



K. K. Wagh Institute of Engineering Education & Research, Nashik
(An Autonomous Institute From A.Y. 2022-23)

	In Sem Examination-I Winter 2023		
	Exam Seat No.:		
	Academic Year:2023-2024	Semester: III	
	Name of Programme: B. Tech (Civil)	Pattern:2022	
	Name of Course: Applied Mathematics-III	Course Code: SMH222301	
	Max. Marks:30	Duration: 1 hr	

	<p>Instructions: Candidates should read carefully the instructions printed on the Question Paper and on the cover page of the Answer Book, which is provided for their use.</p> <ol style="list-style-type: none">1. This question paper contains 3 page(s).2. Answer to each new question is to be started on a new page.3. Assume suitable data wherever required, but justify it.4. Draw the neat labelled diagrams, wherever necessary.5. The last columns indicates the Course Outcome and level of Blooms Taxonomy of the Question/sub-question.6. Use of non programmable pocket calculator is allowed	
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Question No. 1 Attempt following Question

a) Solve: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$ (5) CO2

OR

b) Solve: $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$ (5) CO2

c) Solve: $(D^2 - 4D + 3)y = x^3 e^{2x}$ (5) CO2

OR

d) Solve: $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\log x)$ (5) CO2

- e) Solve by MVP: $(D^2 + 1)y = \operatorname{cosec} x$ (5) CO2

OR

- f) Solve: $\frac{dx}{3x-4y} = \frac{dy}{4x-2z} = \frac{dz}{2y-3x}$ (5) CO2

Question No. 2 Attempt following Question

- a) The deflection of a strut with one end built in and the other supported and subjected to end thrust P, satisfies the equation

$$\frac{d^2 y}{dx^2} + a^2 y = \frac{a^2 R}{P} (l - x). \text{ Given that } \frac{dy}{dx} = y = 0 \text{ when } x=0. \quad (5) \text{ CO4}$$

Prove that $y = \frac{R}{P} \left(\frac{\sin ax}{a} - l \cos ax + l - x \right)$

OR

- b) The differential equation for the elastic curve of a beam is

$$EI \frac{d^2 y}{dx^2} = -\frac{Wx}{2} - Py$$

Where E, I, W and P are constants. Assume that beam to be positioned horizontally with one end at $x=0$ and other end at $x=l$. (5) CO4

With $y(0)=0$ and $\left(\frac{dy}{dx}\right)_{x=\frac{l}{2}} = 0$. Show that the deflection at the centre

is $\frac{W}{2P} \left(\frac{1}{n} \tan \frac{nl}{2} - \frac{l}{2} \right)$ where $n^2 = \frac{P}{EI}$.

- c) A string is stretched and fastened to two points 1 apart. Motion is started by displacing the string in the form $u = a \sin \pi x$ from which it is released at time $t=0$. Find the displacement $u(x, t)$ from one end

using wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. (5) CO4

Prove that $u(x, t) = \sum_{n=1}^{\infty} b_n \sin n\pi x \cos n\pi ct$

OR

- d) If $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibrations of a string of length l fixed at both ends, find the solution with boundary conditions,

i. $y(0, t) = 0$

ii. $y(l, t) = 0$

and initial conditions,

iii. $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

(5) CO4

Prove that $y(x, t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \cos \frac{n\pi ct}{l}$

e)

A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is

$$u(x, 0) = \begin{cases} x, & 0 \leq x \leq 50 \\ 100 - x, & 50 \leq x \leq 100 \end{cases} \quad (5) \text{ CO4}$$

Prove that $u(x, t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{100} e^{-\frac{n^2 \pi^2 c^2 t}{10000}}$

OR

f)

Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ if

- i) u is finite for all t
- ii) $u(0, t) = 0, \forall t$
- iii) $u(l, t) = 0, \forall t$
- iv) $u(x, 0) = u_0$, for $0 \leq x \leq l$, where l being the length of the bar.

(5) CO4

Prove that $u(x, t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} e^{-\frac{n^2 \pi^2 c^2 t}{l^2}}$