



K. K. Wagh Institute of Engineering Education & Research, Nashik
(An Autonomous Institute From A.Y. 2022-23)

SUMMER-2024	
Exam Seat No.:	
Academic Year:2023-2024	Semester:III
Class:SY	Program:B.Tech
Branch Code:MEC/ROB	Pattern:2022
Name of Course:Applied Mathematics III	Course Code:SMH222501
Max. Marks:60	Duration:2.30 Hrs.

Instructions: Candidates should read carefully the instructions printed on the Question Paper and on the cover page of the Answer Book, which is provided for their use.

1. This question paper contains 3 page(s).
2. Answer to each new question is to be started on a new page.
3. Assume suitable data wherever required, but justify it.
4. Draw the neat labelled diagrams, wherever necessary.
5. The last columns indicates the Course Outcome and level of Blooms Taxonomy of the Question/sub-question.

Question No. 1 Attempt following Question

- 1 a) Solve the following differential Equation by Laplace Transform (6) CO3

$$y'' + y = t, y(0) = 1, y'(0) = -2$$

Question No. 2 Attempt following Question

- 2 a) Solve : $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = x^5$ (6) CO2

Question No. 3 Attempt following Question

- 3 a) A flexible string of length π is tightly stretched between $x=0$ and $x=\pi$ on x -axis. Its ends being fixed at these points. When set into small transverse vibration, the displacement $y(x, t)$ from x -axis from any point x at time t is given by $\frac{\partial^2 y}{\partial t^2} = 4 \frac{\partial^2 y}{\partial x^2}$, find the solution of the differential equation which satisfies the conditions:

(i) $u(0, t) = 0$

(ii) $u(\pi, t) = 0$

(iii) $\frac{\partial u}{\partial t} = 0$ when $t = 0$

(iv) $u(x, 0) = 0.1 \sin x + 0.01 \sin 2x$, when $0 < x < \pi$

OR

- 3 b) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, under the conditions: (8) CO2

(i) $u(0, t) = 0$

(ii) $u(\pi, t) = 0$

(iii) $\frac{\partial u}{\partial t} = 0$ when $t = 0$

(iv) $u(x, 0) = 2x$, when $0 < x < \pi$

- 3 c) The temperature at any point of the insulated metal rod of one meter length is governed by the D.E. (8) CO2
 $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, subject to the following conditions the conditions:

(i) $u(0, t) = 0^\circ C$, for all t

(ii) $u(1, t) = 0^\circ C$, for all t

(iii) $u(x, 0) = 50^\circ C$, $0 \leq x \leq 1$

OR

- 3 d) An infinitely long plane uniform plate is bounded by two parallel edges in the y – *direction* and an (8) CO2
 end at right angles to them. The breadth of the plate is π . This end is maintained at temperature u_0
 at all points and other edges at zero temperature. Find the steady state temperature function $u(x, y)$.

Question No. 4 Attempt following Question

- 4 a) Compute correlation coefficient between x and y values using following data. (6) CO4

x	10	11	14	14	20	22	16	12	15	13
y	12	14	15	16	21	26	21	15	16	14

OR

- 4 b) Obtain the regression lines for the following data : (6) CO4

x	15	3	5	7	9	10	12	2
y	16	5	8	10	12	14	15	2

- 4 c) The first four moments about the working mean 3.5 of a distribution are 0.0375, 0.4546, 0.0609 and (5) CO4
 0.5074. Calculate the first four moments about the mean. Also evaluate β_1 , β_2 and comment upon
 the skewness and kurtosis of the distribution.

OR

- 4 d) The first four moments about the working mean 30.2 of a distribution are 0.255, 6.222, 30.211 and (5) CO4
 400.25. Calculate the first four moments about the mean. Also evaluate β_1 , β_2 and comment upon
 the skewness and kurtosis of the distribution.

- 4 e) Number of road accidents on a highway during a month follows a Poisson distribution with mean 5. (5) CO5
Find the probability that in a certain month number of accidents on the highway will be

- (i) Less than 3
(ii) More than 3

OR

- 4 f) In a certain city 4000 tube lights are installed. If the lamps have average life of 1500 burning hours (5) CO5
with standard deviation 100 hours. Assuming normal distribution

- (i) How many lamps will fail in first 1400 hours.
(ii) How many lamps will last beyond 1600 hours.

[Given: Area corresponding to $z=1$ is 0.3413]

Question No. 5 Attempt following Question

- 5 a) Show that $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ is irrotational. Find corresponding scalar (6) CO2
potential ϕ such that $\vec{F} = \nabla\phi$

OR

- 5 b) Show that $\vec{F} = (2xz^3 + 6y)\vec{i} + (6xz^2 - 2y)\vec{j} + (3x^2z^2 - y^2)\vec{k}$ is irrotational. Find (6) CO2
corresponding scalar potential ϕ such that $\vec{F} = \nabla\phi$

- 5 c) Find Directional derivative of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ along the line (5) CO2
 $2(x-2) = (y+1) = (z-1)$

OR

- 5 d) Find Directional derivative of $\phi = xy^2 + yz^3$ at $(1, -1, 1)$ in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$. (5) CO2

- 5 e) Find the work done in moving a particle along the straight line joining the points $(0, 0, 0)$ and $(2, 1, 3)$ (5) CO2
under the field of force given by $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$. Is the field conservative?

OR

- 5 f) Evaluate $\iint_S \nabla \times \vec{F} \cdot d\vec{s}$ for $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$, where S is the surface of the paraboloid (5) CO2
 $z = 9 - x^2 - y^2$, $z \geq 0$.

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