



K. K. Wagh Institute of Engineering Education & Research, Nashik
(An Autonomous Institute From A.Y. 2022-23)

SUMMER-2024	
Exam Seat No.:	
Academic Year:2023-2024	Semester:III
Class:SY	Program:B.Tech
Branch Code:ELE	Pattern:2022
Name of Course:Applied Mathematics-III	Course Code:SMH222601
Max. Marks:60	Duration:2.30 Hrs.

Instructions: Candidates should read carefully the instructions printed on the Question Paper and on the cover page of the Answer Book, which is provided for their use.

1. This question paper contains 2 page(s).
2. Answer to each new question is to be started on a new page.
3. Assume suitable data wherever required, but justify it.
4. Draw the neat labelled diagrams, wherever necessary.
5. The last columns indicates the Course Outcome and level of Blooms Taxonomy of the Question/sub-question.
6. use of non-Programable calculator is allowed.

Question No. 1 Attempt following Question

- 1a) Solve the Cauchy's Differential Equation: $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = x^5$ (6) CO2

Question No. 2 Attempt following Question

- 2a) Solve the differential equation by Laplace Transform: $\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 4y = te^{-2t}$, $y(0) = 0$, $y'(0) = 1$ (6) CO3

Question No. 3 Attempt following Question

- 3a) Find Fourier Transform of: $f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$ Also evaluate $\int_0^\infty \frac{\sin \lambda}{\lambda} d\lambda$ (5) CO3

OR

- 3b) Find Fourier cosine transform of: $f(x) = e^{-x} - 2e^{-2x}$, $x > 0$ (5) CO3

- 3c) Solve the integral equation of: $\int_0^\infty f(x) \cos \lambda x dx = e^{-\lambda}$, $\lambda > 0$ (5) CO3

OR

- 3d) Solve the integral equation of: $\int_0^\infty f(x) \sin \lambda x dx = \begin{cases} 1, & \text{if } 0 \leq \lambda \leq 1 \\ 2, & \text{if } 1 < \lambda \leq 2 \\ 0, & \text{if } \lambda > 2 \end{cases}$ (5) CO3

- 3e) Show that: $\int_0^\infty \frac{(1 - \cos \pi \lambda) \sin \lambda x}{\lambda} d\lambda = \begin{cases} \frac{\pi}{2} & \text{if } 0 \leq x \leq \pi \\ 0 & \text{if } x > \pi \end{cases}$ (6) CO4

OR

- 3f) Find fourier transform of $f(x) = e^{-|x|}$, Also find its integral representation. (6) CO4

Question No. 4 Attempt following Question

- 4a) Find Z-transform of: $f(k) = k^2 5^k$ (5) CO1

OR

- 4b) Find Z-transform of: $f(k) = \frac{\sin \alpha k}{e^{\beta k}}, k \geq 0$ (5) CO1

- 4c) Find Inverse Z-transform by using inversion integral method: $F(z) = \frac{10z}{(z-1)(z-2)}$ (5) CO1

OR

- 4d) Find Inverse Z-transform of: $F(z) = \frac{z}{(z-\frac{1}{2})(z-\frac{1}{3})}, \frac{1}{3} < z < \frac{1}{2}$ (5) CO1

- 4e) Find $f(k)$ if $f(k+2) + 3f(k+1) + 2f(k) = 0, k \geq 0, f(0) = 0, f(1) = 1$ (6) CO3

OR

- 4f) Find $f(k)$ if $f(k+1) + f(k) = 1, k \geq 0, f(0) = 0$ (6) CO3

Question No. 5 Attempt following Question

- 5a) Find Directional derivative of $\phi = x^2 - y^2 - 2z^2$ at $(2, -1, 3)$ in the direction of PQ where $Q(5, 6, 4)$ (5) CO1

OR

- 5b) Find Directional derivative of $\phi = 2x^2 + 3y^2 + z^2$ at $(2, 1, 3)$ along the line $2(x-2) = y-1 = z-3$ (5) CO1

- 5c) Find the work done Under the force field $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$ from $(0, 0, 0)$ to $(1, 1, 1)$. (5) CO4

OR

- 5d) Evaluate: $\int_S \int \text{curl} \vec{F} \cdot \hat{n} dS$, Where $\vec{F} = (x^2 + y - 4)\vec{i} + 3xy\vec{j} + (2xz + z^2)\vec{k}$ and S is the surface of paraboloid $z = 9 - (x^2 + y^2)$ above the XOY plane. (5) CO4

- 5e) Two maxwell's equations are $\nabla \cdot \vec{B} = 0, \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$. Given $\vec{B} = \text{curl}(\vec{A})$ then deduce that $\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\text{grad}(V)$ where V is a scalar point function. (6) CO4

OR

- 5f) Show that: $\vec{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}, \vec{H} = \nabla \times \vec{A}$ are solutions of Maxwell's equations:
 (i) $\nabla \times \vec{H} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t}$, (ii) $\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}$ if 1) $\nabla \cdot \vec{A} + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0$, 2) $\nabla^2 \vec{A} = \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2}$ (6) CO4

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