^2(\sqrt{H+L} - \sqrt{H'+L}) \\ -1.33L\left[(H+L)^{3/2} - (H'+L)^{3/2}\right] \end{array} \right] \tag{13}$$

Substituting Eq. (13) in Eq. (11),

$$0.4\left[(H+L)^{2.5} - (H'+L)^{2.5}\right] + 2L^2(\sqrt{H+L} - \sqrt{H'+L}) - 1.33L\left[(H+L)^{3/2} - (H'+L)^{3/2}\right] = \frac{d^2 H^2}{4R^2} \sqrt{\frac{2g}{1+4f(L/d)}} \times t_{\mathrm{eff}} \tag{14}$$

$$t_{\mathrm{eff}} = \left\{ \begin{array}{l} 0.4\left[(H+L)^{2.5} - (H'+L)^{2.5}\right] + 2L^2(\sqrt{H+L} - \sqrt{H'+L}) \\ -1.33L\left[(H+L)^{3/2} - (H'+L)^{3/2}\right] \end{array} \right\} \times \frac{4R^2 \sqrt{\frac{1+4f(L/d)}{2g}}}{H^2 d^2} \tag{15}$$

The equation (xv) has been written in the form of dimensionless as,

$$t_{eff} = \frac{4R^2}{H^2 d^2} \sqrt{\frac{1+4f(L/d)}{2g}} \left\{ \begin{array}{l} 0.4\left[L^{2.5}\left(1+\frac{H}{L}\right)^{2.5} - \left(1+\frac{H'}{L}\right)^{2.5}\right] + 2L^2\left(\sqrt{1+\frac{H}{L}} - \sqrt{1+\frac{H'}{L}}\right) \\ -1.33L^{2.5}\left[\left(1+\frac{H}{L}\right)^{3/2} - \left(1+\frac{H'}{L}\right)^{3/2}\right] \end{array} \right\}$$

$$t_{eff} = \frac{4R^2}{H^2 d^2} L^{2.5} \sqrt{\frac{1+4f(L/d)}{2g}} \left\{ \begin{array}{l} 0.4\left[\left(1+\frac{H}{L}\right)^{2.5} - \left(1+\frac{H'}{L}\right)^{2.5}\right] + 2\left(\sqrt{1+\frac{H}{L}} - \sqrt{1+\frac{H'}{L}}\right) \\ -1.33\left[\left(1+\frac{H}{L}\right)^{3/2} - \left(1+\frac{H'}{L}\right)^{3/2}\right] \end{array} \right\}$$

$$t_{eff} = \frac{4R^2}{d^2}\left(\frac{L}{H}\right)^2\sqrt{\frac{1+4f(L/d)}{2g}\left(\frac{L}{G}\right)}\left\{\begin{array}{l}0.4\left[\left(1+\frac{H}{L}\right)^{2.5}-\left(1+\frac{H'}{L}\right)^{2.5}\right]+2\left(\sqrt{1+\frac{H}{L}}-\sqrt{1+\frac{H'}{L}}\right)\\-1.33\left[\left(1+\frac{H}{L}\right)^{3/2}-\left(1+\frac{H'}{L}\right)^{3/2}\right]\end{array}\right\}$$

$$t_{eff} = \frac{D^2}{d^2}\left(\frac{L}{H}\right)^2\sqrt{\frac{L}{2g}}\sqrt{(1+4f(L/d))}\left\{\begin{array}{l}0.4\left[\left(1+\frac{H}{L}\right)^{2.5}-\left(1+\frac{H'}{L}\right)^{2.5}\right]+2\left(\sqrt{1+\frac{H}{L}}-\sqrt{1+\frac{H'}{L}}\right)\\-1.33\left[\left(1+\frac{H}{L}\right)^{3/2}-\left(1+\frac{H'}{L}\right)^{3/2}\right]\end{array}\right\}$$

$$t_{\text{eff}} = \left(\frac{L}{H}\right)^2\frac{D^2}{d^2}\sqrt{\left(1+4f\frac{L}{d}\right)}\sqrt{\frac{L}{2g}}\left\{\begin{array}{l}0.4\left[\left(1+\frac{H}{L}\right)^{2.5}-\left(1+\frac{H'}{L}\right)^{2.5}\right]+2\left(\sqrt{1+\frac{H}{L}}-\sqrt{1+\frac{H'}{L}}\right)\\-1.33\left[\left(1+\frac{H}{L}\right)^{3/2}-\left(1+\frac{H'}{L}\right)^{3/2}\right]\end{array}\right\} \quad (16)$$

Results and discussion

The differences between theoretical and actual values are expected because the experimental results are based on fluid motion inside a conical tank with an unstable state condition. As a result, analyses of theoretical and experimental values are conducted. In the following sections, efflux times were calculated using Eq. (16) and compared with the experimental values. Also, the effects of various variables like the liquid level in the tank's height, the length and diameter of the exit pipe on the efflux time are discussed.

Comparison of efflux time data

While calculating the friction factor term (Subbarao et al. 2011) and Reynolds number (3000 to 3 × 10^6) ranges are applying in the Eq. (16) to get the efflux time.

$$f = 0.0014 + \frac{0.125}{\text{Re}^{0.32}} \quad \text{(Known as Drew correlation)} \quad (17)$$

where $\text{Re} = \frac{DV_{2\exp}\rho}{\mu}$.

$V_{2\exp}$ is attained using the measured experimental data

$$V_{2\exp} = \left[\left(\frac{\pi}{3}r^2 \times H - \frac{\pi}{3} \times {r_1}^2 \times H^1\right)/\left[\frac{\pi}{4}d^2 t_{\text{act}}\right]\right] \quad (18)$$

It is assumed that the liquid in the tank has a given density and viscosity to be 1000 kg/m^3 and 10^{-3} kg/m·sec, respectively. Using Eq. 18, the Reynolds number in the pipe was computed, and it was discovered that the flow was turbulent (Peet et al. 2008).

Friction factor (f) and $V_{2\exp}$values are inserted in eqn. (xvi) to attain at t_{eff}. The mathematical equation of efflux time was validated with experimental values of t_{act} and t_{eff}, respectively. Table 3 represents the efflux time comparison of different exit pipe lengths with respect to $D = 0.28$ m and $d = 0.004$ m (Sommerfeld and Stallybrass 1992).

The results had shown an excellent agreement between t_{eff} and t_{act}. This shows that the friction factor equation was successfully represented the experimental efflux time data with a maximum deviation of 1% for all exit pipe lengths.

Following observations can be made from a careful examination of the data.

- The efflux time was increased when the pipe length was increased.
- The efflux time was reduced when the liquid level in the tank's height was reduced.
- The efflux time equation is in good agreement even for a shorter exit pipe length of 0.25 m. Hence, the assumption of fully developed turbulent flow in the pipe is justified in the case of a conical tank with an exit piping system.

The trends are same for other diameters of tank and pipe as well as for other exit pipe lengths and shown in the following Table 4.

Variation of H/L vs efflux time

Asstated in the following efflux time equation by Subbarao et al. (2011), is employed in Eq. 16 to determine theoretical efflux time. The plot of H/L vs. efflux time for a tank witha diameter of 0.28 m and an exit pipe length of 0.25 m is shown in Fig. 2. In the current study, it is assumed that the viscosity and density of liquid water are equal to 0.001 kg/m/sec and 1000kg/m^3. All of the situations under consideration have computed Reynolds numbers that are > 2100. The author represents the efflux time varies linearly with H/L ratio within the range of variables covered. The trend was similar for other exit pipe lengths as shown in Fig. 3.

Variation of 1 + 4fL/d vs. liquid height in the tank (H)

The plot of 1 + 4fL/d vs liquid height in the tank (H) for a 0.28 m dia. tank and a 0.25 m length of exit pipe is shown in Fig. 4. The plot suggests the variation of the friction factor to be negligible with a change in liquid height in the tank. This also justifies the mathematical Eq. (18) was derived under the premise of a constant friction factor. This is additionally confirmed by the following plot of 1 + 4fL/d vs H (Fig. 5), but for an exit pipe length of 0.5 m.

The contraction coefficient value provided by Joye and Barret (2003) is affected by the exit pipe's diameter, according to the difference between 4 mm exit pipe and 6 mm exit pipe experimental and theoretical results, which is valid for average Reynolds numbers > 4200.

Tank draining pattern

The Figs. 6 and 7, the liquid level in the tank varies with regard to time. The figure, according to the author, displays a trend toward slow decline because of the conical tank's tapering cross section. In relation to the tank's exit pipe length, the efflux duration likewise varies (0.25–0.50 m). As the liquid flow level changes, so does the tank's radius and its height as per $V = \frac{\pi}{3} \times r^2 \times h$. Equation 16, is regarded as the friction factor equation at polymer concentrations of 10 ppm since it agrees well with the friction factor equation.

Effect of polymer additives on drag reduction

Polymers are said to function better in turbulent environments than in laminar ones (Snelling 2006). The current analysis demonstrates that the drag reduction is created between the turbulent flow in the exit pipe and laminar flow in the tank (Moin et al., 2004; Peet et al., 2008). The author looked into this situation because using polyethylene oxide polymer solutions significantly reduces drag. According to Mohammed et al. (2021), polymer additives have an impact on how liquid drains from tanks. The optimal polymer concentration for reducing drag is examined and given below.

Table 3 Comparison of efflux time for different exit pipe lengths in case of $D = 0.28$ m, $d = 0.004$ m

S. no.	*L, m*	*H, m*	t_{act}, sec	t_{eff}, sec	%Error
1	1	0.30	390	389	0.257
2		0.28	382	381	0.2617
3		0.26	376	375	0.2659
4		0.24	370	369	0.2702
5		0.22	366	365	0.2732
6	0.75	0.30	366	365	0.2732
7		0.28	360	355	0.2778
8		0.26	352	346	1.7045
9		0.24	340	337	0.8902
10		0.22	328	327	0.3048
11	0.5	0.30	320	319	0.3125
12		0.28	330	326	1.2121
13		0.26	319	316	0.9404
14		0.24	310	305	1.6129
15		0.22	295	294	0.3389
16	0.25	0.30	300	297	1.0101
17		0.28	290	287	1.0344
18		0.26	280	276	1.4285
19		0.24	267	265	0.7490
20		0.22	260	254	1.1811

Table 4 Comparison of efflux time for different exit pipe lengths in case of $D = 0.255$ m, $d = 0.004$ m

S. no.	*L, m*	*H, m*	t_{act}, sec	t_{eff}, sec	%Error
1	1	0.30	387	386	0.2584
2		0.28	380	379	0.2631
3		0.26	373	373	0
4		0.24	368	367	0.2714
5		0.22	365	363	0.5419
6	0.75	0.30	359	353	1.6713
7		0.28	349	344	1.4326
8		0.26	338	334	1.4792
9		0.24	326	325	0.3067
10		0.22	319	317	0.6296
11	0.5	0.30	335	324	0.2985
12		0.28	318	316	0.6289
13		0.26	310	303	2.2580
14		0.24	300	293	2.389
15		0.22	290	282	2.7586
16	0.25	0.30	301	294	2.3255
17		0.28	290	284	2.0689
18		0.26	275	273	0.7272
19		0.24	265	262	1.1320
20		0.22	258	252	2.3255

Studies on tank draining pattern

The Figs. 8, 9 are represents the variation of liquid level in the tank drained with respect to time both with and without polymer solutions. The efflux time was calculated with respect to increasing in the polymer solutions level in the tank as well as variation of the diameter of the tank (D = 0.28 and 0.255) and length of the exit pipe. The author observed that the draining pattern was varied while changing the liquid level in the tank and concentration of the polymer solutions (Reischman et al. 1975). This agrees well with the friction factor Eq. (16), so equation xvi is accepted as the friction factor equation for polymer concentrations of 10ppm.

Influence of polymer concentration

The addition of polymer (PEO) showed the effective draining of solution by reducing the efflux time. The choice of the polymer is based on the observation that it is highly soluble in water. To evaluate the optimum concentration of polymer solution, efflux data were obtained for 10, 20, 30, 60, 65, 70 and 75 ppm shown in Figs. 10, 11.

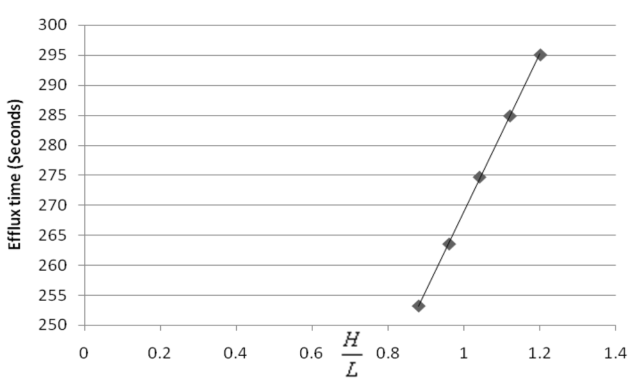

Fig. 2 Variation of H/L vs Efflux time for $D=0.28$ m, $L=0.25$ m

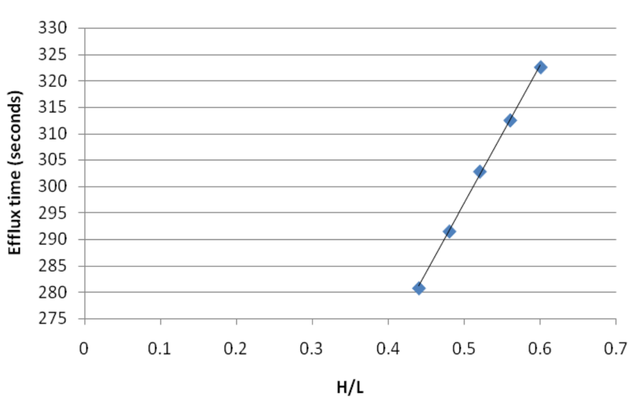

Fig. 3 Variation of H/L vs Efflux time for $D=0.28$ m $L=0.5$ m

The efflux time was decreased due to the increment in polymer concentration from 0 to 65 ppm. On further increment of polymer concentration from 65 to 75 ppm, the efflux time was increased due to the fact that the polymer became more viscous at 70 ppm and at 75 ppm. Thus the optimum concentration of polymer was found as 65 ppm (Viachogiannis and Hanratty 2003).

The above figures show that for a 4 mm exit pipe diameter, the efflux time (or drag) decreases with increasing polymer concentration, reaches its lowest point at 65 ppm and then rises as the concentration is raised to 70 ppm. As a result, it can be deduced that the optimal concentration occurs at 65 ppm and a 4 mm output pipe diameter. Additionally, the plot demonstrates that increasing the diameter of the exit pipe to 6 mm, there is little difference in the efflux time. It might be inferred that polymers only affect the contraction point. The following figure illustrates the trend for tanks with a 0.32 m diameter. Consequently, it may be said that the ideal concentration in instance.

It follows that the optimum concentration for polymer solutions depends only on the tank's diameter has no bearing

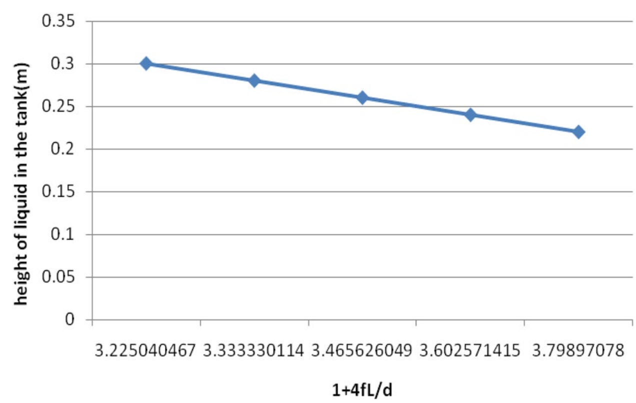

Fig. 4 Variation of 1 + 4f L/d vs liquid height in the tank (H) ($D=0.28$, $L=0.25$ m)

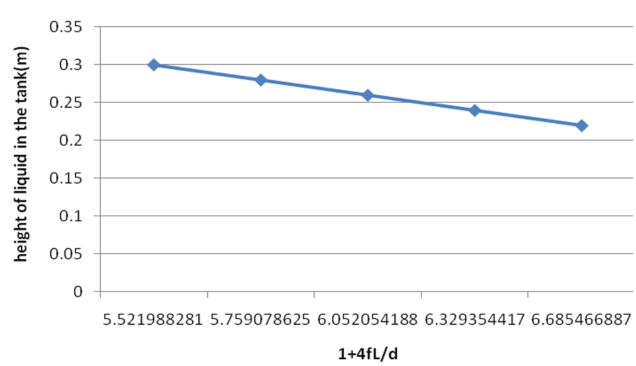

Fig. 5 Variation of 1 + 4f L/d vs liquid height in the tank (H) ($D=0.28$ $L=0.5$ m)

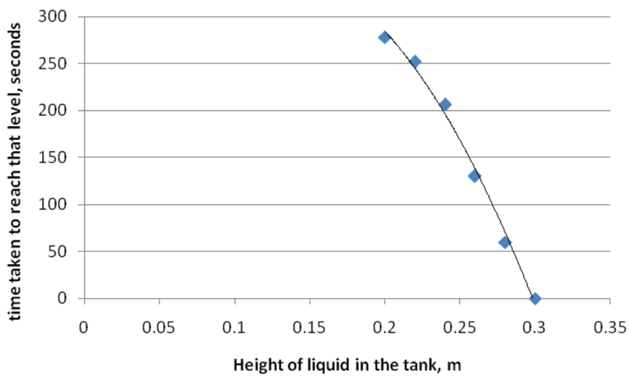

Fig. 6 Variation of tank level with respect to time ($D = 0.28$ m and $L = 0.25$ m)

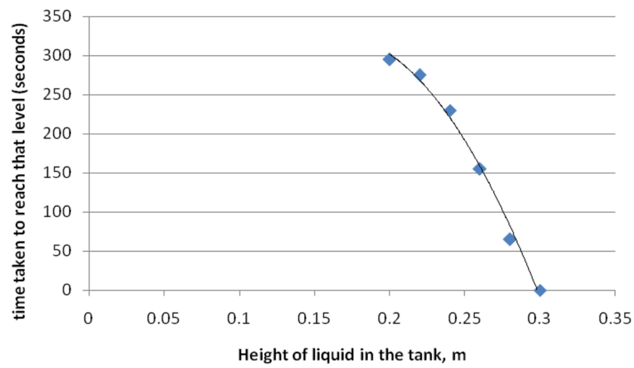

Fig. 7 Variation of tank level with respect to time ($D = 0.28$ m, $L = 0.5$ m)

on the outflow pipe's diameter, either the length of the exit pipe or the height of the liquid in the tank. However, the tank's diameter, initial outflow rate, tank liquid level height, and exit pipe length all influences the polymer drag reduction.

Calculation of % drag reduction

The difference in frictional pressure drop of solutions in the absence and presence of drag reducer along a pipeline segment at a constant flow rate can be yield quantitatively an expression for drag reduction as reported by earlier investigators (Sommerfeld and Stallybrass 1992; Virk 1975).

$$\%DR = \left[\frac{\Delta P_{f,\text{untreated}} - \Delta P_{f,\text{treated}}}{\Delta P_{f,\text{untreated}}}\right] \times 100$$

where $\Delta P_{f,\text{untreated}}$ refers to pressure drop in the absence of polymer and $\Delta P_{f,treated}$ refers to pressure drop in the presence of polymer.

In terms of friction factors, this can also be written as Jones and Moddock (1969)

$$\%DR = \left[\frac{f_{\text{untreated}} - f_{\text{treated}}}{f_{\text{untreated}}}\right] \times 100$$

where $f_{\text{untreated}}$ and f_{treated} refer to friction both with and without polymers.Even though both are same for dilute solutions, they were different for concentrated solution is likely become visco-elastic (Toonder 1995).

The below equation is defined as on the basis of % Drag reduction and efflux time (Kostic 1994):

$$\%DR = \left[\frac{t_{\text{eff,untreated}} - t_{\text{eff,treated}}}{t_{\text{eff,untreated}}}\right] \times 100$$

Efflux time measured based on treated and untreated of polymer are referred as $t_{\text{eff,untreated}}$ and $t_{\text{eff,treated}}$.

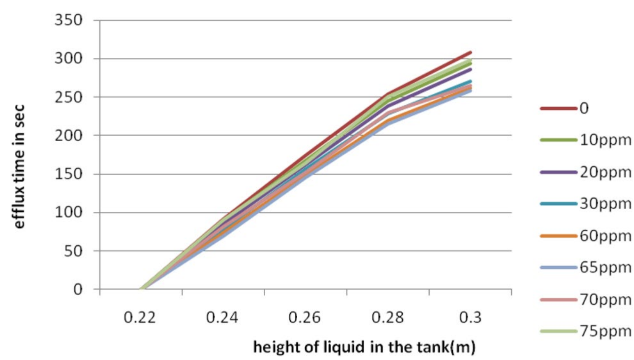

Fig. 8 Variation of efflux time with level of the liquid in the absence and presence of polymer concentration variation used for $D = 0.28$ m, $d = 0.004$ m and $L = 0.75$ m

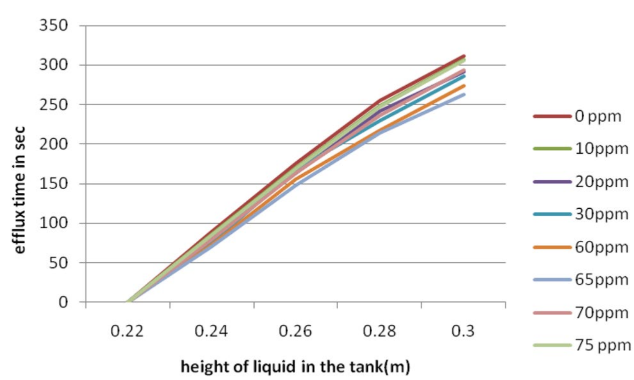

Fig. 9 Variation of efflux time with level of the liquid in the absence and presence of polymer concentration variation used for $D = 0.255$ m, $d = 0.004$ m and $L = 0.75$ m

The calculated values of percentage of drag reduction are shown in Tables 5 and 6, respectively.

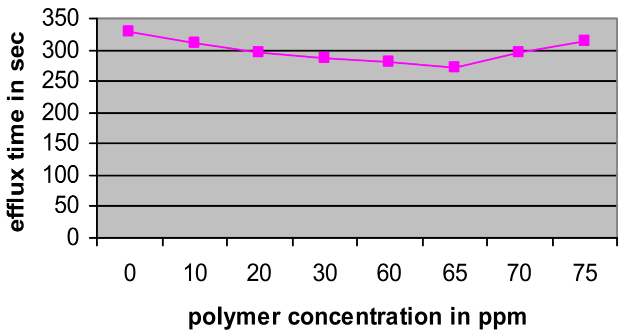

Fig. 10 Influence of polymer concentration on efflux time (for 0.28 m diameter of tank, 4 mm dia of exit pipe, 0.75 m length)

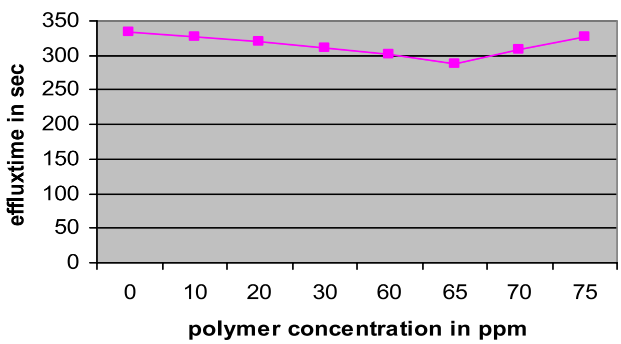

Fig. 11 Influence of polymer concentration on efflux time (for 0.255 m diameter of tank, 4 mm dia of exit pipe, 0.75 m length)

Conclusion

The summary of present work has been illustrating that experimental works were conducted in gravity-driven flow systems to obtain the data on efflux times while draining a storage tank filled with water and water-soluble polymer solutions. The following conclusions have been drawn based on the experimental data analysis obtained from the drag reduction and efflux time calculations, absence and presence of polymer additive:

- The draining pattern of polymer contained liquid was found to be relatively faster than absence of polymer. Mathematical expression was used for evaluating the efflux time.
- Polymer solution concentrations reduce the drag in once through flow system.
- Friction factor equation closely approximates the experimental values of efflux time data.
- For effective draining of tank solutions, an optimum concentration of polymer (PEO) was found to be 65 ppm.

Table 5 % Drag reduction for $d = 0.004$ m and for different exit pipe lengths

S. no.	Polymer concentration (ppm)	%Drag reduction for $d = 0.004$ m, $H = 0.30$ m			
		$L = 1$ m	$L = 0.75$ m	$L = 0.5$ m	$L = 0.25$ m
1	10	3.733333	2.02983	3.476102	3.654485
2	20	1.157125	2.679874	3.892692	3.126426
3	30	1.468968	2.183168	4.09292	3.426407
4	60	1.894901	3.085459	4.41205	3.838589
5	65	2.259187	3.269035	3.243474	4.265779
6	70	1.191463	2.443041	4.128014	3.494113
7	75	0.784375	2.063827	3.543291	3.292147

Table 6 %Drag reduction for $d = 0.006$ m and for different exit pipe lengths

S. no.	Polymer concentration (ppm)	%Drag reduction for $d = 0.006$ m, $H = 0.30$ m			
		$L = 1$ m	$L = 0.75$ m	$L = 0.5$ m	$L = 0.25$ m
1	10	2.431783	2.667382	1.701247	1.717764
2	20	2.964552	3.21281	2.413668	1.937871
3	30	3.424773	4.076147	2.692027	2.393648
4	60	4.194938	4.479183	3.073534	2.629903
5	65	4.906819	5.325646	3.367919	2.954324
6	70	3.238833	3.305958	1.529457	2.163073
7	75	2.607111	2.315875	1.360034	2.012359

- The author observed the gradual increasing of efflux time with an enlarging of the exit pipe length and increase in tank diameter.
- And also observed the gradual reduction of efflux time with an enlarging of the exit pipe diameter and increase in liquid level in the tank's height.

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s40808-022-01518-z.

Data availability statement The present work differs as compared to the below mentioned works in geometry selection, polymer selection (Polyethylene Oxide), modeling of the conical tank, fluid flow and friction factor calculations, etc. The author conducted the detailed study on "Development of Mathematical Model for Efflux Time in Conical Tank using Polyethylene Oxide as Drag Reducing Agent" and influences the various parameters on drag reduction coefficients by changing the tank size, exit pipe length, various exit pipe diameters and various polymer concentrations, etc.

References

Agulilar G, Gaslijevic K, Mathys EF (2006) Reduction of friction in fluid transport: experimental investigation. Revista Mexiciana De Fisca 52:444. https://www.redalyc.org/pdf/570/57065310.pdf

Choi KS, Yang X, Calyton BR, Glover EJ, Alatar M, Semenov BN, Kulik VM (1997) Turbulent drag reduction using complaint surfaces. Proc R Soc Lond 453:2229. https://doi.org/10.1098/rspa.1997.0119

David BV, Roche ECJ (1999) Efflux time from tanks with exit pipes and fittings. Int J Eng Ed 5(3): 206–212. https://www.ijee.ie/articles/Vol15-3/ijee1051.pdf

Joye DD, Barret BC (2003) The tank draining problem revisited: do these equations actually work. Can J Chem Eng 81:1052–1057. https://doi.org/10.1002/cjce.5450810516

Drappie J, Divoux T, Amarouchene Y, Yertrand F, Rodts S, Cadot O (2006) Turbulent drag reduction by surfactants. Europhys Lett J 74(2):362–368. https://doi.org/10.1209/epl/i2005-10519-x

Fortuna G, Hanratty TJ (1972) The influence of drag-reducing polymers on turbulence in the viscous sublayer. J Fluid Mech 53:575–586. https://doi.org/10.1017/S0022112072000321

Hart PW, Sommerfeld JT (1995) Expressions for gravity drainage of annular and Toroidal containers. Process Saf Prog 14(4):238–243. https://doi.org/10.1002/prs.680140406

Henoch C, Krupenkin TN, Olonder PK, Taylor JA, Hodes MS, Lyons AM, Peguero C, Breuer K (2006) Turbulent drag reduction using super hydrophobic surfaces. In: 3rd AIAA Flow Conference. San Francisco, pp 1–5. https://doi.org/10.2514/6.2006-3192

Jones WM, Moddock JL (1969) Relaxation effects in the flow of dilute polymer solutions through tubes of granular beds. Br J App Phys (J. PhyD) 2(2):797–808. https://iopscience.iop.org/article/10.1088/0022-3727/2/6/304/pdf

Jurban BA, Zurigat YH, Al-shukri MS, Al-Busaidim HH (2006) The use of drag reduction agent and a detergent for drag reduction in a circulatory vertical flow. Poly Plast Tech Eng 45:533. https://doi.org/10.1080/03602550600554083

Kostic M (1994) The ultimate asymptote and possible causes of friction drag and heat transfer reduction phenomena. J Energy HMT. 16:1–14. https://www.researchgate.net/publication/251516715_The_Ultimate_Asymptotes_and_Possible_Causes_of_Friction_Drag_and_Heat_Transfer_Reduction_Phenomena

Li FC, Kawaguchi Y, Yu B, Wei JJ, Hishida K (2008) Experimental study of drag reduction mechanism for a dilute surfactant solution flow. Int J Heat Mass Transf 51:835–843. https://doi.org/10.1016/j.ijheatmasstransfer.2007.04.048

MohamedAS BAD, Konstantin M (2021) Particle subgrid scale modeling in hybrid RANS/LES of turbulent channel flow at low to moderate Reynolds number. Powder Techno. https://doi.org/10.1016/j.powtec.2021.11.057

Paschkewitz JS, Dubief YVES, Dimitropoulos CD, Shaqfeh ESG, Moin P (2004) Numerical simulation of turbulent drag reduction using rigid fibers. J Fluid Mech 518:281–317. https://doi.org/10.1017/S0022112004001144

Peet Y, Sagaut P (2008) Turbulent drag reduction using sinusoidal riblets with triangular cross section. In: 38th AIAA Fluid Dynamics Conference and Exhibit, June 23–26 Seattle, WA. https://doi.org/10.2514/6.2008-3745

Reischman MM, Tiederman WG (1975) Laser doppler anemometer measurement in drag-reducing channel flows. J Fluid Mech 70:369–392

Snelling D (2006) Surfactant drag reduction using mixed counter ions, thesis submitted to Department of Chemical Engineering. The Ohio State University.

Sommerfeld JT, Stallybrass MP (1992) Elliptical Integral solutions for drainage of horizontal cylindrical vessels with piping friction. Ind Eng Chem Res 31:743–746. https://doi.org/10.1021/ie00003a015

Subbarao ChV (2011) Comparison of efflux time between cylindrical and conical tanks through an exit pipe. Int J Appl Sci Eng 9: 33. https://gigvvy.com/journals/ijase/articles/ijase-201104-9-1-033.pdf

Subbarao ChV, Divya P, Naidu DA, King P (2013) Drag reduction by anionic surfactant solutions in gravity driven flow system. Iran J Chem Chem Eng 32(1): 95–101. https://doi.org/10.30492/ijcce.2013.5909

Toonder JMJ (1995) Drag reduction by polymer additives in turbulent pipe flow, Ph.D Thesis, submitted to Delft University of Technology

Viachogiannis M, Hanratty TJ (2003) Influence of wavy structure and large scale polymer structures on drag reduction. Experim Fluids 36: 685–700. https://ui.adsabs.harvard.edu/link_gateway/2004ExFl36685V/doi:10.1007/s00348-003-0745-3

Virk PS (1975) Drag reduction fundamentals. AIChE J 21:625–653. https://doi.org/10.1002/aic.690210402

Vov LVS, Pomyalov A, Procaccia I, Tiberkevich V (2005) Drag reduction by micro-bubbles-the limit of minute bubbles. Phys Rev Lett 94:174502-1–174502-5. https://doi.org/10.1103/PhysRevLett.94.174502

Wang Y, Yu B, Zakin LJ, Shi H (2011) Review on Drag Reduction and Its Heat. Add Adv Mech Eng. https://doi.org/10.1155/2011/478749

Xi L (2019) Turbulent drag reduction by polymer additives: Fundamentals and recent advances. Phys Fluids 31:121302. https://doi.org/10.1063/1.5129619