

PHYSICS

Introduction to Electromagnetic Theory

WITH LAB MANUAL

A.B. Bhattacharya

Atanu Nag



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by A. B. Bhattacharya, Atanu Nag

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FOREWORD

Engineering has played a very significant role in the progress and expansion of mankind and society for centuries. Engineering ideas that originated in the Indian subcontinent have had a thoughtful impact on the world.

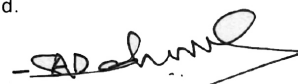
All India Council for Technical Education (AICTE) had always been at the forefront of assisting Technical students in every possible manner since its inception in 1987. The goal of AICTE has been to promote quality Technical Education and thereby take the industry to a greater heights and ultimately turn our dear motherland India into a Modern Developed Nation. It will not be inept to mention here that Engineers are the backbone of the modern society - better the engineers, better the industry, and better the industry, better the country.

NEP 2020 envisages education in regional languages to all, thereby ensuring that each and every student becomes capable and competent enough and is in a position to contribute towards the national growth and development.

One of the spheres where AICTE had been relentlessly working from last few years was to provide high-quality moderately priced books of International standard prepared in various regional languages to all it's Engineering students. These books are not only prepared keeping in mind it's easy language, real life examples, rich contents and but also the industry needs in this everyday changing world. These books are as per AICTE Model Curriculum of Engineering & Technology – 2018.

Eminent Professors from all over India with great knowledge and experience have written these books for the benefit of academic fraternity. AICTE is confident that these books with their rich contents will help technical students master the subjects with greater ease and quality.

AICTE appreciates the hard work of the original authors, coordinators and the translators for their endeavour in making these Engineering subjects more lucid.


(Anil D. Sahasrabudhe)

Acknowledgement

The author(s) are grateful to AICTE for their meticulous planning and execution to publish the technical book for Engineering and Technology students.

We sincerely acknowledge the valuable contributions of the reviewer of the book Prof. R.P Dahiya, for making it students' friendly and giving a better shape in an artistic manner.

This book is an outcome of various suggestions of AICTE members, experts and authors who shared their opinion and thoughts to further develop the engineering education in our country.

It is also with great honour that we state that this book is aligned to the AICTE Model Curriculum and in line with the guidelines of National Education Policy (NEP) -2020. Towards promoting education in regional languages, this book is being translated in scheduled Indian regional languages.

Acknowledgements are due to the contributors and different workers in this field whose published books, review articles, papers, photographs, footnotes, references and other valuable information enriched us at the time of writing the book.

Finally, we like to express our sincere thanks to the publishing house, M/s. Khanna Book Publishing Company Private Limited, New Delhi, whose entire team was always ready to cooperate on all the aspects of publishing to make it a wonderful experience.

A. B. Bhattacharya, Atanu Nag

Preface

The book titled “**Physics - Introduction to Electromagnetic Theory**” is an outcome of the rich experience of our teaching of basic physics courses. The initiation of writing this book is to expose basic science to the engineering students to the fundamentals of physics as well as enable them to get an insight of the subject. Keeping in mind the purpose of wide coverage as well as to provide essential supplementary information, we have included the topics recommended by AICTE, in a very systematic and orderly manner throughout the book. Efforts have been made to explain the fundamental concepts of the subject in the simplest possible way.

During the process of preparation of the manuscript, we have considered the various standard text books and accordingly we have developed sections like critical questions, solved and supplementary problems etc. While preparing the different sections emphasis has also been laid on definitions and laws and also on comprehensive synopsis of formulae for a quick revision of the basic principles. The book covers all types of medium and advanced level problems and these have been presented in a very logical and systematic manner. The gradations of those problems have been tested over many years of teaching a wide variety of students.

Apart from illustrations and examples as required, we have enriched the book with numerous solved problems in every unit for proper understanding of the related topics. Under the common title “Physics” there is a set of four books covering different aspects and applications of physics in engineering. Out of those, the first one covers Introduction to Electromagnetic Theory, the second one is based on Introduction to Mechanics, the third one is related to Quantum Mechanics for Engineers and the fourth one is based on Oscillations, Waves and Optics. It is important to note that in all the books, we have included the relevant laboratory practical. In addition, besides some essential information for the users under the heading “Know More” we have clarified some essential basic information in the appendix and annexure section.

As far as the present book is concerned, “Physics - Introduction to Electromagnetic Theory” is meant to provide a thorough grounding in applied physics on the topics covered. This part of the physics book will prepare engineering students to apply the knowledge of Electromagnetic Theory to tackle 21st century and onward engineering challenges and address the related aroused questions. The subject matters are presented in a constructive manner so that an Engineering degree prepares students to work in different sectors or in national laboratories at the very forefront of technology.

We sincerely hope that the book will inspire the students to learn and discuss the ideas behind basic principles of engineering physics and will surely contribute to the development of a solid foundation of the subject. We would be thankful to all beneficial comments and suggestions which will contribute to the improvement of the future editions of the book. It gives us immense pleasure to place this book in the hands of the teachers and students. It was indeed a big pleasure to work on different aspects covering in the book.

A. B. Bhattacharya, Atanu Nag

Outcome Based Education

For the implementation of an outcome based education the first requirement is to develop an outcome based curriculum and incorporate an outcome based assessment in the education system. By going through outcome based assessments evaluators will be able to evaluate whether the students have achieved the outlined standard, specific and measurable outcomes. With the proper incorporation of outcome based education there will be a definite commitment to achieve a minimum standard for all learners without giving up at any level. At the end of the programme running with the aid of outcome based education, a students will be able to arrive at the following outcomes:

- PO-1: Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
- PO-2: Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
- PO-3: Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
- PO-4: Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
- PO-5: Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
- PO-6: The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
- PO-7: Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
- PO-8: Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
- PO-9: Individual and team work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

- PO-10: Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
- PO-11: Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
- PO-12: Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Course Outcomes

After completion of the course the students will be able to:

- CO-1: Describe different physical concepts of static electromagnetic fields.
- CO-2: Explain the principles of electrostatics and magnetostatics to describe boundary conditions for electric fields and potentials.
- CO-3: Discuss the concepts related to Faraday's law of electromagnetic induction.
- CO-4: Apply the Maxwell's equations to solve problems relating to propagation of waves in electromagnetic field theory.
- CO-5: Analyze different properties of magnetic materials.
- CO-6: Analyze the propagation of electromagnetic waves in different media.

Course Outcomes	Expected Mapping with Programme Outcomes (1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation)											
	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7	PO-8	PO-9	PO-10	PO-11	PO-12
CO-1	3	1	1	-	-	-	-	-	-	-	-	-
CO-2	3	1	2	1	1	-	-	-	-	-	-	-
CO-3	3	2	1	1	-	-	-	-	-	-	-	-
CO-4	3	3	2	1	1	-	-	-	-	-	-	-
CO-5	3	1	3	1	2	-	-	-	-	-	-	-
CO-6	3	3	3	1	1	-	-	-	-	-	-	-

Abbreviations and Symbols

List of Abbreviations

General Terms			
Abbreviations	Full form	Abbreviations	Full form
AC	Alternating Current	<i>emf</i>	electromotive force
BW	Band Width	Ge	Germanium
CO	Course Outcome	LCR	Inductor-Capacitor-Resistor
CRO	Cathode Ray Oscilloscope	<i>mmf</i>	magnetomotive force
CRT	Cathode Ray Tube	PO	Programme Outcome
DC	Direct Current	Q-factor	Quality factor
EM	electromagnetic	UO	Unit Outcome
Units Used			
Abbreviations	Full form	Abbreviations	Full form
A	ampere	nF	nano farad
C	coulomb	Oe	oersted
G	gauss	T	tesla
GHz	Gigahertz	V	volt
Hz	hertz	W	watt
kHz	kilohertz	Wb	weber
mH	milli henry	μ A	micro ampere
nA	nano ampere	μ C	micro coulomb
nC	nano coulomb	μ F	microfarad

List of Symbols

Symbols	Description	Symbols	Description
A	Magnetic vector potential	J	Current density
B	Magnetic induction	J_d	Displacement current density
C	Capacitance of a capacitor	K	Co-efficient of coupling
D	Electric displacement	L	Self-inductance
e	Electronic charge	M	Mutual inductance
E	Electric field intensity	M_s	Saturation magnetization
f_{res}	Resonant frequency	P	Poynting vector
g	Gyromagnetic ratio	p_{eff}	Effective number of Bohr magneton
H	Magnetic intensity	r	Reflection coefficient
I_d	Displacement current		

Symbols	Description	Symbols	Description
R_H	Hall coefficient	ϵ_0	Permittivity of free space
S	Reluctance	ϵ_r, k	Relative permittivity
t	Transmission coefficient	λ	Linear charge density
T_N	Neel temperature	μ_0	Permeability of free space
U	Electromagnetic energy density	ρ	Volume charge density
V_H	Hall voltage	σ	Surface charge density
Z	Impedance of a medium	ϕ	Electric flux
α	Attenuation constant	ϕ_m	Magnetic scalar potential
β	Phase constant	ψ	Wave function
γ	Propagation constant	ω_L	Larmor frequency
δ	Skin depth	χ	Electromagnetic susceptibility

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Guidelines for Teachers

To implement Outcome Based Education (OBE) knowledge level and skill set of the students should be enhanced. Teachers should take a major responsibility for the proper implementation of OBE. Some of the responsibilities (not limited to) for the teachers in OBE system may be as follows:

- Within reasonable constraint, they should manoeuvre time to the best advantage of all students.
- They should assess the students only upon certain defined criterion without considering any other potential ineligibility to discriminate them.
- They should try to grow the learning abilities of the students to a certain level before they leave the institute.
- They should try to ensure that all the students are equipped with the quality knowledge as well as competence after they finish their education.
- They should always encourage the students to develop their ultimate performance capabilities.
- They should facilitate and encourage group work and team work to consolidate newer approach.
- They should follow Blooms taxonomy in every part of the assessment.

Bloom's Taxonomy

Level	Teacher should Check	Student should be able to	Possible Mode of Assessment
Create	Students ability to create	Design or Create	Mini project
Evaluate	Students ability to Justify	Argue or Defend	Assignment
Analyse	Students ability to distinguish	Differentiate or Distinguish	Project/Lab Methodology
Apply	Students ability to use information	Operate or Demonstrate	Technical Presentation/ Demonstration
Understand	Students ability to explain the ideas	Explain or Classify	Presentation/Seminar
Remember	Students ability to recall (or remember)	Define or Recall	Quiz

Guidelines for Students

Students should take equal responsibility for implementing the OBE. Some of the responsibilities (not limited to) for the students in OBE system are as follows:

- Students should be well aware of each UO before the start of a unit in each and every course.
- Students should be well aware of each CO before the start of the course.
- Students should be well aware of each PO before the start of the programme.
- Students should think critically and reasonably with proper reflection and action.
- Learning of the students should be connected and integrated with practical and real life consequences.
- Students should be well aware of their competency at every level of OBE.

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1

Electrostatics in Vacuum

UNIT SPECIFICS

This unit elaborately discusses the following topics:

- Quantization and conservation of charge;
- Coulomb's law and its vector form;
- Superposition principle;
- Gauss law in electrostatics, its integral and differential form;
- Electrostatic field and its curl;
- Electric field intensity and electrostatic potential;
- Poisson's and Laplace's equations;
- Application of Poisson's and Laplace's equations to Cartesian, Spherical and Cylindrical coordinate systems to solve electrostatics problems.

The practical applications of the topics are discussed for generating further curiosity and creativity as well as improving problem solving capacity.

Besides giving a large number of multiple choice questions as well as questions of short and long answer types marked in two categories following lower and higher order of Bloom's taxonomy, assignments through a number of numerical problems, a list of references and suggested readings are given in the unit so that one can go through them for practice. It is important to note that for getting more information on various topics of interest some QR codes have been provided in different sections which can be scanned for relevant supportive knowledge.

After the related practical, based on the content, there is a "Know More" section. This section has been judiciously designed so that the supplementary information provided in this part becomes beneficial for the users of the book. This section mainly highlights the initial activity, examples of some interesting facts, analogy, history of the development of the subject focusing the salient observations and finding, timelines starting from the development of the concerned topics up to the recent time, applications of the subject matter for our day-to-day real life or/and industrial applications on variety of aspects, case study related to environmental, sustainability, social and ethical issues whichever applicable, and finally inquisitiveness and curiosity topics of the unit.

RATIONALE

This fundamental unit on electrostatics helps students to get a primary idea about the quantization and conservation of charge and introduces the concepts of electric field and potential and also has given priority to derive the fundamental equations of electrostatics in vacuum. In addition to some fundamental laws of electrostatics like Coulomb's law, Gauss law as well as Poisson's and Laplace's equations and their applications for solving various electrostatic problems, particularly in Cartesian, Spherical and Cylindrical symmetric systems, practical portion demonstrates the physical concepts and processes. A good number of examples of electrostatic phenomena are there. The integrated concepts exercises for this unit may be considered most interesting when applied to general situations involving more than a narrow set of physical principles. As for example, the electric field exerts force on charges and so the relevance of dynamics comes.

PRE-REQUISITES

Mathematics: Co-ordinate Systems, Vector Calculus (Class XII)

Physics: Electrostatics (Class XII)

UNIT OUTCOMES

List of outcomes of this unit is as follows:

U1-O1: Describe quantization and conservation of charge

U1-O2: Explain Coulomb's law, its vector form and superposition principle

U1-O3: Explain Gauss law in electrostatics, its integral and differential form

U1-O4: Describe about electrostatic field and its curl, electric field intensity and electrostatic potential

U1-O5: Explain Poisson's and Laplace's equations

U1-O6: Apply Laplace's equations to Cartesian, Spherically and Cylindrically symmetric systems to solve electrostatics problems

Unit-1 Outcomes	EXPECTED MAPPING WITH COURSE OUTCOMES (1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation)					
	CO-1	CO-2	CO-3	CO-4	CO-5	CO-6
U1-O1	3	-	1	-	-	-
U1-O2	3	1	-	-	-	-
U1-O3	3	2	-	2	-	-
U1-O4	3	2	1	-	-	1
U1-O5	2	2	-	1	-	1
U1-O6	1	1	-	-	-	-

1.1 INTRODUCTION

Electric charge is the property responsible for electrostatic forces binding the atomic configuration. There are two kinds of electric charges *viz.* the negative charge and the positive charge. The fundamental negatively charged and positively charged particles are well known electrons and protons respectively. While these charges of identical polarity are kept separated by the same distance, the electrostatic force experienced by two electrons and by two protons is exactly the same and is repulsive in nature. On the other hand, electric force between two dissimilar types of charges like an electron and a proton placed at the same distance is attractive in nature. Charge present on a body can be measured by comparing it with the charge of an electron. Magnitude of the charge of an electron and a proton are same, having a value of 1.602×10^{-19} C. The only difference is that for an electron it is negative while for a proton it is positive in nature.

1.2 QUANTIZATION OF CHARGES

A theoretical understanding of electric charge quantization is important if we wish to understand the interactions of elementary particles. Of course, understanding elementary particle interactions is one of the major goals of theoretical physics. It is not known for certain why electric charge is quantized. But all possible charges available in nature can be represented by integral multiples of a unit of charge *e i.e.*, charge of an electron or a proton. Thus charge q on a body can always be denoted by

$$q = ne \quad (1.1)$$

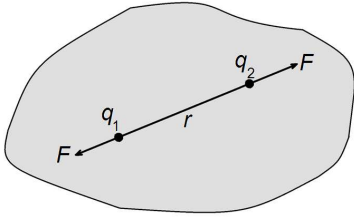
where $e = -1.602 \times 10^{-19}$ C and $n =$ any positive or negative integer. Eq. (1.1) illustrates the basic idea of quantization of charges which can be stated as: ‘charge on a body must always be represented by an integral multiple of e .’

1.3 CONSERVATION OF CHARGE

All atoms are always electrically neutral, which means that they carry the same amount of positive and negative charge, so their net charge is zero. Because electrons are negative, some other part of the atom must contain positive charge. As the fundamental positive and negative units of charge are carried on protons and electrons, we would expect that the total charge cannot change in any system that we define. During mutual interaction, if an object (or part of an object) gains charge, another object (or part of it) must lose charge. In other words, charge cannot be created or destroyed *i.e.*, total charge of an isolated system is always conserved. If two bodies are rubbed against each other capable of transferring charges between them, one will become negatively charged by gaining electrons while the other becomes positively charged by losing those electrons.

1.4 COULOMB’S LAW

Coulomb (1736-1806) carried out his experiments to determine the nature of dependence of electrostatic force on the distance between two charged particles as well on the magnitudes of charges possessed by them. He demonstrated that the electrostatic force increases when the objects acquire more charge and their distance is less and thereby proposed a law regarding the electrostatic force which is known as the Coulomb’s law of electrostatics. It states that, two point charges separated by a fixed distance always exert a force on each other. This force is directly proportional to the product of the charges and inversely proportional to the square of their distance of separation.

**Fig.1.1:** Coulomb's law

This force acts along the line joining the charges. The nature of the force is repulsive for like charges and attractive for unlike charges. Mathematically, if two point charges q_1 and q_2 are separated by a distance r (Fig. 1.1), according to Coulomb's law the force exerted on q_1 due to the charge q_2 will be,

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \quad (1.2)$$

The constant ϵ_0 is called permittivity of free space and in SI system it has the value $8.854 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$. The value of $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$ in SI system. In CGS system, $\frac{1}{4\pi\epsilon_0} = 1$.

EXAMPLE 1.1

Example 1.1 If two different point charges are kept separated by a distance d , the force acting between them is F . Calculate the mutual distance if the effective force between them is reduced to $F/3$.

Solution

If the charges are q_1 and q_2 then initially, $F = \frac{q_1 q_2}{4\pi\epsilon_0 d^2}$

$$\therefore \frac{q_1 q_2}{4\pi\epsilon_0} = Fd^2$$

Finally, let the mutual distance between the charges is x , when the effective force between them is reduced to $F/3$. Thus,

$$\frac{q_1 q_2}{4\pi\epsilon_0 x^2} = \frac{F}{3} \Rightarrow 3Fd^2 = Fx^2 \Rightarrow x = \sqrt{3}d.$$

EXAMPLE 1.2

Example 1.2 Compare the effective electrostatic and gravitational forces acting between the electron and proton in a hydrogen atom. (Given, $m_e = 9.1 \times 10^{-31} \text{ kg}$, $m_p = 1.7 \times 10^{-27} \text{ kg}$ and $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$).

Solution

If the distance between electron and proton is r , the electrostatic force acting

between them will be, $F_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = 9 \times 10^9 \cdot \frac{e^2}{r^2}$

and the gravitational force acting between them will be, $F_g = G \frac{m_e m_p}{r^2}$

$$\therefore \frac{F_e}{F_g} = \frac{9 \times 10^9}{G} \cdot \frac{e^2}{m_e m_p} = 2.26 \times 10^{39}$$

which is the ratio between F_e and F_g .

Example 1.3 The mass of a thermocol ball is 9×10^{-5} kg and it is carrying a $5 \mu\text{C}$ charge. Another similar type of ball is placed at a distance of 2 cm just at the top of it. Calculate the charge of the second ball, so that the system is at equilibrium.

Solution

The two balls will be in equilibrium, when the electrostatic repulsion force between the balls will be balanced by the weight force of the second ball.

$$\therefore \text{At equilibrium } \frac{q_1 q_2}{4\pi\epsilon_0 r^2} = mg$$

$$\text{Now, } mg = 9 \times 10^{-5} \times 9.8 \text{ N; } r = 2 \times 10^{-2} \text{ m; } q_1 = 5 \times 10^{-6} \text{ C and}$$

$$4\pi\epsilon_0 = \frac{1}{9} \times 10^{-9}.$$

$$\therefore q_2 = \frac{9 \times 10^{-5} \times 9.8 \times (2 \times 10^{-2})^2}{5 \times 10^{-6} \times 9} \times 10^{-9} = \frac{9.8 \times 4}{5} \times 10^{-12} = 7.84 \times 10^{-12} \text{ C}$$

which gives the charge of the second ball, so that the said system will be in equilibrium.

EXAMPLE 1.3

Relative permittivity (Dielectric constant): Relative permittivity (ϵ_r) can be defined as the ratio of the force between two point charges placed at a particular distance in free space or vacuum to the force between them in a specific medium, when separated by the equal amount of distance.

$$\text{Thus, } \epsilon_r = \frac{F_{vac}}{F_{med}} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} / \frac{q_1 q_2}{4\pi\epsilon r^2} \quad (1.3)$$

$$\text{or, } \epsilon_r = \frac{\epsilon}{\epsilon_0} \quad (1.4)$$

where ϵ_0 is the free space permittivity and ϵ is the permittivity in that specified medium.

1.4.1 Vector form of Coulomb's Law

The more generalized approach is to use vector notation while deducing the Coulomb's law by considering the direction of the forces between the charge particles. For two like charges q_1 and q_2 separated by a distance r (Fig. 1.2), the force exerted on q_1 due to the charge q_2 may be obtained vectorically as,

$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \hat{r}_{21} \quad (1.5)$$

where \hat{r}_{21} is the unit vector from q_1 to q_2 . Similarly, the force on q_2 due to the charge q_1 is,

$$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \hat{r}_{12} \quad (1.6)$$

where \hat{r}_{12} is the unit vector from q_2 to q_1 .

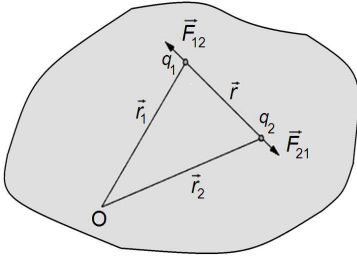


Fig.1.2: Vector illustration of Coulomb's law

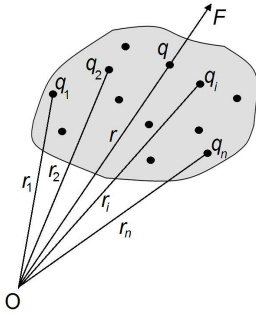


Fig.1.3: Coulomb's law for distributed charges

$$\begin{aligned} \text{As, } \hat{r}_{12} &= \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} \\ &= -\frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} = -\hat{r}_{21} \end{aligned}$$

$$\text{with } r = |\vec{r}_1 - \vec{r}_2| = |\vec{r}_2 - \vec{r}_1|$$

$$\therefore \vec{F}_{12} = -\vec{F}_{21} \quad (1.7)$$

From Eq. (1.7) it is clear that the force on charge 1 due to charge 2 is always equal and opposite of the force on charge 2 due to charge 1. Similarly, the force on i^{th} charge due to j^{th} charge can be generalized as,

$$\vec{F}_{ij} = \frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{r^2} \hat{r}_{ji} = \frac{1}{4\pi\epsilon_0} \frac{q_i q_j (\vec{r}_j - \vec{r}_i)}{|\vec{r}_j - \vec{r}_i|^3} \quad (1.8)$$

In CGS system, the force on i^{th} charge due to j^{th} charge will be

$$\vec{F}_{ij} = \frac{q_i q_j}{r^2} \hat{r}_{ji} = \frac{q_i q_j (\vec{r}_j - \vec{r}_i)}{|\vec{r}_j - \vec{r}_i|^3} \quad (1.9)$$

For n number of charges $q_1, q_2, q_3, \dots, q_n$ located at their respective positions $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$ (Fig. 1.3), the force experienced by the charge q located at \vec{r} due to the rest of the charges will be given by,

$$\vec{F} = \frac{q}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i (\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} \quad (1.10)$$

1.5 SUPERPOSITION PRINCIPLE

If we have a number of point charges in a force field, to determine the force on a particular charge due to the remaining charges we can apply *principle of superposition of charges*. According to this principle, the force \vec{F}_3 on a point charge q_3 (Fig. 1.4) due to the point charges q_1 and q_2 can be determined by the vector addition of the forces \vec{F}_{31} and \vec{F}_{32} acting on q_3 due to q_1 and q_2 respectively *i.e.*,

$$\begin{aligned} \vec{F}_3 &= \vec{F}_{31} + \vec{F}_{32} \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_3}{r_{31}^2} \hat{r}_{31} + \frac{q_2 q_3}{r_{32}^2} \hat{r}_{32} \right] \end{aligned}$$

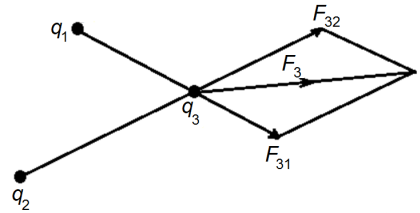


Fig.1.4: Illustrating superposition theorem

In general, for a system of n charges, the force on j^{th} charge due to the remaining charges is

$$\vec{F}_{ij} = \frac{q_j}{4\pi\epsilon_0} \sum_{\substack{i=1 \\ i \neq j}}^n \frac{q_i}{r_{ij}^2} \hat{r}_{ij} \quad (1.11)$$

Example 1.4 4 charges Q , q , Q and q are placed at the corners of a square ABCD. If the net force at C is zero, then find a relationship between the charges Q and q .

Solution

In figure, all the forces acting on the charge Q at C is depicted. For the net force acting on it to be zero we must have, $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

$$\text{or,} \quad F_1 \cos 45^\circ \hat{i} - F_1 \sin 45^\circ \hat{j} - F_2 \hat{j} + F_3 \hat{i} = 0$$

$$\text{or,} \quad (F_1 \cos 45^\circ + F_3) \hat{i} + (-F_1 \sin 45^\circ - F_2) \hat{j} = 0$$

$$\therefore F_1 \cos 45^\circ + F_3 = 0 \Rightarrow F_1 \cos 45^\circ = -F_3$$

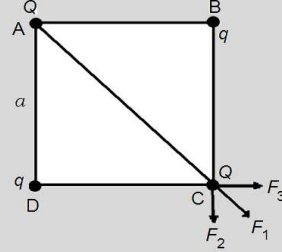
$$\text{and} \quad F_1 \sin 45^\circ + F_2 = 0 \Rightarrow F_1 \sin 45^\circ = -F_2$$

$$\text{Now,} \quad F_1 = k \frac{Q^2}{2a^2} \text{ and } F_3 = k \frac{Qq}{a^2}$$

$$\text{Thus,} \quad k \frac{Q^2}{2a^2} \cdot \frac{1}{\sqrt{2}} = -k \frac{Qq}{a^2}$$

$$\text{or,} \quad Q = -2\sqrt{2}q$$

which is the required relationship.



EXAMPLE 1.4

1.6 CHARGE DENSITIES

As per the charge quantization, charges must exist in a body as multiples of electronic charge (e). However, as the electronic charge is very small, any microscopic charge distribution contains a large number of such electronic charges. So, a charge in a body can be considered to be distributed in terms of a *charge density* function.

In many cases, charged particles (*e.g.*, electrons, protons, positive ions) are unevenly distributed throughout some volume. In this case, we can define a volume charge density. Another possibility is that charge is unevenly distributed across some surface. In this case, we can define a surface charge density. There may also be a case where charge is unevenly distributed over some linear segment of length of a conductor. In this case, we can consider it as a linear charge density.

If Δq be the charge distributed in the conductor volume element ΔV , the volume charge density

$$\text{will be,} \quad \rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta q}{\Delta V} \quad (1.12)$$

On the other hand, if Δq be the charge distributed on a conductor surface element ΔS , the surface charge density will be,

$$\sigma = \lim_{\Delta S \rightarrow 0} \frac{\Delta q}{\Delta S} \quad (1.13)$$

Again, if Δq be the charge distributed in a linear segment of length Δl of the conductor, the linear charge density will be,

$$\lambda = \lim_{\Delta l \rightarrow 0} \frac{\Delta q}{\Delta l} \quad (1.14)$$

In general, the force on a point charge q located at position \vec{r} due to a volume distribution of charge with density $\rho(\vec{r}_0)$, a surface distribution of charge with density $\sigma(\vec{r}_0)$ and a linear distribution of charge with density $\lambda(\vec{r}_0)$ is given by,

$$\vec{F} = \frac{q}{4\pi\epsilon_0} \left[\int_V \frac{\vec{r} - \vec{r}_0}{|\vec{r} - \vec{r}_0|^3} \rho dV_0 + \int_S \frac{\vec{r} - \vec{r}_0}{|\vec{r} - \vec{r}_0|^3} \sigma dS_0 + \int_l \frac{\vec{r} - \vec{r}_0}{|\vec{r} - \vec{r}_0|^3} \lambda dl_0 \right] \quad (1.15)$$

\vec{r}_0 being the space coordinate where the charges are assumed to be distributed for which the force in Eq. (1.15) is calculated.

1.7 ELECTRIC FIELD AND FIELD INTENSITY

In the region surrounding a test charge its electrostatic force can be experienced when another charge is brought close to the test charge. This region is known as the *electric field* of the said test charge. Magnitude of the electric field depends on the material media in which the field is to be calculated. The electric field is measured in N/C or V/m in SI units.

Electric field intensity at a point gives the strength of the electric field at that particular point. The electric field intensity at any particular point can be obtained by the limiting ratio of the force on a test charge situated at that point to the test charge when it approaches zero.

If \vec{F} is the force on a test charge q placed at the point \vec{r} with respect to some origin O the electric field intensity at a position \vec{r} will be,

$$\vec{E}(r) = \lim_{q \rightarrow 0} \frac{\vec{F}}{q} \quad (1.16)$$

The electric field intensity \vec{E} at a point of observation \vec{r} (Fig. 1.5) due to some point charge q located at \vec{r}' where the source point is situated can be given by generalizing Eq. (1.16) as:

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} \quad (1.17)$$

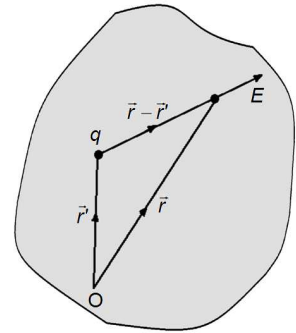


Fig. 1.5: Electric field intensity at a point

If there are n point charges $q_1, q_2, q_3, \dots, q_n$ located at $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$ respectively with a volume distribution of charge with density $\rho(\vec{r}_0)$ in a volume V , a surface distribution of charge with density $\sigma(\vec{r}_0)$ on a surface S and a linear distribution of charge with density $\lambda(\vec{r}_0)$ along a linear segment l , the force exerted on a test charge q located at \vec{r} can be obtained as,

$$\vec{F} = \frac{q}{4\pi\epsilon_0} \left[\sum_{i=1}^n q_i \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3} + \int_V \frac{\vec{r} - \vec{r}_0}{|\vec{r} - \vec{r}_0|^3} \rho dV_0 + \int_S \frac{\vec{r} - \vec{r}_0}{|\vec{r} - \vec{r}_0|^3} \sigma dS_0 + \int_l \frac{\vec{r} - \vec{r}_0}{|\vec{r} - \vec{r}_0|^3} \lambda dl_0 \right] \quad (1.18)$$

and thus the electric field at \vec{r} will be obtained from Eq. (1.16) as

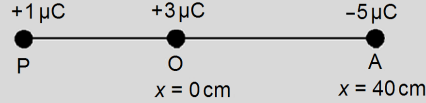
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \left[\sum_{i=1}^n q_i \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3} + \int_V \frac{\vec{r} - \vec{r}_0}{|\vec{r} - \vec{r}_0|^3} \rho dV_0 + \int_S \frac{\vec{r} - \vec{r}_0}{|\vec{r} - \vec{r}_0|^3} \sigma dS_0 + \int_l \frac{\vec{r} - \vec{r}_0}{|\vec{r} - \vec{r}_0|^3} \lambda dl_0 \right] \quad (1.19)$$

Example 1.5 Two charges $+3 \mu\text{C}$ and $-5 \mu\text{C}$ are placed along the X-axis at $x = 0$ cm and $x = 40$ cm. Calculate, at which point the field intensity due to the system will be zero?

Solution

Let, at point P the field intensity is zero *i.e.*, the force on a unit positive charge placed at P is zero. Let $OP = x$.

$$\therefore k \frac{3}{(OP)^2} = k \frac{5}{(AP)^2}$$



$$\text{or,} \quad \frac{3}{x^2} = \frac{5}{(40 + x)^2} \Rightarrow \frac{40 + x}{x} = \sqrt{\frac{5}{3}} = 1.29$$

$$\text{or,} \quad 40 + x = 1.29x$$

$$\text{or,} \quad x = 137.91 \text{ cm.}$$

So, at a distance of 137.91 cm from the point O, the field intensity due to the system will be zero.

EXAMPLE 1.5

Example 1.6 Two charges $5 \times 10^{-11} \text{ C}$ and $-2.7 \times 10^{-11} \text{ C}$ are kept separated by a distance of 20 cm. Along the line joining them, at what point a third charge can be placed so that no net force will be acting on it due to the previous charges.

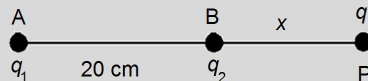
Solution

Let, the charges $5 \times 10^{-11} \text{ C}$ and $-2.7 \times 10^{-11} \text{ C}$ are situated at points A and B respectively. Consider the third charge is to be placed at a distance x , from B.

Now, the force on this charge q due to the charge at A is,

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{5q \times 10^{-11}}{(0.2 + x)^2}$$

and it acts along AB.



EXAMPLE 1.6

EXAMPLE 1.6

and the force on charge q due to the charge at B is,

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{2.7q \times 10^{-11}}{x^2} \text{ along AB.}$$

According to the problem, $\vec{F}_1 = \vec{F}_2$

$$\text{or, } \frac{1}{4\pi\epsilon_0} \left[\frac{5q \times 10^{-11}}{(0.2+x)^2} - \frac{2.7q \times 10^{-11}}{x^2} \right] = 0$$

$$\text{or, } 5x^2 = 2.7(0.2+x)^2$$

$$\therefore x = 55.6 \text{ cm.}$$

EXAMPLE 1.7

Example 1.7 Two point charges $+1 \mu\text{C}$ and $-1 \mu\text{C}$, are placed at the two corners of the base of an equilateral triangle of sides 0.7 m . Calculate the electric field intensity at the third corner of the triangle due to them.

Solution

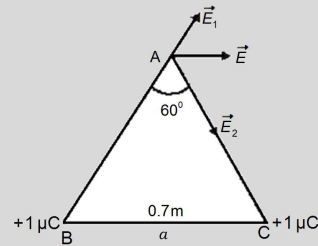
Let, the point charges $+1 \mu\text{C}$ and $-1 \mu\text{C}$ are placed at B and C.

If E be the resultant intensity at A due to them, then

$$E^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos 120^\circ = 2E_1^2 - E_1^2 = E_1^2$$

$$\text{Here } E_1 = E_2 = \frac{kq_1}{a^2}$$

$$\therefore E = 9 \times 10^9 \times \frac{10^{-6}}{(0.7)^2} = 18.37 \times 10^3 \text{ NC}^{-1}.$$



EXAMPLE 1.8

Example 1.8 Four charges $Q_1 = +2 \mu\text{C}$, $Q_2 = +2 \mu\text{C}$, $Q_3 = +5 \mu\text{C}$ and $Q_4 = -5 \mu\text{C}$ are placed at four corners of a square (as shown in the Fig). If the length of the diagonal is 2 m , calculate the field intensity at their intersecting point.

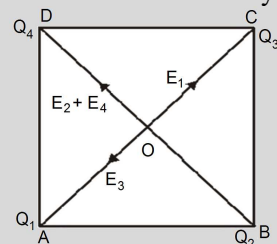
Solution

Let, O be the intersecting point of the diagonals. Now, the electric field intensity at O due to the point charge at A is,

$$E_1 = 9 \times 10^9 \times \frac{Q_1}{(OA)^2} = 9 \times 10^9 \times \frac{2 \times 10^{-6}}{(1)^2} = 18 \times 10^3 \text{ NC}^{-1}$$

the field intensity at O due to the point charge at B is

$$E_2 = 9 \times 10^9 \times \frac{2 \times 10^{-6}}{(1)^2} = 18 \times 10^3 \text{ NC}^{-1}$$



the field intensity at O due to the point charge at C is,

$$E_3 = 9 \times 10^9 \times \frac{5 \times 10^6}{(1)^2} = 45 \times 10^3 \text{ NC}^{-1}$$

and the field intensity at O due to the point charge at D is,

$$E_4 = 9 \times 10^9 \times \frac{5 \times 10^6}{(1)^2} = 45 \times 10^3 \text{ NC}^{-1}.$$

So, the net intensity along OA is $E' = E_3 - E_1 = (45 - 18) \times 10^3 = 2.3 \times 10^4 \text{ NC}^{-1}$.

and the net intensity along OD is $E'' = E_2 + E_4 = (45 - 18) \times 10^3 = 6.3 \times 10^4 \text{ NC}^{-1}$.

Hence the resultant intensity at O is, $E = \sqrt{(E')^2 + (E'')^2}$

$$\therefore E = 10^4 \sqrt{(2.3)^2 + (6.3)^2} = 6.71 \times 10^4 \text{ NC}^{-1}.$$

EXAMPLE 1.8

Example 1.9 Calculate the electric field intensity at $x = 0$, due to infinite charge distribution of equal charges q at $x = 1, x = 2, x = 4, x = 8, \dots$ along the X-axis.

Solution

The electric field intensity at $x = 0$, due to infinite charge distribution of same amount of charge q at respective positions $x = 1, x = 2, x = 4, x = 8, \dots$ along the X-axis will be,

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \left[\frac{q}{1^2} + \frac{q}{2^2} + \frac{q}{4^2} + \frac{q}{8^2} + \dots \right] = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \dots \right] \\ &= \frac{q}{4\pi\epsilon_0} \left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots \right] = \frac{q}{4\pi\epsilon_0} \frac{1}{1 - \frac{1}{4}} = \frac{1}{4\pi\epsilon_0} \frac{4q}{3} \text{ units.} \end{aligned}$$

EXAMPLE 1.9

Example 1.10 A drop of water having the same charge as that of an electron is hanging at equilibrium due to the Earth's electric field of 3 V/m. If electronic charge is $e = 4.805 \times 10^{-10}$ esu, calculate the radius of the water drop.

Solution

At equilibrium, $mg = eE \Rightarrow \frac{4}{3}\pi r^3 \times 1 \times g = eE$ (density of water = 1 gm/cc)

$$\text{or, } r^3 = \frac{3eE}{4\pi g} = \frac{3 \times 4.805 \times 10^{-10}}{4 \times 3.14 \times 980} \times \frac{3}{300 \times 100}$$

$$\therefore r = 0.1054 \text{ cm.}$$

which is the required radius of the water drop.

EXAMPLE 1.10

1.8 CONSERVATION OF ELECTROSTATIC FIELD

The curl of a vector field at a point measures the tendency for the vector field to swirl around that point. When it is zero definitely the force field can be regarded as a conservative force field. In this section we will try to calculate the curl of electrostatic field intensity and interpret whether it is a conservative field or not. To do that we will start by considering a system of n point charges $q_1, q_2, q_3, \dots, q_n$ located at $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$ in an electrostatic force field. The electric field at some position \vec{r}

where a test charge q is placed can be calculated as,
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n q_i \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3}$$

Now, using the vector identity

$$\vec{\nabla} \times \psi \vec{A} = \vec{\nabla} \psi \times \vec{A} + \psi \vec{\nabla} \times \vec{A}$$

we get,

$$\vec{\nabla} \times \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3} = \vec{\nabla} \frac{1}{|\vec{r} - \vec{r}_i|^3} \times (\vec{r} - \vec{r}_i) + \frac{1}{|\vec{r} - \vec{r}_i|^3} \vec{\nabla} \times (\vec{r} - \vec{r}_i)$$

The second term in above expression is zero, since $\vec{\nabla} \times (\vec{r} - \vec{r}_i) = 0$

and also from vector calculus we have,
$$\vec{\nabla} \frac{1}{|\vec{r} - \vec{r}_i|^3} = -\frac{3(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^5}$$

$$\therefore \vec{\nabla} \times \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3} = -\frac{3(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^5} \times (\vec{r} - \vec{r}_i) = 0$$

$$\therefore \vec{\nabla} \times \vec{E}(\vec{r}) = 0$$

As the curl of the electric field is zero so we can conclude that the electric field is conservative. Also, as in a conservative force field the work done is independent of the path traversed, hence in an electric field work done by the field in moving a charge, from one point to the other, depends only on the position of those two points but is independent of the path along which the charge is moved.

Example 1.11 Check whether the field $\vec{E} = axy^2(\hat{y}i + \hat{x}j)$ is conservative or not.

Solution

We have, $\vec{E} = axy^2(\hat{y}i + \hat{x}j)$

Now,

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ axy^3 & ax^2y^2 & 0 \end{vmatrix} = (2axy^2 - 3axy^2)\hat{k} = -axy^2\hat{k}$$

So, the given field is not conservative.

1.9 ELECTRIC FLUX

Electric flux can be defined as the total number of electric lines of forces passing normally through a surface placed in an electric field. Flux leaving a closed surface is positive, whereas flux entering a closed surface is negative. The net flux through the surface is zero if the number of lines that enter the surface is equal to the number that leaves the surface. The electric flux of an electric field \vec{E} , over an elementary surface area $d\vec{S}$ (Fig. 1.6) can be expressed as,

$$d\phi = \vec{E} \cdot d\vec{S} \quad (1.20)$$

and, the total flux, ϕ through the total surface S can be obtained as,

$$\phi = \int d\phi = \oint_S \vec{E} \cdot d\vec{S} = \oint_S E dS \cos \theta \quad (1.21)$$

where θ is the angle between \vec{E} and $d\vec{S}$.

For any linear isotropic medium; the flux density or the electric displacement in terms of electric field intensity is given by:

$$\vec{D} = \epsilon \vec{E} \quad [\text{where } \epsilon \text{ is the permittivity of the medium}] \quad (1.22)$$

and the electric flux ϕ for such a type of medium may be re-defined as

$$\phi = \oint_S \vec{D} \cdot d\vec{S} \quad (1.23)$$

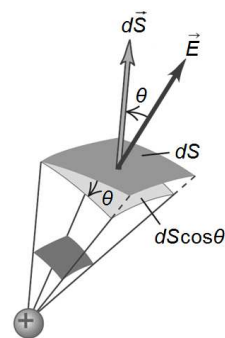


Fig.1.6: Electric flux for an electric field

Example 1.12 In a region of space, the electric field is given by $\vec{E} = 8\hat{i} + 4\hat{j} + 5\hat{k}$. Find the electric flux through any surface of area 100 unit in the Y-Z plane.

Solution

Electric flux,
$$\phi = \oint_S \vec{E} \cdot d\vec{S}$$

Here,
$$\vec{E} = 8\hat{i} + 4\hat{j} + 5\hat{k} \text{ and } d\vec{S} = 100\hat{i}.$$

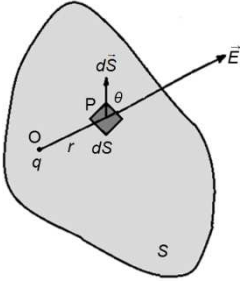
So,
$$\phi = (8\hat{i} + 4\hat{j} + 5\hat{k}) \cdot 100\hat{i} = 800 \text{ units.}$$

EXAMPLE 1.12

1.10 GAUSS LAW

Gauss's law is an alternative to Coulomb's law. While completely equivalent to Coulomb's law, Gauss's law provides a different way to express the relationship between electric charge and electric field. It was formulated by Carl Friedrich Gauss (1777–1855), one of the greatest mathematicians of all time. It states that the total electric flux ϕ i.e. the surface integral of the normal component of electric field intensity over a closed surface is equal to $1/\epsilon_0$ times the total enclosed electric charge by the surface.

Mathematically it may be stated as,



$$\phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} \quad (1.24)$$

where ϵ_0 is the free space permittivity.

Proof: Let, $+q$ point charge is placed at O and is enclosed by the surface S as shown in Fig. 1.7. Let, $d\vec{S}$ be an elementary area surrounding P. Let, \vec{E} be the electric field intensity at P due to charge $+q$ at O along the direction OP and its magnitude can be obtained as,

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

Fig.1.7: Illustration of Gauss's law

If θ be the angle between the vectors \vec{E} and $d\vec{S}$ then

$$\phi = \oint_S \vec{E} \cdot d\vec{S} = \oint_S E dS \cos \theta = \frac{q}{4\pi\epsilon_0} \oint_S \frac{dS \cos \theta}{r^2} = \frac{q}{4\pi\epsilon_0} \int d\omega$$

where $\int d\omega = \oint_S \frac{dS \cos \theta}{r^2}$ is the total solid angle about O subtended by S and is equal to 4π .

$$\text{So,} \quad \phi = \frac{q}{4\pi\epsilon_0} \cdot 4\pi = \frac{q}{\epsilon_0} \quad (1.25)$$

This is Gauss's theorem.

1.10.1 Gaussian Surface

The imaginary surface, in an electrostatic field where there is uniform magnitude of electric field intensity at every point on the surface and is always normal to it, is termed as the *Gaussian surface*.

1.10.2 Gauss's Law in Dielectric Medium

It states that the total amount of electric flux ϕ through any closed surface of a dielectric medium is $1/\epsilon_0$ times to the total charge q enclosed by it.

$$\text{Thus,} \quad \phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

$$\text{or,} \quad \oint_S \vec{D} \cdot d\vec{S} = q$$

where, $\vec{D} = \epsilon_0 \vec{E}$ is known as the electric displacement vector or the flux density.



1.10.3 Differential form of Gauss's Law

If a charge q is distributed over a volume V enclosed by a surface S and if ρ be the volume density of charge over a small elemental volume dV within it, then

$$q = \int_V \rho dV$$

Again from Gauss's law in electrostatics we have,

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

Using Gauss's theorem the above equation may be written as,

$$\int_V \vec{\nabla} \cdot \vec{E} dV = \frac{q}{\epsilon_0} = \frac{1}{\epsilon_0} \int_V \rho dV$$

Now equating the integrands on both sides of the above equation for any arbitrary volume element dV we have,

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (1.26)$$

This is the differential form of Gauss's law. It states that the divergence of the electric field intensity at any point is equal to $1/\epsilon_0$ times the charge density at that point.

In terms of electric displacement vector $\vec{D} = \epsilon_0 \vec{E}$ Eq. (1.26) may be written as,

$$\vec{\nabla} \cdot (\epsilon_0 \vec{E}) = \rho$$

$$\text{or,} \quad \vec{\nabla} \cdot \vec{D} = \rho \quad (1.27)$$

This is differential form of Gauss's law for any dielectric medium.

Example 1.13 If the electric field is given by $\vec{E} = \frac{1}{\epsilon_0} (x\hat{i} + y\hat{j} - 2z\hat{k})$ then find the charge density.

Solution

We have, $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

Here, $\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} (1 + 1 - 2) = 0$

So, $\rho = 0$, which is the required charge density.

EXAMPLE 1.13

1.11 APPLICATION OF GAUSS'S LAW

Gauss's law is valid for any distribution of charges and for any closed surface. The law can be used in two ways. If we know the charge distribution, and if it has enough symmetry to let us evaluate the integral in Gauss's law, we can find the field. Or if we know the field, we can use Gauss's law to find the charge distribution, such as charges on conducting surfaces. Gauss law can be applied to calculate the electric flux and hence the electric field intensity due to a uniform distribution of charge over a surface. In this section we present examples of both kinds of applications.

As you study them, watch for the role played by the symmetry properties of each system. We will use Gauss's law to calculate the electric fields caused by several simple charge distributions.

1.11.1 Electric Field for a Charged Cylinder

Let us consider a uniformly charged cylinder of radius a . It can be treated as an infinite line charge distribution. Let λ be the charge per unit length of the cylinder. To calculate the electric field for such distribution Gauss law can be applied. To calculate the field intensity at a distance r from the axis of the cylinder, let us draw a coaxial Gaussian cylindrical surface with length L and radius r (Fig. 1.8).

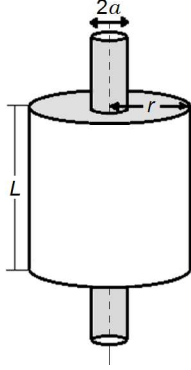


Fig.1.8: Gaussian field around a charged cylinder

The magnitude of the field is the same and is outward at all points on the Gaussian surface. The total enclosed charge in it will be, $q = \lambda L$.

Now, from Gauss law we have the electric flux

$$\phi = \oint_S \vec{E} \cdot d\vec{S} = \oint_S E dS = E \oint_S dS = E \cdot 2\pi r L$$

$$\text{or, } \frac{q}{\epsilon_0} = E \cdot 2\pi r L$$

$$\text{or, } E = \frac{q}{2\pi\epsilon_0 r L} = \frac{\lambda L}{2\pi\epsilon_0 r L}$$

$$\text{or, } E = \frac{\lambda}{2\pi\epsilon_0 r} \quad [\text{for } r > a] \quad (1.28)$$

Now to calculate the field intensity at any point inside the cylinder:

i) If all the charges lie on the surface of the cylinder *i.e.* if the cylinder is hollow, then the charge inside the cylinder is zero and the electric flux,

$$\phi = \int E \cdot dS = E \cdot 2\pi r L = 0$$

$$\therefore E = 0 \quad (1.29)$$

ii) If the charge is distributed uniformly throughout the cylinder then the enclosed charge within the inner cylinder with radius $r < a$, will be

$$q' = \frac{q}{\pi a^2} \cdot \pi r^2$$

$$\text{or, } q' = \frac{\lambda L r^2}{a^2}$$

So, using Gauss's law, the flux over the inner cylindrical surface will be,

$$E \cdot 2\pi r L = \frac{q'}{\epsilon_0}$$

$$\therefore E = \frac{q'}{2\pi\epsilon_0 r L}$$

$$\text{or, } E = \frac{\lambda r}{2\pi\epsilon_0 a^2} \quad [\text{for } r < a] \quad (1.30)$$

Variations of electric field E with distance r for both hollow and solid cylinder are shown respectively in Fig. 1.9 (a) and Fig. 1.9 (b).



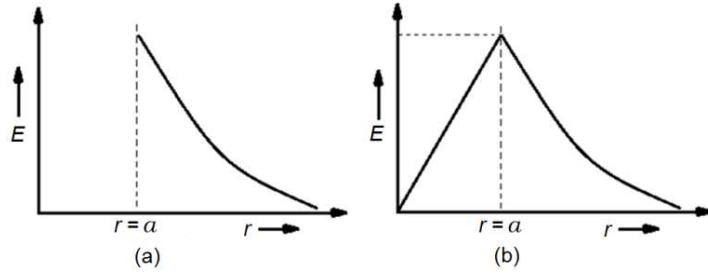


Fig.1.9: Variation of E with r around a charged (a) hollow and (b) solid cylinder

1.11.2 Electric Field Due to a Charged Solid Sphere

Consider that q amount of charge is distributed over a uniformly charged solid sphere of radius a . Now electric field can be calculated for the cases below:

a) At a point inside the surface ($r < a$): Here all the charge q is uniformly distributed throughout the volume of the sphere [Fig. 1.10 (a)] and the amount enclosed charge by the Gaussian sphere of radius r

is,

$$q' = \frac{q}{\frac{4}{3}\pi a^3} \cdot \frac{4}{3}\pi r^3 = \frac{qr^3}{a^3}$$

Using Gauss's law the field intensity inside the sphere is,

$$E \cdot 4\pi r^2 = \frac{q'}{\epsilon_0}$$

or,

$$E = \frac{q'}{4\pi\epsilon_0 r^2} = \frac{qr^3}{4\pi\epsilon_0 r^2 a^3} = \frac{qr}{4\pi\epsilon_0 a^3} = \frac{\rho r}{3\epsilon_0} \quad [\text{where } \rho = q / \frac{4}{3}\pi a^3]$$

\therefore

$$E = \frac{\rho r}{3\epsilon_0} \quad [\text{for } r < a] \quad (1.31)$$

b) At a point outside the sphere ($r > a$): We draw a Gaussian surface of radius $r > a$ [Fig. 1.10 (b)] so that the flux of the field over the surface is,

$$\phi = \int_S \vec{E} \cdot d\vec{S} = \int_S E \cdot dS = E \int_S dS$$

or,

$$\phi = E \cdot 4\pi r^2$$

or,

$$\frac{q}{\epsilon_0} = E \cdot 4\pi r^2$$

or,

$$E = \frac{q}{4\pi\epsilon_0 r^2} \quad [\text{for } r > a] \quad (1.32)$$

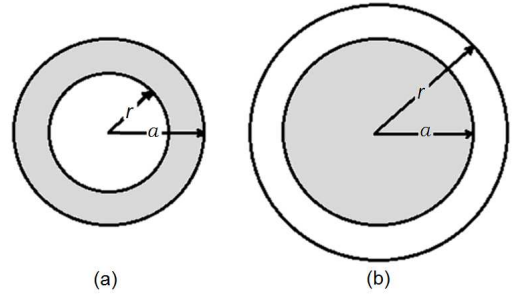


Fig.1.10: Gaussian surface (a) inside and (b) outside of a charged solid sphere

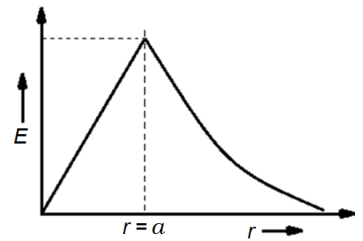


Fig.1.11: Variation of E with r for a charged solid sphere

From Eq. (1.31) and Eq. (1.32) it is clear that $E \propto r$ when $r < a$ and $E \propto 1/r^2$ when $r > a$. The variation of electric field E with r for the charge distribution is shown in Fig. 1.11.

Example 1.14 For a spherically symmetric charge distribution as given by

$$\rho(r) = \rho_0 \left(1 - \frac{r^3}{a^3} \right) \text{ for } 0 < r < a$$

and $\rho(r) = 0$ for $r > a$

Calculate the i) total charge, ii) electric field intensity \vec{E} for $r < a$ and $r > a$.

Solution

$$\begin{aligned} \text{i) Total charge will be } Q &= \int_V \rho dV = \rho_0 \int_{r=0}^a \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \left(1 - \frac{r^3}{a^3} \right) r^2 \sin \theta d\theta d\phi dr \\ &= 4\pi\rho_0 \left(\frac{a^3}{3} - \frac{a^3}{6} \right) = \frac{2\pi\rho_0}{3} a^3. \end{aligned}$$

ii) As per Gauss's law we can write

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0}$$

$$\text{For } 0 < r < a, E \times 4\pi r^2 = \frac{2\pi\rho_0}{3\epsilon_0} a^3$$

$$\text{or, } E = \frac{\rho_0 a^3}{6\epsilon_0 r^2}.$$

$$\text{At } r = a, E = \frac{\rho_0 a}{6\epsilon_0}.$$

$$\text{For } r > a, E = 0.$$

EXAMPLE 1.14

Example 1.15 Calculate the total charge for a spherically symmetric charge distribution given by $\rho(r) = \rho_0 \left(1 - \frac{r^2}{a^2} \right)$.

Solution

$$\begin{aligned} \text{Total charge will be, } Q &= \int_V \rho dV = \rho_0 \int_{r=0}^a \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \left(1 - \frac{r^2}{a^2} \right) r^2 \sin \theta d\theta d\phi dr \\ &= 4\pi\rho_0 \left(\frac{a^3}{3} - \frac{a^3}{5} \right) = \frac{8\pi\rho_0}{15} a^3. \end{aligned}$$

EXAMPLE 1.15

1.11.3 Electric Field Due to an Infinite Charged Sheet

For an infinitely extended sheet of charge, density of charge is due to the charges situated on the surface of the sheet. Now, to find the electric field at P (Fig. 1.12), consider the point Q which is at an equal distance from O *i.e.* $OP = OQ$. The symmetry of the problem suggests that the Gaussian surface will be cylindrical and carries the points P and Q at its opposite bases. Since the normal to the curved surface is perpendicular to the electric field, flux through the curved surface must be zero.

Thus, the total flux

$$\phi = 2\vec{E} \cdot \vec{\Delta S} = 2E \cdot \Delta S \quad [\text{As } E \parallel \Delta S]$$

where ΔS is the area of each bases. Thus using Gauss's law we

have,

$$2E \cdot \Delta S = \frac{q}{\epsilon_0} = \frac{\sigma \cdot \Delta S}{\epsilon_0}$$

where σ is the surface charge density.

$$\therefore E = \frac{\sigma}{2\epsilon_0} \quad (1.33) \text{ (a)}$$

In vector form, $\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$ [\hat{n} being a unit vector] (1.33) (b)

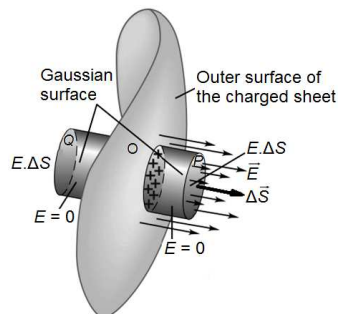


Fig.1.12: Infinitely charged sheet

Example 1.16 Derive an expression for the electric field between two infinitely extended parallel plate capacitors carrying charge with density σ . Assume the mutual separation between the plates is d .

Solution

Let us consider two infinitely extended plane parallel sheets as shown in Fig. Let us first consider a point P lying anywhere in the region between the two plates with charge densities $+\sigma$ and $-\sigma$.

We draw a normal cylinder of cross sectional area S through P having one of its ends in the conducting plate. Then we have from Gauss's law (as in the case in the infinite plane sheet).

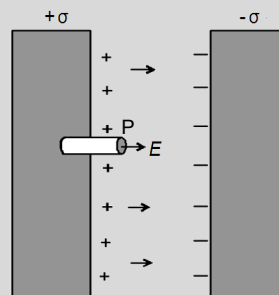
$$\int_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} \quad (\epsilon_0 \text{ is the permittivity})$$

or,

$$ES = \frac{\sigma S}{\epsilon_0}$$

hence

$$E = \frac{\sigma}{\epsilon_0}$$



So, the field between the two plates is uniform and is independent of their distance of separation. But any point lying outside the plate field will be zero as per the Gauss law (no charge enclosed within).

1.12 COULOMB'S LAW FROM GAUSS'S LAW

In electrostatics, Coulomb's law and Gauss's law are interrelated and one can be deduced from the other. To arrive at Coulomb's law from Gauss's law let us consider two point charges q_1 and q_2 separated by a distance r in an electrostatic region. To calculate the electric field intensity E due to q_1 let us draw a Gaussian sphere of radius r with q_1 at its center O (Fig. 1.13). Thus from Gauss's theorem the flux through the sphere is,

$$\iint_S \vec{E} \cdot d\vec{S} = \frac{q_1}{\epsilon_0}$$

By symmetry electric field at any point on the surface of the sphere is along the outward normal at that point and has the same magnitude at any point on the surface.

Thus,
$$\iint_S \vec{E} \cdot d\vec{S} = \iint_S E \cdot dS = \frac{q_1}{\epsilon_0}$$

or,
$$E \cdot 4\pi r^2 = \frac{q_1}{\epsilon_0}$$

or,
$$E = \frac{q_1}{4\pi\epsilon_0 r^2}$$

Thus the force experienced by the point charge q_2 due to the electric field of charge q_1 will be,

$$F = q_2 E = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}, \text{ which is Coulomb's law.}$$

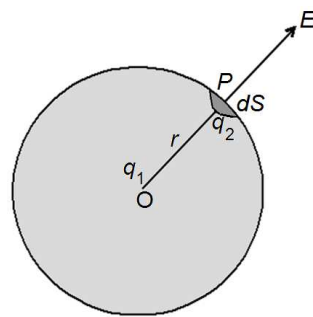


Fig.1.13: Coulomb's law from Gauss law

1.13 ELECTROSTATIC POTENTIAL

When a charged particle moves in an electric field, the field exerts a force that can do work on the particle. The work can be expressed in terms of electric potential energy. Hence the *electrostatic potential* at any point in an electrostatic field is related to the work done in carrying a unit positive charge from some other point to that point against that electric field. In other word, electric potential at any point in an electrostatic field can be defined as the amount of work done required to bring a unit positive charge from infinity to that particular point against the field. Potential is a property of a point in space and depends only on the source charges.

Now, if we wish to move a positive test charge ($+q$) from one fixed point to another fixed point in an electric field, the work done by this external agent in moving the charge q by a distance $d\vec{r}$ between the points will be given by,

$$dW = -q\vec{E} \cdot d\vec{r}$$

The negative sign is due to the fact that work is to be done on the system by the external agent. So, the total work done to move the charge q from some initial position i to a final position f in the electric field will be,

$$W = -q \int_i^f \vec{E} \cdot d\vec{r}$$

Now, according to the definition of electric potential at a point, as it represents the amount of work done required to bring a unit positive charge from infinity to that particular point against the electric field, the electric potential at a distance r from a charge q can be obtained by putting $i = \infty$ and $f = r$ as,

$$\begin{aligned} V &= -\int_{\infty}^r \vec{E} \cdot d\vec{r} \\ &= -\int_{\infty}^r \frac{q}{4\pi\epsilon_0 r^2} dr \end{aligned}$$

or,
$$V = \frac{q}{4\pi\epsilon_0 r} \quad (1.34)$$

The SI unit of electrostatic potential is J/C or volts (V).

1.14 ELECTRIC POTENTIAL DIFFERENCE

When a coulomb of charge (or any given amount of charge) possesses a relatively large quantity of potential energy at a given location, then that location is said to be a location of high electric potential. Similarly, if a coulomb of charge (or any given amount of charge) possesses a relatively small quantity of potential energy at a given location, then that location is said to be a location of low electric potential. The *potential difference* between any two points (A and B) in an electric field can be defined as the amount of work done required to bring a unit positive charge from the point B to A against the field of the charge q (Fig. 1.15).

So, potential difference between A and B is given by,

$$V_{AB} = \frac{W}{q} = \int_A^B dV = -\int_A^B \vec{E} \cdot d\vec{r}$$

\therefore
$$\begin{aligned} V_{AB} &= V_A - V_B = -\int_{r_A}^{r_B} \frac{q}{4\pi\epsilon_0 r^2} dr \\ &= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right] \end{aligned} \quad (1.35)$$

The potential difference is also measured in J/C or V. Also, from the definition of potential difference we can write

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz = -\vec{E} \cdot d\vec{l}$$

or,
$$\left(\hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z} \right) \cdot (\hat{i} dx + \hat{j} dy + \hat{k} dz) = -\vec{E} \cdot d\vec{l}$$

or,
$$\vec{\nabla} V \cdot d\vec{l} = -\vec{E} \cdot d\vec{l}$$

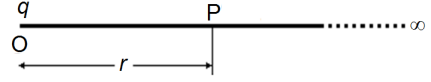


Fig.1.14: Electric field intensity

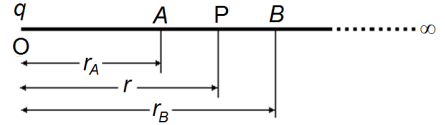


Fig.1.15: Potential difference

From which we obtain,

$$\vec{E} = -\vec{\nabla}V \quad (1.36)$$

So, the electric field intensity at any point is the negative gradient of the potential at that point. The negative sign indicates that \vec{E} is directed from higher to lower potential. It may also be noted that:

- i) while moving a charge from one point to another point if the potential difference is found to be positive, there will be a gain in potential energy due to that movement and external agent must perform work against the electric field. On the other hand, if the potential difference will have a negative value, work will be done by the field;
- ii) the potential difference between two points in an electrostatic field depends only on the positional coordinates of the points and so it may be regarded as a point function and
- iii) the potential difference between two points is path independent.

Considering the path independence of potential difference we can assume, $V_{AB} = -V_{BA}$, over any chosen closed path, *i.e.*,

$$V_{BA} + V_{AB} = \oint_c \vec{E} \cdot d\vec{r} = 0 \quad (1.37)$$

Applying Stoke's theorem, we can write:

$$\oint_c \vec{E} \cdot d\vec{r} = \iint_S \vec{\nabla} \times \vec{E} \cdot d\vec{S} = 0 \quad (1.38)$$

From which it follows that for electrostatic field,

$$\vec{\nabla} \times \vec{E} = 0 \quad (1.39)$$

Again any vector field \vec{A} that satisfies $\vec{\nabla} \times \vec{A} = 0$ is called an irrotational field. So, electrostatic field is irrotational also.

1.15 ELECTRIC POTENTIAL CALCULATIONS

1.15.1 For a Point Charge

Let us consider two points A and B as shown in the Fig. 1.15. If now a charge q is displaced from B to A , then the expression for the potential difference between the points will be obtained as:

$$V_{AB} = -\int_A^B \vec{E} \cdot d\vec{r} = -\int_{r_A}^{r_B} \frac{q}{4\pi\epsilon_0 r^2} dr = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right] = V_A - V_B$$

Usually the potential at infinity is assumed to be zero. So, the potential at any point due to a charge will be the amount of work done to bring the charge from infinity to that point.

So, using $r_A = r$ and $r_B = 0$ in above equation we get,

$$V = \frac{q}{4\pi\epsilon_0 r}$$

This gives the potential at r due to a charge q situated at some chosen origin O . Alternatively, we can also define potential by the following equation,

$$V = - \int_{\infty}^r \vec{E} \cdot d\vec{r} \quad (1.40)$$

If now the point charge is not situated at the origin O (Fig. 1.16) but at some other point with position \vec{r}' , then the potential at point P can be generalized as,

$$V(r) = \frac{q}{4\pi\epsilon_0} \frac{1}{|\vec{r} - \vec{r}'|} \quad (1.41)$$

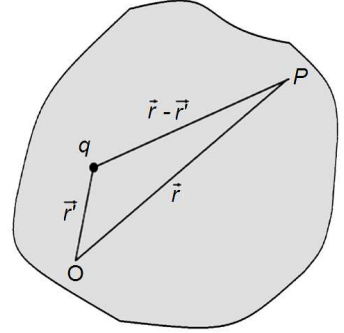


Fig.1.16: Potential due to a charge not at the origin

1.15.2 For Distributed Charges

As of now, we have considered the potential due to point charges only. Now, if there is n point charges $q_1, q_2, q_3, \dots, q_n$ located respectively at their positions $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$, the potential at any point at a position \vec{r} can then be written as:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{|\vec{r} - \vec{r}_1|} + \frac{q_2}{|\vec{r} - \vec{r}_2|} + \dots + \frac{q_n}{|\vec{r} - \vec{r}_n|} \right]$$

or,

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{|\vec{r} - \vec{r}_i|} \quad (1.42)$$

For continuous charge distribution, replacing point charges q_n by corresponding charge elements λdl or σdS or ρdV depending on whether the distribution is a linear, a surface or a volume charge distribution we get after replacing the summation by an integration:

For line charge,

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_l \frac{\lambda dl'}{|\vec{r} - \vec{r}_n|} \quad (1.43)$$

for surface charge,

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_l \frac{\sigma dS'}{|\vec{r} - \vec{r}_n|} \quad (1.44)$$

and for volume charge,

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_l \frac{\rho dV'}{|\vec{r} - \vec{r}_n|} \quad (1.45)$$

In the above expressions, the primed coordinates represent the *source coordinates* linear charge distribution $\lambda(\vec{r}')$, surface charge distribution $\sigma(\vec{r}')$ and volume charge distribution $\rho(\vec{r}')$ and the unprimed coordinates corresponds to the *field coordinates*.



1.16 RELATION BETWEEN FIELD INTENSITY AND POTENTIAL

To find a relationship between the electric field intensity and the electric potential at a point, let us start with the expression for the electric field produced by a charge $+q$ placed at some origin O at a distance r from it. This is given by,

$$\vec{E}(r) = \frac{q}{4\pi\epsilon_0 r^2} \hat{r} = \frac{q\vec{r}}{4\pi\epsilon_0 r^3}$$

Now,
$$\vec{\nabla}\left(\frac{1}{r}\right) = \vec{\nabla}\left(x^2 + y^2 + z^2\right)^{-1/2}$$

or,
$$\begin{aligned} \vec{\nabla}\left(\frac{1}{r}\right) &= \hat{i} \frac{\partial}{\partial x} \left(x^2 + y^2 + z^2\right)^{-1/2} + \hat{j} \frac{\partial}{\partial y} \left(x^2 + y^2 + z^2\right)^{-1/2} + \hat{k} \frac{\partial}{\partial z} \left(x^2 + y^2 + z^2\right)^{-1/2} \\ &= -\frac{\hat{i}x + \hat{j}y + \hat{k}z}{\left(x^2 + y^2 + z^2\right)^{3/2}} = -\frac{\vec{r}}{r^3} \end{aligned}$$

\therefore
$$\vec{E}(r) = \frac{q\vec{r}}{4\pi\epsilon_0 r^3} = -\frac{q}{4\pi\epsilon_0} \vec{\nabla}\left(\frac{1}{r}\right) = -\vec{\nabla}\left(\frac{q}{4\pi\epsilon_0 r}\right) = -\vec{\nabla}V$$

Thus,
$$\vec{E} = -\vec{\nabla}V$$

which is the same relation between electric field intensity and electric potential energy as obtained in Eq. (1.36). The negative sign indicates that the direction of E is along the direction of decreasing V .

Example 1.17 The electric potential for an electric field is given by

$$V(x, y, z) = (2x^2 + 2y^2 + 3z^2)^{-\frac{1}{2}}. \text{ Find the electric field intensity at } (1, 1, -1).$$

Solution

Here
$$V(x, y, z) = (2x^2 + 2y^2 + 3z^2)^{-\frac{1}{2}}$$

So, the necessary electric field intensity will be

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

or,
$$\begin{aligned} \vec{E} &= -\hat{i} \frac{\partial}{\partial x} \left\{ (2x^2 + 2y^2 + 3z^2)^{-\frac{1}{2}} \right\} - \hat{j} \frac{\partial}{\partial y} \left\{ (2x^2 + 2y^2 + 3z^2)^{-\frac{1}{2}} \right\} \\ &\quad - \hat{k} \frac{\partial}{\partial z} \left\{ (2x^2 + 2y^2 + 3z^2)^{-\frac{1}{2}} \right\} \\ &= (2x^2 + 2y^2 + 3z^2)^{-\frac{3}{2}} (2x\hat{i} + 2y\hat{j} + 3z\hat{k}) \end{aligned}$$

So, the field at a point $(1, 1, -1)$ is
$$\vec{E}\Big|_{(1,1,-1)} = 7^{-\frac{3}{2}} (2\hat{i} + 2\hat{j} - 3\hat{k}).$$

Example 1.18 Calculate the electric potential at the centre of a square with 1 m side, if q , $4q$, $-3q$ and $2q$ charges are placed at its corner (Given $q = 1 \text{ nC}$).

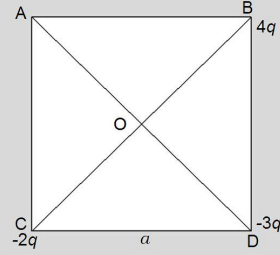
Solution

Each side of the square (a) is 1 m, as shown in the Fig.

$$\text{Now, } AO = BO = CO = DO = \frac{a}{\sqrt{2}} = x \text{ (say)}$$

Potential at the centre at O can be calculated as

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{x} + \frac{4q}{x} - \frac{3q}{x} - \frac{2q}{x} \right] = 0.$$



EXAMPLE 1.18

Example 1.19 If in a region space electric field is always in the X-direction then prove that (i) the potential is independent of Y and Z coordinates and (ii) if the field is constant, there is no free charge in that region.

Solution

i) As per the given condition electric field is $\vec{E} = E\hat{i}$, where the symbols have their usual meaning.

Again we can use $\vec{E} = -\vec{\nabla}\phi$ where ϕ is the potential.

$$\text{So, } E\hat{i} = -\hat{i} \frac{\partial \phi}{\partial x} - \hat{j} \frac{\partial \phi}{\partial y} - \hat{k} \frac{\partial \phi}{\partial z}$$

Comparing co-efficient of unit vectors we have,

$$E = -\frac{\partial \phi}{\partial x}$$

$$\text{and } \frac{\partial \phi}{\partial y} = \frac{\partial \phi}{\partial z} = 0$$

As there is no variation along y and z so we can say potential is independent of Y and Z coordinates.

ii) If the electric field is constant *i.e.*, $E = \text{constant}$

$$\text{so on taking the derivative with respect to } x \text{ we have } \frac{\partial^2 \phi}{\partial x^2} = -\frac{\partial E}{\partial x} = 0$$

$$\text{Also } \frac{\partial^2 \phi}{\partial y^2} = \frac{\partial^2 \phi}{\partial z^2} = 0$$

So in totality we can write $\nabla^2 \phi = 0$ which is the Laplace's equation and which holds good in charge free region. So it can be stated that the region is charge free.

EXAMPLE 1.19

1.17 ELECTROSTATIC POTENTIAL ENERGY

Any charge placed in an electric field will experience a force, as will any mass placed in a gravitational field. Just as mass in a gravitational field has some potential energy, so does a charge in an electric field. The *electric potential energy* of a system of point charges is the amount of work to bring a unit positive test charge from infinity to any point in the field to form the said system. Initially, let us consider a system of two charges q_1 and q_2 separated by a distance r_{12} as shown in Fig. 1.17 (a).

Now, the amount of work done to bring the charge q_2 from infinity to its respective position to form the system against the electric field produced by q_1 is given by,

$$W = -q_2 \int_{\infty}^{r_{12}} \vec{E} \cdot d\vec{r} = -\frac{q_1 q_2}{4\pi\epsilon_0} \int_{\infty}^{r_{12}} \frac{dr}{r^2} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} \quad (1.46)$$

This, work done will be stored in the two charge system in the form of electric potential energy U .

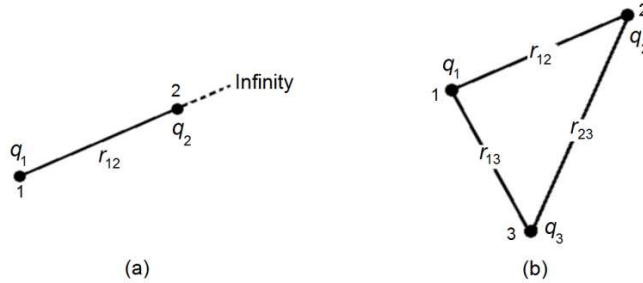


Fig.1.17: Electric potential energy for (a) two charge system and (b) three charge system

$$\text{Thus,} \quad U = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} \quad (1.47) \text{ (a)}$$

Similarly, if a system of three charges is there with their position as shown in Fig. 1.17 (b), the potential energy of the system will be,

$$U = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right] \quad (1.47) \text{ (b)}$$

1.18 ELECTRIC DIPOLE

Two equal and opposite point charges are separated by a distance small compared to the distance of observation are said to constitute an *electric dipole*.

The moment of the dipole can be defined as the product of the magnitude of any one of the charges and the distance of separation between them. It is a vector quantity and is always directed from the negative charge to the positive charge.

Let, the charges are $+q$ and $-q$ and their distance of separation is \vec{d} .

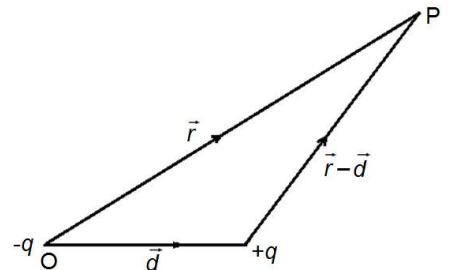


Fig.1.18: Electric dipole

To calculate the potential at the point of observation, for simplicity let us take the origin at the location of the charge $-q$ and also let the position of the point of observation P from the origin O is \vec{r} [Fig. 1.18]. Now, the potential at P due to the combination of the charges $+q$ and $-q$ (*i.e.*, the dipole as a whole) is,

$$\varphi = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\sqrt{r^2 + d^2 - 2\vec{r} \cdot \vec{d}}} - \frac{1}{r} \right]$$

Now from Fig. 1.18 we have,

$$\sqrt{r^2 + d^2 - 2\vec{r} \cdot \vec{d}} = \frac{1}{r} \left[1 - \frac{2\vec{r} \cdot \vec{d}}{r^2} + \frac{d^2}{r^2} \right]^{\frac{1}{2}} \approx \frac{1}{r} \left[1 - \frac{2\vec{r} \cdot \vec{d}}{r^2} \right]^{\frac{1}{2}} \approx \frac{1}{r} \left[1 + \frac{\vec{r} \cdot \vec{d}}{r^2} \right]$$

So,

$$\varphi = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} + \frac{\vec{r} \cdot \vec{d}}{r^3} - \frac{1}{r} \right] = \frac{q}{4\pi\epsilon_0} \frac{\vec{r} \cdot \vec{d}}{r^3} = \frac{\vec{r} \cdot q\vec{d}}{4\pi\epsilon_0 r^3} = \frac{\vec{r} \cdot \vec{p}}{4\pi\epsilon_0 r^3}$$

where, $\vec{p} = q\vec{d}$

is the electric dipole moment and the electric potential for the dipole is given by,

$$\varphi = \frac{\vec{r} \cdot \vec{p}}{4\pi\epsilon_0 r^3} = -\frac{\vec{p} \cdot \vec{\nabla} \left(\frac{1}{r} \right)}{4\pi\epsilon_0} \quad (1.48)$$

Thus, the electric field intensity will be

$$\vec{E} = -\vec{\nabla}\varphi = -\vec{\nabla} \left(\frac{\vec{r} \cdot \vec{p}}{4\pi\epsilon_0 r^3} \right) = -\frac{1}{4\pi\epsilon_0} \vec{\nabla} \left(\frac{\vec{r} \cdot \vec{p}}{r^3} \right)$$

Now,

$$\vec{\nabla} \left(\frac{\vec{r} \cdot \vec{p}}{r^3} \right) = \frac{1}{r^3} \vec{\nabla} (\vec{r} \cdot \vec{p}) + (\vec{r} \cdot \vec{p}) \vec{\nabla} \left(\frac{1}{r^3} \right)$$

Again,

$$\begin{aligned} \vec{\nabla} (\vec{r} \cdot \vec{p}) &= \left[\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right] (xp_x + yp_y + zp_z) \\ &= \hat{i}p_x + \hat{j}p_y + \hat{k}p_z = \vec{p} \end{aligned}$$

Also,

$$\vec{\nabla} \left(\frac{1}{r^3} \right) = -\frac{3\vec{r}}{r^5}$$

Therefore,

$$\begin{aligned} \vec{E} &= -\frac{1}{4\pi\epsilon_0} \left[\frac{1}{r^3} \vec{\nabla} (\vec{r} \cdot \vec{p}) + (\vec{r} \cdot \vec{p}) \vec{\nabla} \left(\frac{1}{r^3} \right) \right] \\ &= -\frac{1}{4\pi\epsilon_0} \left[\frac{\vec{p}}{r^3} - \vec{r} \cdot \vec{p} \frac{3\vec{r}}{r^5} \right] \end{aligned}$$

or,

$$\vec{E} = -\frac{1}{4\pi\epsilon_0 r^3} \left[\vec{p} - \frac{3(\vec{p} \cdot \vec{r})\vec{r}}{r^2} \right] = -\frac{q}{4\pi\epsilon_0 r^3} \left[\vec{d} - \frac{3(\vec{d} \cdot \vec{r})\vec{r}}{r^2} \right] \quad (1.49)$$

1.19 POISSON'S AND LAPLACE'S EQUATIONS

The Gauss's law as in Eq. (1.26) and the scalar potential equation given by Eq. (1.36) can be combined into a set of partial differential equations for the scalar potential.

Poisson's equation: Differential form of Gauss's law is, $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$, where ρ is the volume charge density. Again the electric field is related to the potential as, $\vec{E} = -\vec{\nabla}V$.

$$\therefore \vec{\nabla} \cdot (-\vec{\nabla}V) = \frac{\rho}{\epsilon_0}$$

$$\text{or, } \nabla^2 V = -\frac{\rho}{\epsilon_0} \quad (1.50) \text{ (a)}$$

This is known as Poisson's equation in electrostatics.

Laplace's equation: For a charge free region the charge density, $\rho = 0$ and thus from Poisson's equation,

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

$$\text{we have, } \nabla^2 V = 0 \quad (1.50) \text{ (b)}$$

This is known as Laplace's equation. From this equation integrating we have, $V = \text{constant}$. So, we can conclude that any equi-potential surface satisfies Laplace's equation.

EXAMPLE 1.20

Example 1.20 Show that the potential function $V = V_0 (x^2 - 2y^2 + z^2)$ satisfies Laplace's equation (V_0 is a constant).

Solution

Let the potential function is $V = V_0 (x^2 - 2y^2 + z^2)$

$$\text{Now we have } \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = V_0 (2 - 4 + 2) = 0.$$

So, V satisfies Laplace's equation.

EXAMPLE 1.21

Example 1.21 Show that the function $V = 2x^2 + 7y - 2z^2$ represents the potential function in charge-free region.

Solution

Given potential $V = 2x^2 + 7y - 2z^2$

$$\text{So, } \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 4 + 0 - 4 = 0$$

The expression satisfies Laplace's equation which leads to the conclusion that the charge density $\rho = 0$.

Hence the region of space is charge free.

Example 1.22 The electrostatic potential in free space is given by

$\varphi = \alpha - \beta(x^2 + y^2) - \gamma \ln \sqrt{x^2 + y^2}$ where α , β and γ are constants. Find the charge density in the region.

Solution

From the given expression of φ we have

$$\frac{\partial \varphi}{\partial x} = -2\beta x - \frac{\gamma}{\sqrt{x^2 + y^2}} \frac{x}{\sqrt{x^2 + y^2}} = -2\beta x - \frac{\gamma x}{x^2 + y^2}$$

or,
$$\frac{\partial^2 \varphi}{\partial x^2} = -2\beta - \frac{\gamma [x^2 + y^2 - x \times 2x]}{x^2 + y^2} = -2\beta - \gamma \frac{y^2 - x^2}{x^2 + y^2}$$

Similarly,
$$\frac{\partial^2 \varphi}{\partial y^2} = -2\beta - \gamma \frac{x^2 - y^2}{x^2 + y^2}$$

Again,
$$\frac{\partial^2 \varphi}{\partial z^2} = 0$$

So,
$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = -4\beta$$

or,
$$\nabla^2 \varphi = -4\beta$$

Now comparing the above expression with Poisson's equation $\nabla^2 \varphi = -\frac{\rho}{\epsilon_0}$ we have,

$$\frac{\rho}{\epsilon_0} = 4\beta$$

or,
$$\rho = 4\beta \epsilon_0$$

which is the required charged density for the said region.

EXAMPLE 1.22

1.19.1 Laplacian Operator

In Cartesian coordinate system (x, y, z) it can be written as,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (1.51)$$

In spherical polar coordinate (r, θ, φ) it can be written as,

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial^2}{\partial \varphi^2} \quad (1.52)$$

In cylindrical coordinate system (ρ, φ, z) it can be written as,

$$\nabla^2 = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2} \quad (1.53)$$

1.20 UNIQUENESS THEOREM

Statement: Two solutions of Laplace's equation obeying the same boundary conditions differ at best by a constant.

Proof: To prove the uniqueness theorem we assume that ϕ_1 and ϕ_2 are the two solutions of Laplace's equation in a volume V exterior to the surface S_1, S_2, \dots, S_n of the different conductors and bounded on the outside by a surface S . Let, ϕ_1 and ϕ_2 satisfy the same boundary conditions on the various surfaces i.e., S_1, S_2, \dots, S_n .

These boundary conditions involve either the *Dirichlet condition* which is the specifications of the potential ϕ on the bounding surfaces or the *Neumann condition* giving the specifications of the normal derivative of ϕ , i.e., $\frac{\partial \phi}{\partial n}$ on the bounding surfaces.

$$\text{Let } \phi = \phi_1 - \phi_2$$

$$\text{and also let } \nabla^2 \phi = \nabla^2 \phi_1 - \nabla^2 \phi_2 = 0.$$

Since both ϕ_1 and ϕ_2 satisfy Laplace's equation, if Dirichlet condition is satisfied, we have $\phi_1 = \phi_2$ on the bounding surfaces. That is, $\phi = 0$ on the bounding surfaces.

If on the other hand, Neumann condition is satisfied $\frac{\partial \phi_1}{\partial n} = \frac{\partial \phi_2}{\partial n}$ on the bounding surfaces.

Therefore $\frac{\partial \phi}{\partial n}$ or $\hat{n} \cdot \vec{\nabla} \phi$ must vanish on the boundaries, \hat{n} being a unit vector along the normally outward direction to the given surface.

Applying Gauss's divergence theorem in vector analysis to the vector $\phi \vec{\nabla} \phi$, we obtain,

$$\int_V \vec{\nabla} \cdot (\phi \vec{\nabla} \phi) dV = \int_{S+S_1+S_2+\dots+S_n} \phi \vec{\nabla} \phi \cdot \hat{n} dS$$

As, $\phi = 0$ on the boundaries (Neumann condition), the right hand side of the equation should vanish.

We have the vector identity

$$\vec{\nabla} \cdot (\phi \vec{\nabla} \phi) = \phi \nabla^2 \phi + |\vec{\nabla} \phi|^2$$

Also $\nabla^2 \phi = 0$ in the region V .

$$\text{So, } \vec{\nabla} \cdot (\phi \vec{\nabla} \phi) = |\vec{\nabla} \phi|^2$$

Hence the equation gives,

$$\int_V |\vec{\nabla} \phi|^2 dV = 0$$

Now $|\vec{\nabla}\phi|^2$, being a perfect square, is entirely positive or zero. But since its integral vanishes, $\vec{\nabla}\phi$ must be zero at each point in V . Therefore, $\phi = \phi_1 - \phi_2 = c$, where c is a constant *i.e.*, the function ϕ at each point in the region V will have the same value it has on the bounding surfaces.

From Dirichlet condition $\phi = 0$ on the bounding surfaces. Thus in this case $\phi = 0$, throughout V , or, $\phi_1 = \phi_2$ throughout V , clearly $c = 0$.

For Neumann condition, $\vec{\nabla}\phi \cdot \hat{n} = 0$ on the boundaries and $\vec{\nabla}\phi = 0$ for all points in V . So, the only possibility is that ϕ is a constant. This proves the uniqueness theorem.

1.21 LAPLACE'S EQUATION IN CARTESIAN COORDINATE

Laplace's equation in Cartesian coordinate can be written as,

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \quad (1.54)$$

Let us assume that, $V(x, y, z) = X(x) Y(y) Z(z)$ (1.55)

where $X(x)$, $Y(y)$ and $Z(z)$ are functions of x , y and z respectively. From Eq. (1.54) using Eq. (1.55)

we can write
$$Y(y)Z(z) \frac{\partial^2 X(x)}{\partial x^2} + Z(z)X(x) \frac{\partial^2 Y(y)}{\partial y^2} + X(x)Y(y) \frac{\partial^2 Z(z)}{\partial z^2} = 0$$

Dividing throughout by $X(x) Y(y) Z(z)$ we get

$$\frac{1}{X(x)} \frac{\partial^2 X(x)}{\partial x^2} + \frac{1}{Y(y)} \frac{\partial^2 Y(y)}{\partial y^2} + \frac{1}{Z(z)} \frac{\partial^2 Z(z)}{\partial z^2} = 0$$

In the above equation, all terms are function of one coordinate variably only, hence we can write them separately to be equal to zero and then rearranging we can write,

$$\frac{\partial^2 X(x)}{\partial x^2} + k_x^2 X(x) = 0 \quad (1.56) \text{ (a)}$$

$$\frac{\partial^2 Y(y)}{\partial y^2} + k_y^2 Y(y) = 0 \quad (1.56) \text{ (b)}$$

$$\frac{\partial^2 Z(z)}{\partial z^2} + k_z^2 Z(z) = 0 \quad (1.56) \text{ (c)}$$

where k_x , k_y and k_z are separation constants satisfying

$$k_x^2 + k_y^2 + k_z^2 = 0 \quad (1.57)$$

All the 2nd order differential Eqs. (1.56) have the same form.

Let us find out a possible solution for the Eq. (1.56) (a). Depending upon the value of k_x , we can have different possible solutions.

$$\text{if } k_x = 0; \quad X(x) = Ax + B \quad (1.58)$$

$$\text{if } k_x = k; \quad X(x) = A \sin kx + B \cos kx = Ce^{ikx} + De^{-ikx} \quad (1.59)$$

$$\text{and if } k_x = ik; \quad X(x) = A \sinh kx + B \cosh kx = Ce^{kx} + De^{-kx} \quad (1.60)$$

where A , B , C and D are constants and can be calculated from the boundary conditions. Similarly we have the possible solutions for other two differential equations. The particular solutions for $X(x)$, $Y(y)$ or $Z(z)$ to be used may be chosen based on the nature of the problems.

1.22 APPLICATION OF LAPLACE'S EQUATION

These equations are very essential for solving practical electrostatic problems with known values of electrostatic potential at some boundaries. Using the boundary values, these equations help to find out a possible solution for electrostatic field and potential throughout the volume. We shall consider such applications in terms of the following boundary value problems:

1.22.1 Parallel Plate Capacitor

Let us imagine a parallel plate capacitor along the Z -axis with plates at $z = 0$ and $z = d$. Let, the upper plate has the potential V_1 and the lower plate is grounded. Since the plates are along the Z -axis, so the components of potential along X - and Y -axis are zero and hence from Laplace's equation we have,

$$\frac{d^2 V}{dz^2} = 0$$

Integrating we get,

$$\frac{dV}{dz} = A$$

Again integrating we get,

$$V = Az + B \quad (1.61)$$

where A and B are integrating constants.

Now at $z = 0$, $V = 0$. Thus from Eq. (1.61) $B = 0$

$$\therefore V = Az$$

and at $z = d$, $V = V_1$

$$\therefore V_1 = Ad$$

$$\text{or, } A = \frac{V_1}{d}$$

$$\text{Thus } V = \frac{V_1 z}{d}$$

$$\text{Further, } \vec{E} = -\hat{z} \frac{\partial V}{\partial z} = -\hat{z} \frac{V_0}{d} \quad (1.63)$$

$$\text{Hence, } \rho_s = \epsilon_0 E = \epsilon_0 \frac{V_0}{d} \quad (\rho_s = \text{surface charge density}) \quad (1.64)$$

So, the enclosed charge within an area α of the capacitor will be,

$$Q = -\epsilon_0 \frac{V_0 \alpha}{d}$$

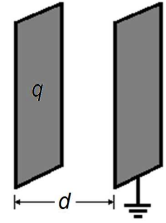


Fig.1.19: Parallel plate capacitor



(1.62)

and the capacitance will be,

$$C = \frac{|Q|}{V_0} = \frac{\epsilon_0 \alpha}{d} \quad (1.65)$$

Special case: Region between the plates filled up with two dielectric layers

Consider that there is no variation of the field along X- or Y-axis and there is no free charge at the interface (Fig. 1.20). Let us imagine that one electrode is grounded and the other is maintained at a potential V . For the two regions, the solution for Laplace's equation can be written as:

$$V_1 = A_1 z + B_1 \quad \text{for } z \leq a \quad (1.66)$$

and
$$V_2 = A_2 z + B_2 \quad \text{for } a \leq z \leq d \quad (1.67)$$

Now, applying the boundary conditions $V_2 = 0$ for $z = 0$ and $V_1 = V_0$ for $z = d$, we can write

$$V_0 = A_1 d + B_1$$

And
$$B_2 = 0$$

So,
$$V_1 = A_1 (z - d) + V_0$$

and
$$V_2 = A_2 z$$

As both the solution should give the same potential at $z = a$.

Thus,
$$A_1 (a - d) + V_0 = A_2 a \quad (1.68)$$

Also, at the dielectric boundary at $z = a$, the normal component of electric flux density vector is constant.

Noting that $\vec{D} = -\epsilon \vec{\nabla} V$, we can write

$$\epsilon_1 A_1 = \epsilon_2 A_2 \quad (1.69)$$

Solving Eq. (1.68) and Eq. (1.69) for A_1 and A_2 , we get

$$A_1 = \frac{V_0}{(a - d) + \frac{\epsilon_1}{\epsilon_2} a} \quad \text{and} \quad A_2 = \frac{V_0}{(a - d) \frac{\epsilon_2}{\epsilon_1} + a} \quad (1.70)$$

Using the above expressions for A_1 and A_2 , we can find V_1 and V_2 as,

$$V_1 = \frac{V_0 (z - d)}{(a - d) + \frac{\epsilon_1}{\epsilon_2} a} + V_0 \quad (1.71)$$

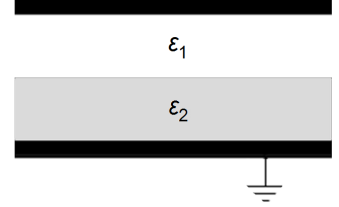


Fig.1.20: Parallel plate capacitor with two dielectric layers

and

$$V_2 = \frac{V_0 z}{(a-d) \frac{\epsilon_2}{\epsilon_1} + a} \quad (1.72)$$

1.22.2 Spherical Capacitor

Let us consider, a spherical capacitor of inner radius a and outer radius b as shown in Fig. 1.21. Let the inner sphere is at potential V_0 and the outer sphere is grounded. As the potential will be a function of radial coordinates only using spherical polar coordinate and considering the symmetry of the problem the from Laplace's equation may be written as,

$$\frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) = 0$$

Integrating we have, $r^2 \frac{\partial V}{\partial r} = A$

or,

$$\frac{\partial V}{\partial r} = \frac{A}{r^2}$$

Again, integrating we get,

$$V = -\frac{A}{r} + B \quad (1.73)$$

where A and B are two constants. Now, at $r = b$, $V = 0$

$$\therefore 0 = -\frac{A}{b} + B \Rightarrow B = \frac{A}{b}$$

So,

$$V = -\frac{A}{r} + \frac{A}{b} = A \left(\frac{1}{b} - \frac{1}{r} \right) \quad (1.74)$$

Again at $r = a$, $V = V_0$

$$\therefore V_0 = A \left(\frac{1}{b} - \frac{1}{a} \right) \Rightarrow A = \frac{V_0}{\left(\frac{1}{b} - \frac{1}{a} \right)}$$

Thus from Eq. (1.74) we have,

$$V = \frac{V_0}{\left(\frac{1}{b} - \frac{1}{a} \right)} \left(\frac{1}{b} - \frac{1}{r} \right) = \frac{V_0 a}{r} \frac{(r-b)}{(a-b)} = \frac{V_0 a}{r} \frac{(b-r)}{(b-a)} \quad (1.75)$$

Further,

$$\vec{E} = -\hat{a}_r \frac{\partial V}{\partial r} = \frac{V_0 ab}{r^2 (b-a)} \hat{a}_r \quad (1.76)$$

Hence,

$$\rho_s = \epsilon_0 E = \frac{\epsilon_0 V_0 ab}{r^2 (b-a)} \quad (\rho_s = \text{surface charge density}) \quad (1.77)$$

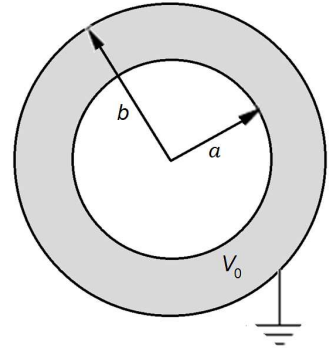


Fig.1.21: Spherical capacitor

$$\begin{aligned}
 \text{So, } Q &= \frac{\epsilon_0 V_0 ab}{(b-a)} \int_0^{2\pi} \int_0^\pi \frac{r^2}{r^2} \sin \theta d\theta d\varphi \\
 &= \frac{4\pi\epsilon_0 V_0 ab}{(b-a)}
 \end{aligned}$$

and the capacitance will be,

$$C = \frac{|Q|}{V_0} = \frac{4\pi\epsilon_0 ab}{(b-a)} \quad (1.78)$$

1.22.3 Cylindrical Capacitor

Consider a cylindrical capacitor of inner radius a and outer radius b as shown in Fig. 1.22. Let, the inner cylinder be at potential V_0 and the outer cylinder be grounded. Now since the cylinder has radial symmetry thus from Laplace's equation for cylindrical coordinate system we may write,

$$\frac{1}{r} \frac{d}{dr} \left(\frac{\partial V}{\partial r} \right) = 0$$

$$\text{or, } r \frac{\partial V}{\partial r} = A$$

$$\text{or, } \frac{\partial V}{\partial r} = \frac{A}{r}$$

$$\text{or, } V = A \ln r + B \quad (1.79)$$

Now, at $r = b$, $V = 0$

$$\therefore 0 = A \ln b + B$$

$$\text{or, } B = -A \ln b$$

$$\text{or, } V = A \ln r - A \ln b = A \ln \frac{r}{b} \quad (1.80)$$

Again at $r = a$, $V = V_0$

$$\therefore V_0 = A \ln \frac{a}{b} \Rightarrow A = \frac{V_0}{\ln \frac{a}{b}}$$

So, from Eq. (1.80) we have,

$$V = \frac{V_0}{\ln \frac{a}{b}} \ln \frac{r}{b} = \frac{V_0}{\ln \frac{b}{a}} \ln \frac{b}{r} \quad [\text{as } a \leq r \leq b] \quad (1.81)$$

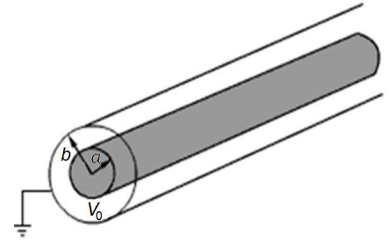


Fig.1.22: Cylindrical capacitor



$$\text{Further, } \vec{E} = -\hat{r} \frac{\partial V}{\partial r} = -\frac{\hat{r}}{r} \frac{V_0}{\ln \frac{b}{a}} \quad (1.82)$$

$$\begin{aligned} \text{Hence, } \rho_s &= \epsilon_0 E_{r=a} \\ &= \frac{\epsilon_0}{a} \frac{V_0}{\ln \frac{b}{a}} \quad (\rho_s = \text{surface charge density}) \end{aligned} \quad (1.83)$$

So, for a length L of the coaxial conductor, the capacitance will be,

$$C = \frac{Q}{V_0} = \frac{2\pi aL}{V_0} \rho_s = \frac{2\pi\epsilon_0 L}{\ln \frac{b}{a}} \quad (1.84)$$

1.23 FARADAY CAGE

In Faraday's experiments on charge, magnetism and their mutual interaction, it was noted that charge on a conductor resides on the outer surface only and nothing inside that conductor is affected by any change in the outside region. Subsequently, field theory was developed on Faraday's work and it was accepted that an electric field can be extended into space beyond a charge. There is a redistribution of charge to the outside of a conductor due to the electrostatic repulsion of like charges. This results zero net electrostatic field within the conductor that means any space which is enclosed by a continuously conducting layer. Any electronic noise that exists outside the cage is completely cancelled within that space. This mechanism can be used to justify disregarding electrostatic fields in highly conductive electrolyte solutions. All the noise created inside the cage is also prevented from escaping to the outside of the cage.

There are few points to consider. First one is that breaks in the cage causing gaps that allow for penetration by outside electromagnetic fields. For a mesh the penetration of electromagnetic radiation is limited to oscillations that have wavelengths less than two times the diameter of the opening. Hence a 0.01 m opening would permit 0.02 m and shorter wavelengths that correspond to 150 GHz noise. The second one is a long conductive material. Accessing the Faraday cage through a door or opening can create the possibility for a break in the continuity. If one side is discontinuous then redistribution of charge may not properly occur. Under those circumstances the cancelling effect will not exist and a non-zero field will exist inside the cage. Low frequency work can be assisted by connecting wire to discontinuous edges but it is insufficient for experiments operating in the higher frequencies. The final thing is to consider the conductivity of the cage. When the size of the cage increases it can become a larger concern. The more resistive the conducting layer is the slower charge redistributes, causing a non-cancelling field.

Applications: Faraday cage always reduces noise, particularly power line noise with an AC power grid. Some experiments deal with low currents and or high frequencies. Physical electrochemistry falls into the latter category and when micro or nano-electrodes are involved, then both have important roles. Corrosion may not require much precision and accuracy, but corrosion resistant alloys can lead to measured currents in nA range, where a Faraday cage is required. If the cell current does not exceed 1 μA , a Faraday cage is crucial to be used.

1.24 COFFEE RING EFFECT

A *coffee ring* is a pattern left by a puddle of particle-laden liquid after it evaporates. The phenomenon is termed as the characteristic ring-like deposit along the perimeter of a spill of coffee. It is also found after spilling red wine. The mechanism behind the formation of these rings is known as the *coffee ring effect* or the *coffee stain effect*, or simply *ring stain*.

The coffee-ring pattern originates from capillary flow which is induced by differential evaporation rates across the drop. Liquid evaporating from the edge is replenished by liquid from interior. The resulting flow can carry nearly all the dispersed material to the edge.

With time, this process shows a rush-hour effect, causing a rapid acceleration of the edgeward flow at the final stage when drying process is maintained. The stronger flow redistributes particles to the center of the droplet. For particles to accumulate at the edges, the liquid must have a weak flow or to disrupt the flow. Surfactants can be added to reduce the liquid's surface tension gradient, disrupting the induced flow. Water has a weak flow to begin with, which is then reduced largely by natural surfactants. The coffee ring effect is used in convective deposition to order particles on a substrate using capillary-driven assembly, replacing a stationary droplet. Convective deposition can control particle orientation, afforded by the system to reach a state of maximum packing of the particles in the thin meniscus layer over which evaporation occurs. It was shown that convective assembly could control particle orientation in assembling multi-layers, resulting in long-range 3D colloidal crystals from dumbbell shaped particles. Recent advances have increased the application of coffee-ring assembly from colloidal particles to organized patterns of inorganic crystals.

1.25 METHOD OF IMAGES

Solution of Laplace's and Poisson's equation can be obtained in a number of ways. For a given set of boundary conditions, to find a solution of the applicable Poisson's equation or Laplace's equation, we have to first establish the fact that the solution is a unique solution regardless of the used method. Uniqueness theorem thus can be stated as *for a given set of boundary conditions the solution to the Poisson's equation is unique*.

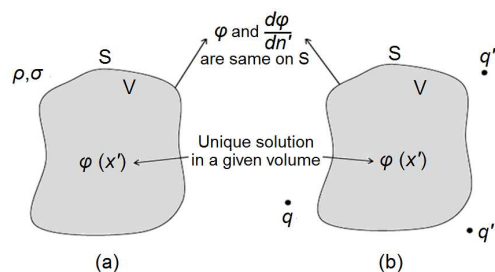


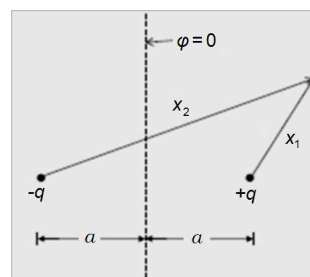
Fig.1.23: Uniqueness theorem

Fig. 1.23 compares two different external charge distributions as specified by (a) ρ, σ and (b) q, q', q'', \dots , but the boundary conditions of those systems are identical. Then, the potentials are same inside the regions. The image charges must be external to the volume of interest.

1.25.1 Point Charge in Front of a Conducting Plane

Let a point charge q is placed near a conducting plane of infinite extent as shown in Fig. 1.24. The boundary condition is that $\phi = 0$ on the surface of the conducting plane. Let the conducting plane coincide with the YZ-plane and the point charge line on the X-axis at $x = a$. Consider now a system of two point charges at a distance $2a$ apart (Fig. 1.24).

Potential: The potential at (x, y, z) due to the two charges will be,



$$\varphi(x, y, z) = \frac{q}{4\pi\epsilon_0 r_1} + \frac{-q}{4\pi\epsilon_0 r_2}$$

Fig.1.24: Point charge in front of a conducting plane

or,

$$\varphi(x, y, z) = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\sqrt{(x-a)^2 + y^2 + z^2}} - \frac{1}{\sqrt{(x+a)^2 + y^2 + z^2}} \right] \quad (1.85)$$

satisfies not only (i) Poisson equation for $x > 0$ and (ii) the boundary at all points exterior to the charges, but also the boundary condition of the original problem. Therefore, Eq. (1.85) is the correct potential in the entire half-space exterior to the conducting plane ($x > 0$).

Induced surface charge: The surface charge density induced on the conductor is

$$\begin{aligned} \sigma(y, z) &= \epsilon_0 E_x|_{x=0} = -\epsilon_0 \left. \frac{\partial \varphi}{\partial x} \right|_{x=0} \\ &= -\frac{qa}{2\pi(a^2 + y^2 + z^2)^{3/2}} \end{aligned} \quad (1.86)$$

The total charge induced on the plane is

$$\begin{aligned} Q &= \iint \sigma(y, z) dy dz \\ &= -\int_0^{2\pi} \int_0^\infty \frac{qa}{2\pi(a^2 + r^2)^{3/2}} r dr d\phi \\ &= -\left. \frac{qa}{\sqrt{a^2 + r^2}} \right|_0^\infty = -q \end{aligned} \quad (1.87)$$

where $y^2 + z^2 = r^2$. Hence, the charge q is attracted toward the plane because of the negative induced charge. The force acting on the charge will be

$$F = -\frac{q^2}{4\pi\epsilon_0 (2a)^2} \quad (1.88)$$

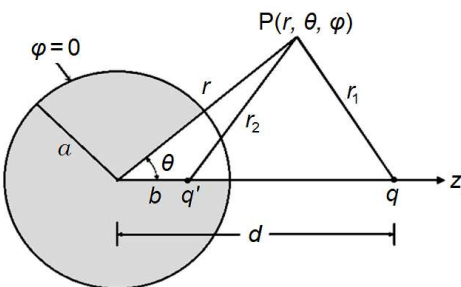


Fig.1.25: Point charge in front of a grounded conducting sphere

1.25.2 Point Charge in Front of a Grounded Conducting Sphere

Fig. 1.25 illustrates a point charge in the vicinity of a grounded conducting sphere of radius a . It is convenient to formulate the problem by means of spherical coordinates, with the origin of coordinates at the center of the sphere.

Let the charge q is situated at $z = d$ on the z -axis. The boundary condition, $\varphi(r = a) = 0$ can be satisfied by an image charge q' inside the sphere ($z = b$).

Potential: The potential due to the charges q and q' is

$$\varphi(r, \theta, \varphi) = \frac{q}{4\pi\epsilon_0 r_1} + \frac{q'}{4\pi\epsilon_0 r_2}$$

$$\text{or, } \varphi(r, \theta, \varphi) = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{\sqrt{r^2 + d^2 - 2rd \cos \theta}} - \frac{q'}{\sqrt{r^2 + b^2 - 2rb \cos \theta}} \right] \quad (1.89)$$

Now, $\varphi(r = a, \theta, \varphi) = 0$ for all θ only if

$$\sqrt{\frac{a^2 + b^2 - 2ab \cos \theta}{a^2 + d^2 - 2ad \cos \theta}} = -\frac{q'}{q} = \text{constant}$$

This is the case if

$$b = \frac{a^2}{d} \quad (1.90)$$

$$\text{then } \sqrt{\frac{a^2 + b^2 - 2ab \cos \theta}{a^2 + d^2 - 2ad \cos \theta}} = \frac{a}{d}$$

$$\text{and } q' = -\frac{a}{d} q \quad (1.91)$$

These Eqs. (1.90) and (1.91) specify the location and magnitude of the image charge q' .

Induced surface charge: The surface charge density induced on the conductor will be,

$$\sigma(\theta, \varphi) = -\epsilon_0 \left. \frac{\partial \varphi}{\partial r} \right|_{r=a} = -\frac{q}{4\pi a^2} \frac{a}{d} \frac{1 - \frac{a^2}{d^2}}{2\pi \left(1 + \frac{a^2}{d^2} - 2\frac{a}{d} \cos \theta \right)^{3/2}} \quad (1.92)$$

The total charge induced on the plane is

$$\begin{aligned} Q &= \iint \sigma(\theta, \varphi) a^2 \sin \theta d\theta d\varphi \\ &= \int_0^{2\pi} \int_0^\pi -\frac{qa}{2d} \frac{1 - \frac{a^2}{d^2}}{\left(1 + \frac{a^2}{d^2} - 2\frac{a}{d} \cos \theta \right)^{3/2}} \sin \theta d\theta d\varphi \\ &= -\frac{qa}{d} = q' \end{aligned} \quad (1.93)$$

The charge q is attracted toward the sphere because of the negative induced charge. So, the force of attraction on the charge is

$$\begin{aligned}
 F &= -\frac{qq'}{4\pi\epsilon_0(d-b)^2} \\
 &= -\frac{q^2 a}{4\pi\epsilon_0 d^3} \left(1 - \frac{a^2}{d^2}\right)^{-2} \quad (1.94)
 \end{aligned}$$

UNIT SUMMARY

- **Quantization of charge**

$$q = ne$$

- **Conservation of charge**

Charge cannot be created or destroyed *i.e.*, charge of an isolated system is conserved.

- **Coulomb's law**

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \quad [\text{for 2 charges } q_1 \text{ and } q_2 \text{ separated by a distance } r]$$

- **Superposition principle**

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_3}{r_{31}^2} \vec{r}_{31} + \frac{q_2 q_3}{r_{32}^2} \vec{r}_{32} \right] \quad [\text{for 3-charge system}]$$

- **Charge densities**

$$\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta q}{\Delta V} \quad \sigma = \lim_{\Delta S \rightarrow 0} \frac{\Delta q}{\Delta S} \quad \lambda = \lim_{\Delta l \rightarrow 0} \frac{\Delta q}{\Delta l}$$

- **Electric field intensity**

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

- **Curl of Electrostatic field: Conservation of electric field**

$$\vec{\nabla} \times \vec{E}(\vec{r}) = 0$$

- **Electric flux**

$$\phi = \int d\phi = \oint_S \vec{E} \cdot d\vec{S} = \oint_S E dS \cos \theta$$

- **Gauss's law in Electrostatics**

$$\phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

- **Application of Gauss's law**

Electric field around a charged cylinder: $E = \frac{\lambda}{2\pi\epsilon_0 r}$

Electric field due to a charged solid sphere: $E = \frac{\rho r}{3\epsilon_0}$ [for $r < a$]

$E = \frac{q}{4\pi\epsilon_0 r^2}$ [for $r > a$]

Electric field due to a non-conducting infinite charged sheet: $E = \frac{\sigma}{2\epsilon_0}$

- **Coulomb's law from Gauss's law**

$$\iint_S \vec{E} \cdot d\vec{S} = \frac{q_1}{\epsilon_0} \Leftrightarrow F = q_2 E = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

- **Electrostatic potential**

$$V = \frac{q}{4\pi\epsilon_0 r} \quad [\text{for a charge } q \text{ at a distance } r]$$

- **Electric potential difference**

$$V_{AB} = V_A - V_B = - \int_{r_A}^{r_B} \frac{q}{4\pi\epsilon_0 r^2} dr = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right]$$

- **Electrostatic potential calculations**

For line charge distribution, $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_l \frac{\lambda(\vec{r}') d\vec{l}'}{|\vec{r} - \vec{r}_n|}$

For surface charge distribution, $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_l \frac{\sigma(\vec{r}') dS'}{|\vec{r} - \vec{r}_n|}$

For volume charge distribution, $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_l \frac{\rho(\vec{r}') dV'}{|\vec{r} - \vec{r}_n|}$

- **Relationship between electric field intensity and electric potential**

$$\vec{E} = -\vec{\nabla}V$$

- **Electrostatic potential energy**

$$U = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right]$$

- **Electric Dipole**

$$\vec{E} = -\frac{1}{4\pi\epsilon_0 r^3} \left[\vec{p} - \frac{(\vec{p} \cdot \vec{r}) \vec{r}}{r^2} \right] = -\frac{q}{4\pi\epsilon_0 r^3} \left[\vec{d} - \frac{(\vec{d} \cdot \vec{r}) \vec{r}}{r^2} \right]$$

- **Poisson's and Laplace's equations**

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \quad \nabla^2 V = 0$$

- **Uniqueness theorem**

Two solutions of Laplace's equation obeying the same boundary conditions differ at best by constant.

- **Solution of the Laplace's equation in Cartesian coordinate system**

$$\text{if } k_x = 0; X(x) = Ax + B$$

$$\text{if } k_x = k; X(x) = A \sin kx + B \cos kx = Ce^{ikx} + De^{-ikx}$$

$$\text{if } k_x = ik; X(x) = A \sinh kx + B \cosh kx = Ce^{kx} + De^{-kx}$$

- **Applications of Poisson's and Laplace's equations to Cartesian, Spherical and Cylindrical systems**

$$\text{Solution of Laplace's equation for a parallel plate capacitor: } V = \frac{V_1 z}{d}$$

$$\text{Solution of Laplace's equation for a spherical capacitor: } V = \frac{V_0 a}{r} \frac{(b-r)}{(b-a)}$$

Solution of Laplace's equation for a cylindrical capacitor:

$$V = \frac{V_0}{\ln \frac{b}{a}} \ln \frac{b}{r} \quad [\text{as } a \leq r \leq b]$$

- **Faraday Cage**

Faraday Cage always reduces noise, particularly power line noise with an AC power grid.

- **Coffee ring effect**

A coffee ring is a pattern left by a puddle of particle-laden liquid after it evaporates. The phenomenon is termed as the characteristic ring-like deposit along the perimeter of a spill of coffee. It is also found after spilling red wine. The mechanism behind the formation of these rings is known as the coffee ring effect

- **Method of images**

Point charge in front of a conducting plane

$$F = -\frac{q^2}{4\pi\epsilon_0 (2a)^2}$$

Point charge in front of a grounded conducting sphere

$$q' = -\frac{a}{d} q$$

$$F = -\frac{qq'}{4\pi\epsilon_0(d-b)^2} = -\frac{q^2a}{4\pi\epsilon_0d^3}\left(1 - \frac{a^2}{d^2}\right)^{-2}$$

EXERCISES

Multiple Choice Questions

- 1.1 If (r, θ, ϕ) represents the spherical polar co-ordinate of a point in a region where the electrostatic potential is given by $V = k\phi^2$, the charge density associated with that region will be
 (a) a function of ϕ only (b) constant in the region
 (c) a function of all the co-ordinate (r, θ, ϕ) (d) a function of (r, ϕ) only
- 1.2 The electric flux through each of the faces of a cube of side 1 m if a charge q coulomb is placed at its centre is
 (a) $\frac{q}{4\epsilon_0}$ (b) $4\epsilon_0 q$ (c) $\frac{q}{6\epsilon_0}$ (d) $\frac{q}{\epsilon_0}$
- 1.3 The electric flux through the surface vector \vec{S} equal to $6\hat{j}$ in a region of electric field $3\hat{i} + \hat{j}$ is
 (a) $10 \text{ Nm}^2\text{C}^{-1}$ (b) $6 \text{ Nm}^2\text{C}^{-1}$ (c) $15 \text{ Nm}^2\text{C}^{-1}$ (d) none of these
- 1.4 In free space Poisson's equation reduces to
 (a) $\nabla^2 V = 0$ (b) $\nabla^2 V = \frac{\rho}{\epsilon_0}$ (c) $\nabla^2 V = -\frac{\rho}{\epsilon_0}$ (d) $\nabla^2 V = \infty$
- 1.5 Electrostatic field may be obtained as the gradient of a scalar potential
 (a) because it is conservative field (b) because it is always solenoidal
 (c) when the field is linear in x, y and z (d) where there is no magnetic field
- 1.6 The torque acting on the polar molecules of a dipole when subjected to an electric field of intensity E is given by,
 (a) $\tau = p_0 E \tan \theta$ (b) $\tau = p_0 E \sin \theta$ (c) $\tau = p_0 E \cos \theta$ (d) none of these
- 1.7 Two conducting spheres of radii r and $2r$ contain charges q and $2q$. If the electric fields just outside the surface of these two spheres are E_1 and E_2 respectively, then
 (a) $E_1 = E_2$ (b) $E_1 = 2E_2$ (c) $2E_1 = E_2$ (d) $E_1 = 4E_2$
- 1.8 Torque ($\vec{\tau}$) on an electric dipole having dipole moment (\vec{p}) in an electric field \vec{E} can be expressed as
 (a) $\vec{\tau} = -\vec{p} \times \vec{E}$ (b) $\vec{\tau} = -\vec{p} \cdot \vec{E}$ (c) $\vec{\tau} = \vec{p} \times \vec{E}$ (d) $\vec{\tau} = \vec{p} \cdot \vec{E}$
- 1.9 The electric field between two oppositely charged plates with equal charge density σ is

- (a) 0 (b) $\frac{2\sigma}{\epsilon_0}$ (c) $\frac{\sigma}{\epsilon_0}$ (d) $\frac{\sigma}{2\epsilon_0}$

1.10 In a certain region, if the electric field $E = 0$, corresponding potential will be-
 (a) zero (b) constant (c) function of position (d) infinity

1.11 The unit of $\int_S \vec{D} \cdot d\vec{S}$ is

- (a) C (b) F (c) W (d) V

1.12 The electrostatic potential energy of a system of two charges, q_1 and q_2 separate by a distance r is

- (a) $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$ (b) $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$ (c) $\frac{1}{4\pi\epsilon_0} \frac{q_1^2 q_2}{r^2}$ (d) $\frac{\epsilon_0}{4\pi} \frac{q_1 q_2}{r^2}$

1.13 Work done to displace a charge q from a point P_1 having potential V_1 to a point P_2 having potential V_2 is

- (a) $q(V_1 - V_2)$ (b) $q(V_1 V_2)$ (c) $q(V_1 + V_2)$ (d) $q(V_1 / V_2)$

1.14 If a proton of charge $+q$ is accelerated through the potential V then the kinetic energy of it is

- (a) $\frac{1}{2}qV^2$ (b) qV^2 (c) qV (d) $\frac{1}{2}qV$

1.15 Electric field in a region containing space charge can be obtained by using

- (a) Laplace equation (b) Poisson's equation
 (c) Helmholtz equation (d) Coulomb's law

1.16 Which of the following statements is not correct regarding electrostatic field vector \vec{E} ?

- (a) $\oint_c \vec{E} \cdot d\vec{r} = 0$, where c is a simple closed curve
 (b) $\int_a^b \vec{E} \cdot d\vec{r}$ is independent of the path for given end points a and b
 (c) $\vec{E} = \vec{\nabla} \times \vec{A}$ for some vector potential \vec{A} (d) $\vec{E} = \vec{\nabla} \phi$ for some scalar field ϕ

1.17 Total electric flux (ϕ) of electric field (\vec{E}) over a surface S is

- (a) $\vec{E} \cdot d\vec{S}$ (b) $\oint_S \vec{E} \cdot d\vec{S}$ (c) $\vec{E} \times d\vec{S}$ (d) $\oint_S \vec{E} \times d\vec{S}$

1.18 The Poisson's equation in a region of space with volume charge density ρ is

- (a) $\nabla^2 V = \frac{\rho}{\epsilon_0}$ (b) $\nabla^2 V = -\frac{\rho}{\epsilon_0}$ (c) $\nabla^2 V = 0$ (d) $\nabla^2 V = \frac{\rho^2}{\epsilon_0}$

1.19 The ratio of acceleration of an electron and a proton placed in a uniform electric field is,

- (a) 0 (b) 1 (c) $\frac{m_p}{m_e}$ (d) $\frac{m_e}{m_p}$
- 1.20 If the electric field intensity at a distance of 0.25 m from a point charge q is 2NC^{-1} , the value of the point charge q is
 (a) $1.39 \times 10^{-5} \mu\text{C}$ (b) $1.39 \times 10^5 \mu\text{C}$ (c) $13.9 \times 10^{-5} \text{C}$ (d) $10^{-6} \mu\text{C}$
- 1.21 The surface charge density of a charged metallic sphere placed in vacuum is σ . The SI unit of electric field intensity at any point near it will be
 (a) $\sigma\epsilon_0$ (b) $\frac{\epsilon_0}{\sigma}$ (c) $\frac{\sigma}{\epsilon_0}$ (d) $\epsilon_0^2 \sigma$
- 1.22 To balance a liquid drop of mass $2 \times 10^{-4} \text{kg}$ and charge $9.8 \times 10^{-2} \mu\text{C}$, the required field should be
 (a) 10^4NC^{-1} (b) $2 \times 10^4 \text{NC}^{-1}$ (c) $4 \times 10^4 \text{NC}^{-1}$ (d) $5 \times 10^4 \text{NC}^{-1}$
- 1.23 The ratio of the attractive forces between two point charges in vacuum and in a medium of dielectric constant k is
 (a) $1 : k$ (b) $1 : k^2$ (c) $k^2 : 1$ (d) $k : 1$
- 1.24 The magnitude of the image charge for a point charge q placed at a distance d from the centre of a grounded conducting sphere of radius a is
 (a) $q' = -\frac{d}{a}q$ (b) $q' = -\frac{a}{d^2}q$ (c) $q' = -\frac{a^2}{d}q$ (d) $q' = -\frac{a}{d}q$

Answers of Multiple Choice Questions

1.1 (d), 1.2 (c), 1.3 (b), 1.4 (c), 1.5 (a), 1.6 (b), 1.7 (b), 1.8 (c), 1.9 (c), 1.10 (b), 1.11 (a), 1.12 (b), 1.13 (a), 1.14 (c), 1.15 (b), 1.16 (c), 1.17 (b), 1.18 (b), 1.19 (d), 1.20 (a), 1.21 (c), 1.22 (b), 1.23 (d), 1.24 (d)

Short and Long Answer Type Questions

Category I

- 1.1 Illustrate Gauss's law of electrostatics using the concept of electric flux.
- 1.2 Verify whether the potential $V(x, y)$ satisfy Laplace's equation or not. Find also the charge density associated with the given potential.
- 1.3 Arrive at Coulomb's law by the appropriate use of Gauss's law.
- 1.4 Investigate that the electric field $\vec{E} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ is solenoidal as well as conservative.
- 1.5 Try to obtain the differential form of Gauss law in electrostatics from its integral form.
- 1.6 Arrive at Laplace equation starting from Poisson's equation for cylindrical and spherical co-ordinate system.
- 1.7 Establish Poisson's equation from Gauss's law in electrostatics.

- 1.8 Evaluate the electric field between infinitely extended parallel plate capacitor carrying charge density σ by applying Gauss' law . Assume the mutual separation between the plates is d .
- 1.9 Check whether the potential function $x^2 - y^2 + z$ satisfies Laplace's equation or not.
- 1.10 Starting from the initial condition properly emphasize on the usefulness of uniqueness theorem in electrostatic problems.
- 1.11 Solve Laplace's equation to find the potential at a distance r from the axis of an infinitely long conducting cylinder of radius a charged with a surface charge density σ . Take the potential of the cylinder to be zero.
- 1.12 If in a region of space electric field is always in the X-direction then prove that
 - i) the potential is independent of y and z coordinates and ii) if the field is constant, there is no free charge in that region.
- 1.13 Suppose that electric field in some region is found to be $\vec{E} = \alpha r^3 \hat{r}$ in spherical coordinates (α is a constant). Find the corresponding charge density.
- 1.14 A very long cylindrical object carries charge distribution proportional to the distance from the axis (r). If the cylinder is of radius a , then find the electric field both at $r > a$ and $r < a$, by the application of Gauss' law in electrostatics.
- 1.15 Try to arrive at Poisson's equation and Laplace's equation from the fundamentals.
- 1.16 Physically describe the electric field intensity at a point. "Electric field intensity at a point 10^4 NC^{-1} " what do you mean by this?
- 1.17 a) The potential in a medium is given by $\phi(r) = \frac{qe^{-r/\lambda}}{4\pi\epsilon_0 r}$. Obtain corresponding electric field.
 b) Also obtain the charge density that may produce the potential mentioned above.
- 1.18 Evaluate the potential of a uniformly charged sphere of radius R having a constant charge density ρ at a distance r ($r > R$) from the centre of the sphere.
- 1.19 A particle is acted upon by two constant forces $\hat{i} + 4\hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$, so that it is displaced from the point $\hat{i} + 3\hat{j} + 3\hat{k}$ to the point $4\hat{i} + 5\hat{j} + \hat{k}$. Find the total work done.

Category II

- 1.20 Would electric potential energy be meaningful if the electric field were not conservative?
- 1.21 Discuss how potential difference and electric field strength are related. Give an example.
- 1.22 What is the strength of the electric field in a region where the electric potential is constant?
- 1.23 If the voltage between two points is zero, can a test charge be moved between them with zero net work being done?
- 1.24 Would Gauss's law be helpful for determining the electric field of a dipole? Why?
- 1.25 Prove that the electric field outside the conductor is directed perpendicular to the surface.

- 1.26 Two concentric spheres of radii a and b are kept at potentials V_a and V_b . If the intervening space is vacuum, then write the appropriate differential that the electrostatic potential satisfies. Solve this equation to find the potential at any point between the spheres and also calculate the total charge outside the surface.

Numerical Problems

- 1.1 If in a region of space the electric field is given by $\vec{E} = 2\hat{i} - \hat{j} + \hat{k}$, find the electric flux through a surface area 40 unit in YZ plane. [Ans: 80 unit]
- 1.2 Q charge is supplied to a conducting rod of length L . For uniform charge distribution calculate the electric field intensity at a distance a from one end of the rod. [Ans: $\frac{1}{4\pi\epsilon_0} \frac{Q}{a(L+a)}$]
- 1.3 Find the potential at the centre of a square of side 1 m with charges 1nC, 4 nC, -3 nC, and -2 nC at its corner
- 1.4 At each corners of an equilateral triangle of side a , a point charge q is located. Find the electrostatic energy of the system?
- 1.5 3 equal positive charges are placed at three corners of a square of side a . Determine the magnitude and direction of electric field intensity at the 4th corner of the square.
- 1.6 A capacitor is made with two infinitely long cylindrical conductor of radii a and b ($a > b$), with vacuum in the intervening space. If the internal cylinder is kept at ground and the outer cylinder has a charge density σ then solve Laplace equation to find the electrostatic potential in the space between the cylinders.
- 1.7 If the electric field is given by $\vec{E} = \frac{1}{\epsilon_0} (x\hat{i} + y\hat{j} - 2z\hat{k})$, then find the charge density.
- 1.8 Find the unit vectors perpendicular to $2x^2 + y^2 - 2z^2 = 100$ at the point $(1, -2, 1)$.
- 1.9 The electrostatic potential in free space is given by $\phi = \alpha - \beta(x^2 + y^2) - \gamma \ln \sqrt{x^2 + y^2}$ where α, β and γ are constants. Find the charge density in the region.
- 1.10 A spherically symmetric charge distribution is given by,

$$\rho(r) = \rho_0 \left(1 - \frac{r^2}{a^2} \right) \text{ for } 0 \leq r \leq a, \rho_0 \text{ is a constant}$$

$$= 0 \quad \text{for } r > a$$

Calculate the i) total charge, ii) the electric field intensity and potential V both inside ($r < a$) and outside ($r > a$) regimes.

PRACTICAL

Determination of specific charge (e/m) of electron by J. J. Thomson's method

Theory

The specific charge of any fundamental particle is a characteristic property. The specific charge of an electron is defined as its charge per unit mass (e/m). The unit of e/m is C/Kg. In a Cathode Ray Tube (CRT) the beam of electrons moving with velocity v is deflected by vertical electric field E .

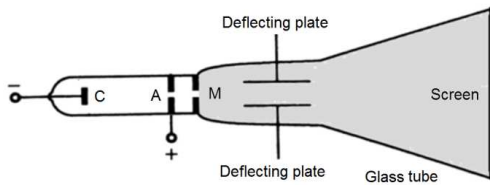


Fig. (i)

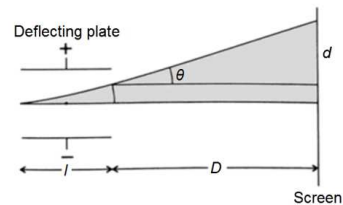


Fig. (ii)

The deflection of the spot of light on the screen is given by,

$$\frac{d}{D} = \frac{y}{l/2} = \frac{1}{2} \frac{eE}{m} \left(\frac{l}{v} \right)^2 \frac{2}{l} = \frac{eEl}{mv^2}$$

$$\text{or, } \frac{e}{m} = \frac{v^2 d}{DIE} \quad (i)$$

If the effect the electric field can be cancelled by a magnetic field B then,

$$evB = eE$$

$$\text{or, } v = \frac{E}{B} \quad (ii)$$

From equations (i) and (ii) we have,

$$\frac{e}{m} = \frac{E^2 d}{DIEB^2} = \frac{Ed}{DIB^2}$$

$$\text{since, } E = \frac{V}{h}$$

$$\text{therefore, } \frac{e}{m} = \frac{Vd}{hlDB^2} \quad (iii)$$

where h is the distance between two horizontal plates, l is the extension of the electric field, V is the applied voltage and D is the distance of the screen from the deflecting plates.

If θ be the angular deflection of the magnetic needle when the bar magnets are properly placed to set up the magnetic field B and the field of the bar magnets is perpendicular to the horizontal component of the earth's magnetic field H , then we have

$$B = H \tan \theta \quad (iv)$$

From equations (iii) and (iv) we have,

$$\frac{e}{m} = \frac{d}{h l D} \frac{V}{H^2 \tan^2 \theta} \quad (v)$$

Procedure

1. First keep the arms of the magnetometer perpendicular to the magnetic meridian.
2. Then keep the deflecting plates of the CRT horizontal. Under this condition deflection of the spot on the screen due to application of the voltage will be vertical.
3. Positive and negative voltages are applied to the deflecting plates and mean deflection of the spot in the screen corresponding to a definite voltage is then measured.
4. A bar magnet is placed on the arm of the magnetometer to nullify the deflection of the spot due to electric field.
5. To calculate the strength of the applied magnetic field, the CRT is removed and the circular disc containing the magnetic needle is placed at the position of the CRT (central region). The deflection of the magnetic needle is calculated by using the bar magnet on the scale of the magnetometer.

Experimental Data

Given: $H = 0.37 \times 10^{-4} \text{ T}$.

Table 1: Determination of deflection of the light spot and the magnetic needle for different positions of the magnet

No of Obs.	Vertical Voltage	Vertical deflection of the spot (cm)		Position of the magnets to balance electric field (cm)		Deflection of the needle of the magnetometer (deg)		Mean θ (deg)	e/m in C/kg
		y_1	y_2	x_1	x_2	θ_1	θ_2		
1									
2									
3									
4									
5									

Calculation and Result

Mean $e/m = \dots\dots\dots \text{C/kg}$.

Discussion

1. The axis of the magnetometer arms is to be kept perpendicular to the earth's magnetic meridian plane.
2. The voltage should be adjusted to a suitable value to get a large possible limiting value of d .

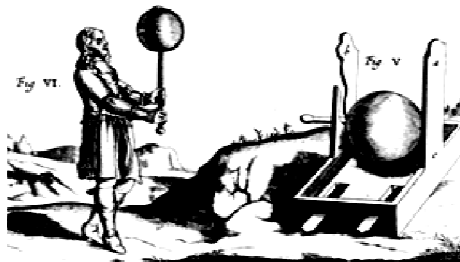
3. To get accurate result the deflection of both ends of the needle is to be measured.

KNOW MORE

Charles Augustine de Coulomb proved his law (Coulomb's Law) and this law became the fundamental principles to establish electrostatics. The word electricity came from the Greek 'elektron' which means amber. The 'amber effect' is called static electricity.

Activity

Watch "Spark: Shock and Awe – The story of Electricity" and then write a summary of the topics discussed in the episode. Also on the basis of the knowledge gathered design an experiment on "Electrostatics and Batteries".



Interesting facts

Otto von Guericke (1601-1674) invented an electrostatic generator. In it a Sulphur ball was rotated in a wooden cradle and it was rubbed by hand. The ball so charged was transported at the end of an insulating rod. Benjamin Franklin, starting from his famous kite experiment made major scientific contributions in electrostatics.



Analogy

Due to triboelectric effect electrostatic charges are produced to build up on the surface of the fur when the cat moves (Reference of the figure: Wikipedia).

History

Benjamin Franklin (1706-1790): He was a physical scientist. His 1740's work on electricity changed unrelated observations into the field of coherent science.

Charles Coulomb (1736-1806): He studies both electrostatic and magnetism. He investigated strengths of materials. In this area he was able to identify forces acting on beams.

Timelines

- 600 BC Greeks first identified attractive properties of amber when rubbed
- 1600 AD Electric bodies both repel and attract
- 1735 AD *du Fay*: Two distinct type of electricity
- 1750 AD *Franklin*: Positive and Negative charge
- 1770 AD *Coulomb*: “Inverse square law”
- 1890 AD *J. J. Thompson*: Quantization of electric charge – “Electron”

Applications (Real Life / Industrial)

Today in every sphere we are applying the discovery of electricity. The production of static electricity in 1705, in fact, resulted in a very fascinating journey. We started with scientists like Francis Hauksbee and Benjamin Franklin and end with Thomas Edison and Nikola Tesla. One very important component that emerged today is the capacitor that has wide applications in many areas of electronics and electrical engineering. Electrostatic induction is the phenomenon used for electro-mechanic precipitation or projection. In such technologies, charged particles of small sizes are deposited or collected intentionally on surfaces. There are a lot of applications ranging from electrostatic coating to inkjet printing.

Case Study (Environmental / Sustainability / Social / Ethical Issues)

Some quotes

“And we daily in our experiments electrise bodies *plus* or *minus*, as we think proper. [These terms we may use till your Philosophers give us better.] To electrise *plus* or *minus*, no more needs to be known than this, that the parts of the Tube or Sphere, that are rubb’d, do, in the Instant of Friction, attract the Electrical Fire, and therefore take it from the Thin rubbing; the same parts immediately, as the Friction upon them ceases, are disposed to give the fire they have received, to any Body that has less”. —*Benjamin Franklin*

Inquisitiveness and Curiosity Topics

The title and abstract of an early paper with interesting finding is given below:

Reference: Letter 25 May 1747. Quoted in I. Bernard Cohen, *Franklin and Newton: An Enquiry into Speculative Newtonian Experimental Science and Franklin’s Work in Electricity as an Example Thereof* (1956), 439.

“We call that fire of the black thunder-cloud “electricity,” and lecture learnedly about it, and grind the like of it out of glass and silk: but what is it? What made it? Whence comes it? Whither goes it?”

—*Thomas Carlyle*

From history of Electrical Engineering II: 18-th Century Electrostatic Experiments and a First Current Source Creation

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Abstract — The paper covers the electrical science development from the middle of the 18-th century to the beginning of the 19-th century. It overviews main inventions made by P. Mushenbroek (Leiden jar), B. Franklin (lightning rod), Ch. Coulomb (Coulomb's balance), A. Volta (Volta's cell, condenser, electrometer) and his followers. It clarified the nature of electricity, created a new electrical discipline – electrostatic and also the sound background for the following important discovery of 19-th century – electromagnetism.

Index Terms — Electrostatics, Leiden jar, lightning rod, Coulomb's balance, Volta's cell, History of electrical engineering.



Pieter van Musschenbroek (1692 – 1761).

Reference: The Carlyle Anthology (1876), 230.

“When the face, the back of the hand, or another part of the body the sensitivity of which is not too weakened by touch is brought near an electrified conductor, there is felt the impression of a fresh breeze, of a light breath, or of a cobweb.”
— Pierre Bertholon

Reference: In biography article by Louis Dulieu, in Charles Coulston Gillispie, *Dictionary of Scientific Biography* (1980), Vol. 2, 83.

The concept of electrostatic induction was used in the past for building high-voltage generators called influence machines. In recent years a new wireless power transfer technology has been developed based on electrostatic induction between oscillating distant dipoles.

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2

Electrostatics in a Linear Dielectric Medium

UNIT SPECIFICS

In this unit we have considered the following aspects:

- Dielectrics;
- Dielectric polarization;
- Polarizability and charge density;
- Classifications of dielectrics;
- Different types polarization: electronic, ionic, orientation and space charge polarization; relationship among D, E and P;
- Constitutive relations;
- Behaviour of dielectric under alternating field and dielectric losses.

The practical applications of the topics are discussed for generating further curiosity and creativity as well as improving problem solving capacity.

Besides giving a large number of multiple choice questions as well as questions of short and long answer types marked in two categories following lower and higher order of Bloom's taxonomy, assignments through a number of numerical problems, a list of references and suggested readings are given in the unit so that one can go through them for practice. It is important to note that for getting more information on various topics of interest some QR codes have been provided in different sections which can be scanned for relevant supportive knowledge.

After the related practical, based on the content, there is a "Know More" section. This section has been judiciously designed so that the supplementary information provided in this part becomes beneficial for the users of the book. This section mainly highlights the initial activity, examples of some interesting facts, analogy, history of the development of the subject focusing the salient observations and finding, timelines starting from the development of the concerned topics up to the recent time, applications of the subject matter for our day-to-day real life or/and industrial applications on variety of aspects, case study related to environmental, sustainability, social and ethical issues whichever applicable, and finally inquisitiveness and curiosity topics of the unit.

RATIONALE

This unit on Electrostatics in a Linear Dielectric Medium will help our students to get a clear idea about the dielectrics, its polarization and charge density as well as classification of dielectrics and polarizations. It establishes some theoretical relations showing the behaviour of dielectric when alternating field is applied and also the dielectric losses.

Typically by the term dielectric we mean materials having high polarizability. With this there is an associated term known as the relative permittivity. It is important to mention that the term insulator is usually used to represent electrical obstruction while on the other hand the terminology dielectric is concerned with the energy storing capacity of the material through the system of polarization. A very popular example of a dielectric is the electrically insulating material which is kept within the metallic plates of a capacitor. With the application of the electric field, polarization of the dielectric enhances the surface charge of the capacitor due to the contribution of the strength of given electric field.

PRE-REQUISITES

Mathematics: Vector Calculus (Class XII)

Physics: Electrostatics (Class XII)

UNIT OUTCOMES

List of outcomes of this unit is as follows:

U2-O1: Describe dielectrics, dielectric polarization, polarizability and charge density

U2-O2: Describe classifications of dielectrics

U2-O3: Explain different types polarization: electronic, ionic, orientation and space charge polarization

U2-O4: Explain relationship among D, E and P

U2-O5: Describe constitutive relations

U2-O6: Identify the behaviour of dielectric under alternating field

Unit-2 Outcomes	EXPECTED MAPPING WITH COURSE OUTCOMES (1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation)					
	CO-1	CO-2	CO-3	CO-4	CO-5	CO-6
U2-O1	3	1	-	1	-	-
U2-O2	3	-	-	-	-	-
U2-O3	3	-	-	-	-	-
U2-O4	2	1	-	1	-	1
U2-O5	2	-	-	1	-	-
U2-O6	1	1	-	-	-	-

2.1 INTRODUCTION

For conductors even in the presence of a feeble electric field, a number of free charge carriers can move. But another class of substances called as the dielectrics or insulators do not possess any free electron inside them. In this type of materials the electrons are strongly bound to the atoms or molecules composing the material and cannot be detached by the applied electric field below a *break down voltage* from the materials. So, normally they do not conduct electricity. But they have some electric effect which is different from that of a conductor and so these types of materials are very useful in many engineering applications. Thus, the study of dielectrics in an electric field is equally important as that of a conductor.

2.2 DIELECTRICS

The materials, in which all of the electrons are bound to the atoms of the material, are termed as the *dielectric* materials. In this type of material, no free electrons are available to conduct electricity. So, they behave like an insulator. Under the influence of an electric field, the dielectric ions or molecules may be polarized. As, these types of materials exhibit electrostatic field for a long time in it, they play major role in many electronic devices. The resistivity of dielectric substances is in the range $10^6 \Omega m - 10^{16} \Omega m$. Generally a dielectric substance is characterized by a parameter, called the *dielectric constant*, defined as the ratio of permittivity of the dielectric medium to that of the free space. It is also termed as the *relative permittivity* of the medium and is a dimensionless quantity. Mathematically it can be expressed as,

$$k = \epsilon_r = \frac{\epsilon}{\epsilon_0} \quad (2.1)$$

where ϵ is the permittivity of the dielectric medium and ϵ_0 is the free space permittivity. In SI unit system, $\epsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$.

Alternatively, *dielectric constant* of a material may be defined as the ratio of the capacitances of a given capacitor completely filled with the material ($C = \epsilon A / d$) to the capacitance of the same capacitor in vacuum ($C_0 = \epsilon_0 A / d$).

$$\text{Hence,} \quad k = \epsilon_r = \frac{\epsilon}{\epsilon_0} = \frac{C}{C_0} \quad (2.2)$$

Air, wax, glass, mica, paper, *etc.* are examples of such dielectric substances.

Example 2.1 An air capacitor is given a charge raising its potential to 250 V. If on inserting a dielectric medium its potential falls to 50 V, what is the dielectric constant of the medium?

Solution

We have dielectric constant of the medium, $k = \frac{C_2}{C_1}$

Again, $C \propto \frac{1}{V}$

So, $k = \frac{V_1}{V_2} = \frac{250}{50} = 5.$

Example 2.2 A parallel plate capacitor, of area 0.0005 m^2 and a plate separation of 5 mm , has a charge of 0.1 nC , when a voltage of 50 V is applied between the plates. Calculate the dielectric constant of the material used to form the capacitor.

Solution

Given, plate area $A = 0.0005 \text{ m}^2$, plate separation $d = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$, voltage $V = 50 \text{ V}$ and charge $Q = 0.1 \text{ nC} = 0.1 \times 10^{-9} \text{ C}$.

Thus, the dielectric constant of the used material will be

$$\begin{aligned} k &= \frac{Cd}{\epsilon_0 A} = \frac{Qd}{\epsilon_0 AV} \\ &= \frac{0.1 \times 10^{-9} \times 5 \times 10^{-3}}{8.854 \times 10^{-12} \times 0.0005 \times 50} \\ &= \frac{0.5}{8.854 \times 0.025} = 2.26. \end{aligned}$$

2.2.1 Classifications

The large number of molecules present in a material medium consists of the nucleus and the electrons revolving around it. If we can replace all the positive and negative charges in a substance by an equivalent positive and negative charge located at the respective centre of gravity of the existing charge systems, then depending on the separation between the centre of gravities of the positive and negative charge systems, dielectric substances may be classified as, *polar* and *non-polar*.

Polar dielectrics: When the centre of gravities of the positive and negative charge systems of the dielectric substance is separated by a finite distance, the dielectric substance is termed as the polar dielectric material with an intrinsic *dipole moment*. Thus for polar dielectric molecule we have a permanent dipole moment.

H_2O , HCl etc are examples of polar dielectric material.

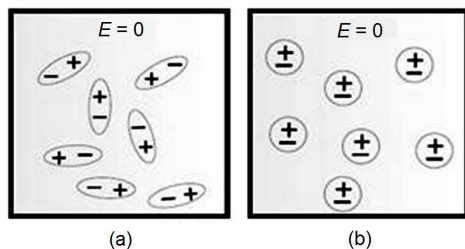


Fig. 2.1: (a) Polar and (b) non-polar dielectrics

In normal state the charges (positive and negative) of a polar dielectric molecule are randomly oriented [Fig. 2.1 (a)] and compensate each other so that the resultant dipole moment per unit volume of a polar dielectric molecule is zero, though it possesses a permanent dipole moment. In case of an isolated atom, the center of gravity of positive charges coincides with the center of gravity of electron cloud of the atom. So the electric dipole moment is zero. Hence an isolated atom cannot have a permanent electric dipole.

Non-polar dielectrics: When the centre of gravities of the positive and negative charge systems of the dielectric substance coincides exactly, the dielectric substance is termed as the non-polar dielectric material having zero electric dipole moment in absence of any electric field. These types of molecules possess symmetric structure with no permanent dipole moment [Fig. 2.1 (b)].

H_2 , O_2 , N_2 , He , Ne , Ar , etc. are examples of non-polar dielectric material.

The basic difference between polar and non-polar dielectric materials is as given below:

Polar dielectrics	Non-polar dielectrics
i) They have permanent dipole moment even in the absence of electric field.	i) They do not have permanent dipole moment in the absence of electric field.
ii) Polarization is temperature dependent.	ii) Polarization is temperature independent.
iii) Examples are: HCl, CO, H ₂ O <i>etc.</i>	iii) Examples are: O ₂ , N ₂ , H ₂ <i>etc.</i>

2.3 DIELECTRIC POLARIZATION

In dielectric material, electrons of the outermost orbit are not able to move freely as they are rigidly bound to the nucleus with definite forces, and so they are referred to as the *bound charges*. When an external electric field is applied to this dielectric medium, these bound charges will experience electrostatic forces. For polar dielectric, the permanent dipole associated with the molecule experiences a torque which tends to align them along the direction of the electric field [Fig. 2.2 (a)].

It is then said to be polarized, possessing a net dipole moment. When no electric field is applied to a non-polar dielectric material, as discussed earlier, the positively charged nucleus and negatively charge electrons are so close to each other that their action is neutralized [Fig. 2.2 (b)]. But when an electric field is applied externally to the non-polar dielectric material, the positive charge is displaced a little along the direction of the applied electric field and the negative charge is displaced a little opposite to the direction of the applied electric field, forming an electric dipole [Fig. 2.2 (b)]. So, the molecules of a non-polar dielectric material acquires an induced electric dipole moment along the direction of the external electric field and this phenomena is called *polarization* or more specifically *dielectric polarization* of the non-polar dielectric material.

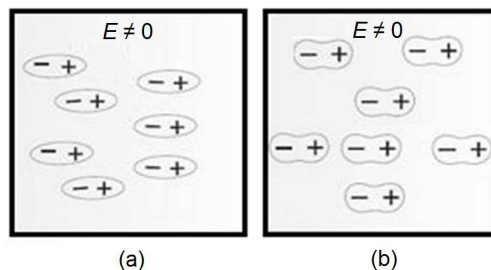


Fig. 2.2: (a) Polar and (b) non-polar dielectrics in presence of an electric field



2.4 ELECTRIC DIPOLE MOMENT

When externally an electric field is applied to a dielectric material, the equal and opposite positive and negative charges separated by a distance in the dielectric material will constitute an *electric dipole*. The electric dipole moment is basically a measure of positive and negative electrical charge separation within a system. Electric dipole moment of the electric dipole thus developed can be defined as the product of any one of the charges and the distance of separation between them.

Thus the induced electric dipole moment will be,

$$\vec{p} = q\vec{d} \quad (2.3)$$

It is a vector quantity and is directed from the negative to the positive charge. It measures the overall polarity of the system.

Practical unit of electric dipole moment is *debye*.

$$1 \text{ debye} = 10^{-8} \text{ statC-cm} = 3.33 \times 10^{-30} \text{ C-m.}$$

2.5 POLARIZATION VECTOR

The electric dipole moment per unit volume of a dielectric substance is termed as the *electric polarization vector*. Alternately, induced dipole moment per unit volume or the induced charge per unit area of a dielectric substance is known as electric polarization vector. It is a vector quantity, directed from the negative to the positive charge.

If n is the number of molecules per unit volume and \vec{p} is the component of the electric dipole moment of every molecule along the external electric field, the electric polarization vector is given by,

$$\vec{P} = n\vec{p} \quad (2.4)$$

Considering a continuously polarized dielectric, the electric polarization vector associated with an infinitesimal volume element ΔV can be obtained as,

$$\vec{P} = \lim_{\Delta V \rightarrow 0} \frac{\Delta \vec{p}}{\Delta V} = \frac{d\vec{p}}{dV} \quad (2.5)$$

2.6 ELECTRIC SUSCEPTIBILITY

When external electric field is applied to a dielectric material, the dielectric molecules are oriented along the direction of the electric field and get polarized. The electric polarization vector for a linear homogeneous dielectric medium is proportional to the external electric field and may be expressed as,

$$\vec{P} \propto \vec{E}$$

$$\text{or,} \quad \vec{P} = \epsilon_0 \chi \vec{E} \quad (2.6)$$

where χ is the electric susceptibility of the dielectric medium and ϵ_0 is the permittivity of the free space. Electric susceptibility effectively represents a dimensionless proportionality constant and it indicates the degree of polarization of a dielectric material in presence of an external electric field. It can be obtained from Eq. (2.6) as,

$$\chi = \frac{P}{\epsilon_0 E} \quad (2.7)$$

and so it indeed represents the ratio of polarization per unit volume per unit electric field of the dielectric substance. It is a positive constant for a particular dielectric material.

2.7 ATOMIC POLARIZABILITY

When an electric field is applied externally to a dielectric substance, the dielectric molecules are oriented along the direction of the given electric field and get polarized. The net induced dipole moment of an atom in a dielectric medium is proportional to the external electric field and may be expressed as,

$$\vec{P} \propto \vec{E}$$

$$\text{or,} \quad \vec{p} = \alpha \vec{E} \quad (2.8)$$

where α is the proportionality constant and is known as the atomic polarizability of the dielectric medium. It can be defined as the induced dipole moment in the atom in response to the application of electric field. Basically, it is the ratio of induced electric dipole moment per unit electric field of an atom of the dielectric medium.

Hence it can be mathematically expressed as,

$$\alpha = \frac{p}{E} \quad (2.9)$$

The SI unit of atomic polarizability is found to be Fm^2 .

Now, the electric polarization vector can be expressed in terms of atomic polarizability as,

$$\vec{P} = n\vec{p} = n\alpha\vec{E} \quad (2.10)$$

and hence the electric susceptibility of the dielectric medium is given by,

$$\chi = \frac{P}{\epsilon_0 E} = \frac{n\alpha}{\epsilon_0} \quad (2.11)$$

Eq. (2.11) gives the relationship between electric susceptibility and polarizability.

Example 2.3 Calculate the induced dipole moment per unit volume of He gas if it is placed in an electric field of 6000 Vcm^{-1} . Take the atomic polarizability of He as $0.18 \times 10^{-40} \text{ Fm}^2$ and density of He as $2.6 \times 10^{25} \text{ atoms/m}^3$.

Solution

Here, the atomic polarizability of He = $\alpha = 0.18 \times 10^{-40} \text{ Fm}^2$, density of He = $N = 2.6 \times 10^{25} \text{ atoms/m}^3$ and electric field = $E = 6000 \text{ Vcm}^{-1} = 6 \times 10^5 \text{ Vm}^{-1}$.

So, the induced dipole moment per unit volume of He gas

$$p = N\alpha E = 2.6 \times 10^{25} \times 0.18 \times 10^{-40} \times 6 \times 10^5$$

$$\text{or, } p = 2.81 \times 10^{-10} \text{ C/m}^2.$$

EXAMPLE 2.3

Example 2.4 Calculate the polarizability of a gas if its susceptibility is 4×10^{-4} . Given the density and molecular weight of the gas are 1.7 kg/m^3 and 49 respectively.

Solution

Given, susceptibility of the gas = $\chi = 4 \times 10^{-4}$, its density = 1.7 kg/m^3 and molecular weight = 49.

So, the number of atoms per unit volume is

$$n = \frac{\text{Avogadro number} \times \text{density}}{\text{Atomic weight}} = \frac{6.02 \times 10^{26} \times 1.7}{49} = 2.089 \times 10^{25}.$$

Thus, the polarizability of the gas will be

$$\begin{aligned} \alpha &= \frac{\epsilon_0 \chi}{n} = \frac{8.854 \times 10^{-12} \times 4 \times 10^{-4}}{2.089 \times 10^{25}} \\ &= 1.695 \times 10^{-40} \text{ Fm}^2. \end{aligned}$$

EXAMPLE 2.4

2.8 DIELECTRIC SUBSTANCE IN ELECTRIC FIELD

Let us discuss the behavior of a dielectric substance when placed in an electric field. For this, we initially consider a parallel plate capacitor which is placed in an initially uniform electric field so that

one plate of the capacitor acquires positive charges and the other plate of the capacitor acquires negative charges (Fig. 2.3).

Now, if a dielectric substance is placed in between the plates of the parallel plate capacitor, there will be a slight shift of its positive charges towards the negative plate and negative charges towards the positive plate; so that the slab of dielectric substance gets polarized with the formation of molecular dipole in it.

Due to this polarization, and hence the formation of the dipole an electric field will be induced inside the dielectric slab, opposite to the direction of external electric field. If E is the intensity of the initial electric field and E_p is the intensity of the electric field produced inside the dielectric material, the resultant electric field inside the dielectric material will be,

$$\vec{E}' = \vec{E} - \vec{E}_p \quad (2.12)$$

Thus when a dielectric is placed in an initially uniform electric field, the field within the dielectric is reduced due to the dielectric polarization.

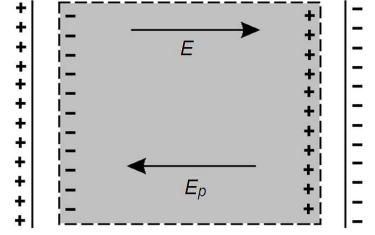


Fig. 2.3: Dielectric substance in an electric field

2.9 DIELECTRIC UNDER ALTERNATING FIELD

While placed in an alternating electric field, the dipoles in the dielectric material get aligned along the applied field and a part of the applied energy gets lost. These losses of energy are termed as the *dielectric losses*. The dielectric losses depend on the frequency of the alternating electric field and the method of polarization. To calculate an expression for the loss let us consider a parallel plate capacitor with capacitance C , plate area A and separation between the plates d .

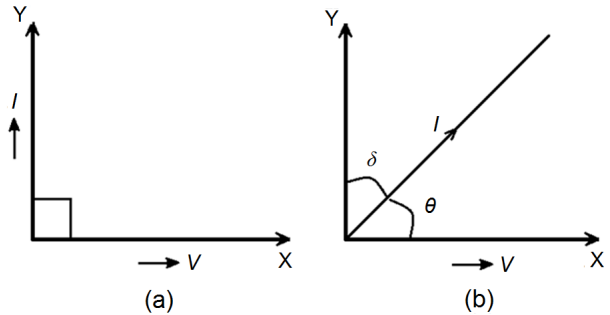


Fig. 2.4: V-I phase dependence for (a) ideal, (b) real dielectrics

When placed in an alternating electric field, the current through the capacitor will be the sum of the conduction current and the displacement current and is given by,

$$I = I_d + I_c = \frac{q}{t} + \frac{V}{R} = \frac{CV}{t} + \frac{V}{R} = \omega CV + \frac{V}{R}$$

For an ideal dielectric material $I_c = 0$ and the resultant current is given by,

$$I = I_d = \omega CV$$

Again the capacitance of the capacitor is given by

$$C = \frac{\epsilon A}{d} \quad (2.13)$$

Thus,
$$I_d = \frac{\omega \epsilon AV}{d} \quad (2.14)$$

Now, the loss angle will be

$$\tan \delta = \frac{I_c}{I_d} \quad [\text{from Fig. 2.4 (b)}] \quad (2.15)$$

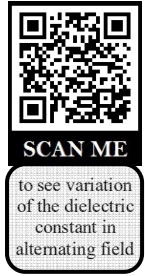
or,

$$\begin{aligned} I_c &= I_d \tan \delta = \frac{\omega \varepsilon A V}{d} \tan \delta \\ &= \omega \varepsilon A E \tan \delta \quad [\text{as } V = Ed] \end{aligned} \quad (2.16)$$

Thus, the actual power loss is given by using Eq. (2.16) as,

$$\begin{aligned} P_a &= VI_c = \omega \varepsilon A E V \tan \delta = \omega \varepsilon A E^2 d \tan \delta \\ &= \omega \varepsilon v E^2 \tan \delta = 2\pi \nu \varepsilon v E^2 \tan \delta \end{aligned} \quad (2.17)$$

From Eq. (2.17), it is clear that the actual power loss is proportional to the volume ($v = Ad$) of the dielectric material, frequency $\left(\nu = \frac{\omega}{2\pi}\right)$ of the alternating electric field, dielectric constant (ε) of the dielectric material, square of the alternating electric field (E) and the loss tangent ($\tan \delta$).



2.10 HOMOGENEITY, LINEARITY AND ISOTROPY

A dielectric medium is called as a *homogeneous* medium if its physical characteristics like mass, density, permittivity *etc.* remain same at all points in a given direction, *i.e.*, the physical characteristics of the medium is independent of the space coordinates.

On the other hand, a dielectric medium is said to be *inhomogeneous* if its physical characteristics like mass, density, permittivity *etc.* changes with the change in positions, *i.e.*, the physical characteristics of the medium is dependent on the space coordinates.

If the permittivity of a dielectric medium does not change with the applied electric field, the medium is said to be *linear* and if the permittivity of a dielectric medium changes with the applied electric field, the medium is said to be *non-linear*.

A dielectric medium is called as an *isotropic* medium if its physical characteristics like mass, density, permittivity *etc.* remains same at all points in every directions and the dielectric medium is said to be *anisotropic* if its physical characteristics like mass, density, permittivity *etc.* changes with the change in directions.

2.11 RELATION OF FLUX DENSITY, INTENSITY AND POLARIZATION

In this section we wish to find a relationship among flux density, field intensity and polarization of a dielectric medium. For this, let us consider a homogeneous, isotropic dielectric slab of face area A and thickness t , placed between the plates of a parallel plate capacitor (Fig. 2.5). Now, if the parallel plate capacitor is placed in an initially uniform electric field, one plate of the capacitor will acquire positive charges and the other plate of the capacitor will acquire negative charges and the dielectric slab will become polarized as discussed earlier.

Due to this polarization let, q_p and $-q_p$ be the bound charges induced on the end faces of the slab producing their own electric field intensity \vec{E}_p . Thus, for the whole slab the dipole moment is

$q_p t$ and since the volume of the slab is tA so, the magnitude of electric polarization may be obtained

$$\text{as, } |\vec{P}| = \frac{q_p t}{tA} = \frac{q_p}{A} = \sigma_p \quad (2.18)$$

where $\sigma_p = \frac{q_p}{A}$ is the surface density of polarization charges of the dielectric slab.

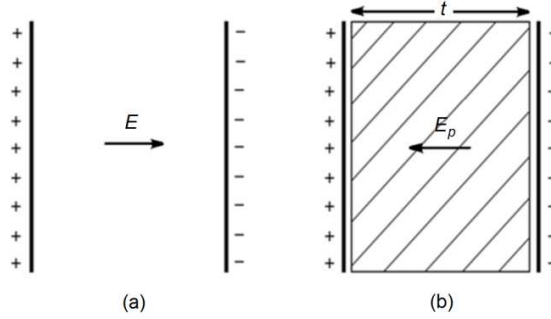


Fig. 2.5: Parallel plate capacitor with (a) no dielectric and (b) given dielectric

Now, if \vec{E} be the electric field intensity of the initial electric field and \vec{E}_p be the electric field intensity produced inside the dielectric material, the resultant electric field inside the dielectric material will be,

$$\vec{E}' = \vec{E} - \vec{E}_p \quad (2.19)$$

Again if σ and σ_p be the surface density of charges on the capacitor plate and the surface density of induced (polarization) charges on the dielectric slab respectively then,

$$E_p = \frac{\sigma_p}{\epsilon_0} \quad (2.20) \text{ (a)}$$

$$\text{and } E = \frac{\sigma}{\epsilon_0} \quad (2.20) \text{ (b)}$$

$$\text{So, } E' = \frac{\sigma}{\epsilon_0} - \frac{\sigma_p}{\epsilon_0} \quad (2.21)$$

$$\text{or, } \sigma = \epsilon_0 E' + \sigma_p = \epsilon_0 E' + P \quad (2.22)$$

The quantity $\epsilon_0 E' + P$ is called the electric flux density or electric displacement and is denoted by D . This basically represents the number of lines of force that passes normally through unit area. More generally, in vector form the expression for the electric displacement vector is given by,

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad (2.23)$$

For free space *i.e.*, in vacuum $\vec{P} = 0$ and the electric displacement vector is given by,

$$\vec{D} = \epsilon_0 \vec{E} \quad (2.24) \text{ (a)}$$

2.12 FLUX DENSITY AND ELECTRIC FLUX

Electric flux density represents the number of electric lines crossing normally through a unit area and for a linear isotropic medium under consideration with permittivity ϵ ; the flux density vector is defined as,

$$\vec{D} = \epsilon \vec{E} = \epsilon_0 \epsilon_r \vec{E} \quad (2.24) \text{ (b)}$$

Surface integration of the normal component of the flux density vector over a closed surface (S) may be regarded as the electric flux (ϕ) enclosed by the surface. Hence the electric flux may be expressed as

$$\phi = \int_S \vec{D} \cdot d\vec{S} \quad (2.25)$$

2.13 GAUSS'S LAW IN DIELECTRICS

It is similar to Gauss's law in electrostatics and applicable for any dielectric medium. It states that the total electric flux *i.e.*, the surface integration of the normal component of the flux density vector over a closed surface of a dielectric medium is equal to $\frac{1}{\epsilon_0}$ times the total charge enclosed by the surface.

$$\text{Thus,} \quad \phi = \int_S \vec{D} \cdot d\vec{S} = q \quad (2.26)$$

$$\text{where} \quad \vec{D} = \epsilon \vec{E} = \epsilon_0 k \vec{E} \quad (2.27)$$

is the electric displacement vector. Here, k is the dielectric constant of the dielectric medium.

2.14 CONSTITUTIVE RELATIONS

Different physical parameters like electric susceptibility, dielectric constant, atomic polarizability, electric polarization of a dielectric medium are related with each other through some relations commonly referred as the constitutive relations for dielectrics. In the subsequent sections we will derive a few such relations.

2.14.1 Electric Susceptibility and Dielectric Constant

We know that the electric displacement vector, $\vec{D} = \epsilon \vec{E} = \epsilon_0 k \vec{E}$ and the electric polarization vector, $\vec{P} = \epsilon_0 \chi \vec{E}$.

So, from the expression for the electric displacement vector, $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ we get

$$\epsilon_0 k \vec{E} = \epsilon_0 \vec{E} + \epsilon_0 \chi \vec{E}$$

Simplifying we get,

$$k = 1 + \chi \quad (2.28)$$

This gives the relationship between electric susceptibility and dielectric constant of the dielectric medium.

EXAMPLE 2.5

Example 2.5 If the dielectric constant of a gas at NTP is 1.000056, find its electric susceptibility.

Solution

The dielectric constant of a gas at NTP is 1.0000456.

So, its susceptibility will be $\chi = \epsilon - 1 = 1.000056 - 1 = 0.000056 = 5.6 \times 10^{-5}$.

2.14.2 Electric Susceptibility and Atomic Polarizability

The electric susceptibility of the dielectric medium is given by,

$$\chi = \frac{P}{\epsilon_0 E} = \frac{n\alpha}{\epsilon_0}$$

$$\therefore k = 1 + \chi = 1 + \frac{n\alpha}{\epsilon_0} \quad (2.29)$$

This gives the relationship between electric susceptibility and atomic polarizability of the dielectric medium.

EXAMPLE 2.6

Example 2.6 Assuming that the electric polarizability of an Argon atom is $1.43 \times 10^{-40} \text{ F.m}^2$, find the dielectric constant of solid Argon. Given density of Argon is 1.8 g.cm^{-3} and atomic mass of Argon is 39.95 g.mol^{-1} .

Solution

Given, the electric polarizability of an Argon atom = $1.43 \times 10^{-40} \text{ F.m}^2$, density of Argon = 1.8 g.cm^{-3} and atomic mass of Argon = 39.95 g.mol^{-1} .

So, the number of Ar atoms per unit volume is

$$n = \frac{\text{Avogadro number} \times \text{density}}{\text{Atomic weight}} = \frac{6.02 \times 10^{26} \times 1.8}{39.95} = 2.7 \times 10^{25}.$$

Thus, the dielectric constant of solid Argon is

$$k = 1 + \frac{n\alpha}{\epsilon_0} = 1 + \frac{2.7 \times 10^{25} \times 1.43 \times 10^{-40}}{8.854 \times 10^{-12}} = 1.000436.$$

2.14.3 Electric Polarization and Dielectric Constant

We have, from $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$,

$$\epsilon_0 k \vec{E} = \epsilon_0 \vec{E} + \vec{P}$$

$$\text{or,} \quad \vec{P} = (k - 1)\epsilon_0 \vec{E} \quad (2.30)$$

This gives the relationship between electric polarization and dielectric constant of the medium under consideration.

Now, if \vec{E} be the electric field intensity of the initial electric field and \vec{E}_p be the electric field intensity produced inside the dielectric material, the resultant electric field inside the dielectric material will be,

$$\vec{E}' = \vec{E} - \vec{E}_p = \vec{E} - \frac{\vec{P}}{\epsilon_0} = \vec{E} - \chi \vec{E}'$$

$$\text{or, } \vec{E}' = \frac{\vec{E}}{1 + \chi} = \frac{\vec{E}}{k} \quad (2.31) \text{ (a)}$$

$$\text{or, } \frac{\vec{E}'}{\vec{E}} = 1 + \chi = k \quad (2.31) \text{ (b)}$$

Thus the ratio of the applied electric field \vec{E} and the net electric field \vec{E}' within the dielectric is the dielectric constant of the medium.

2.15 TYPES OF POLARIZATION

If a dielectric substance is placed in a uniform electric field, the field re-aligns the dipoles resulting in polarization. These may be produced,

- i) when the electric field distorts the charge distribution of molecules and produces induced dipole moment in each molecule and,
- ii) when the electric field exerts a torque on the randomly oriented permanent dipoles in place of polar molecules.

In the subsequent sections some basic types of dielectric polarization are illustrated extensively.

2.15.1 Induced (Electronic) Polarization

If a dielectric substance is placed in an external electric field, the electrons of the atoms of the dielectric substance are relatively displaced from the heavily fixed nuclei. Due to this an induced electric dipole moment is produced along the direction of the external electric field and the induced dipole moment per unit volume of the dielectric substance gives the induced (electronic) polarization. Due to the displacement of electrons with respect to the nucleus an induced electric dipole moment is produced along the direction of the external field.

It is independent of temperature of the dielectric substance but depends on the atomic radius. To investigate the nature of dependence let us consider an atom of atomic number Z and radius r , with uniform distribution of the charge cloud as illustrated in Fig. 2.6 (a). So, the charge density will be,

$$\rho = -\frac{Ze}{\frac{4}{3}\pi r^3} \quad (2.32)$$

Now when this atom is placed in an electric field \vec{E} , let the nucleus and the electron cloud will be displaced by an amount x [Fig. 2.6 (b)].

So, the Lorentz force along the direction of the electric field will be,

$$F_L = ZeE \quad (2.33)$$

and the force of attraction between the nucleus and the electron cloud will be,

$$F_G = \frac{Ze \cdot \frac{4}{3}\pi x^3 \rho}{4\pi\epsilon_0 x^2} = -\frac{Z^2 e^2 x}{4\pi\epsilon_0 r^3} \quad (2.34)$$

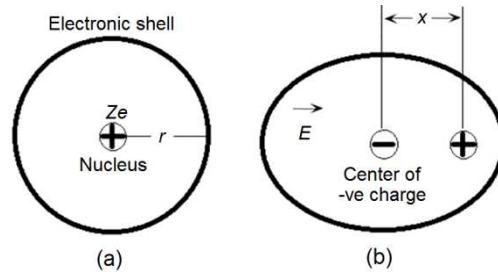


Fig. 2.6: (a) Neutral atom ($E = 0$) and (b) induced polarization in presence of E -field

Now, for equilibrium the net force on the nucleus must be zero. So, equating Eq. (2.33) with Eq. (2.34) we get,

$$ZeE = \frac{Z^2 e^2 x}{4\pi\epsilon_0 r^3}$$

or,

$$x = \frac{4\pi\epsilon_0 r^3 E}{Ze} \quad (2.35)$$

So, the displacement of electron cloud is proportional to the applied electric field. Now the induced electric dipole moment will be,

$$\begin{aligned} p &= Zex \\ &= 4\pi\epsilon_0 r^3 E \end{aligned} \quad (2.36)$$

Thus the induced electric dipole moment is also proportional to the applied electric field.

Again the electric polarizability is given by,

$$\alpha = \frac{p}{E} = 4\pi\epsilon_0 r^3 \quad (2.37)$$

If the number of atoms per unit volume be n , the electric polarization will be,

$$\begin{aligned} P &= np = n\alpha E \\ &= 4\pi\epsilon_0 nr^3 E \end{aligned} \quad (2.38)$$

This gives the relationship between electronic polarizability and atomic radius.

Also we have dielectric constant of the medium as,

$$k = 1 + \frac{n\alpha}{\epsilon_0}$$

$$\therefore n\alpha = \epsilon_0 (k - 1)$$

or,

$$4\pi\epsilon_0 r^3 n = \epsilon_0 (k - 1)$$

or,

$$(k - 1) = 4\pi nr^3$$

or,

$$k = 1 + 4\pi nr^3 \quad (2.39)$$

Eq. (2.39) gives the relationship between the dielectric constant and atomic radius.

Example 2.7 At 0°C and 1 atmospheric pressure, the dielectric constant of He is 1.0000684 and the gas contains 2.7×10^{25} atoms/ m^3 under these conditions. Calculate the radius of electron cloud and the displacement when the He atom is subjected to a field of 10^6 V/m.

Solution

Here, dielectric constant $= k = 1.0000684$, no of atoms per unit volume $= n = 2.7 \times 10^{25}$ atoms/ m^3 and applied electric field $= E = 10^6$ V/m.

Now we have, $k = 1 + 4\pi n r^3$

$$\text{or, } a^3 = \frac{k-1}{4\pi n} = \frac{1.0000684-1}{4\pi \times 2.7 \times 10^{25}} = 20.2 \times 10^{-36}$$

$$\text{or, } a = 2.72 \times 10^{-12} \text{ m.}$$

which gives the radius of the electron cloud.

Again, the displacement of the electron cloud is

$$\begin{aligned} x &= \frac{4\pi\epsilon_0 a^3 E}{Ze} \\ &= \frac{4\pi \times 8.854 \times 10^{-12} \times 20.2 \times 10^{-36} \times 10^6}{2 \times 1.6 \times 10^{-19}} = 0.7 \times 10^{-22} \text{ m.} \end{aligned}$$

EXAMPLE 2.7

2.15.2 Atomic Polarization

The dielectric materials which possesses ionic bond (*e.g. NaCl, KCl etc.*), can generate atomic or ionic polarization. In this type of a crystal there happens a transfer of electron from one atom to another. When an electric field is applied to this type of crystal, the positive and negative ions are respectively displaced along and opposite to the direction of the electric field [Fig. 2.7 (b)], until the force of the ionic bonding can stop it. Due to this an induced electric dipole moment and hence induced electric polarization (p_i) is produced. This induced electric polarization is due to the mutual displacement of the positive and negative ions of the lattice and is independent of temperature. The induced dipole moment of the ionic crystal is proportional to the applied electric field and thus, $p_i \propto E$

$$\text{or, } p_i = \alpha_i E \quad [\text{where } \alpha_i \text{ is the ionic polarizability}] \quad (2.40)$$

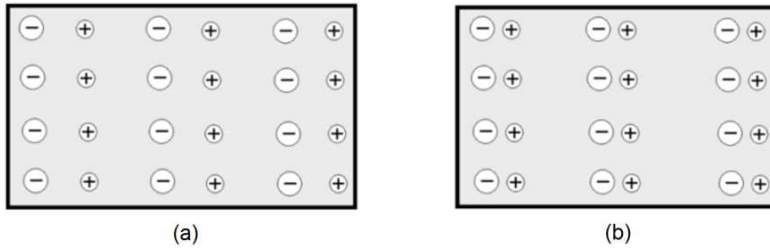


Fig. 2.7: Ionic polarization with (a) no field ($E = 0$), (b) applied field ($E \neq 0$)

If the number of dipoles per unit volume of the crystal is n , the ionic polarization is given by,

$$P_i = np_i = n\alpha_i E \quad (2.41)$$

Apart from ionic polarization, an ionic molecule also possesses electronic polarization due to the movement of electron cloud under the action of the electric field. So, total polarization will be the sum of ionic and electronic polarizations and is given by,

$$P = P_e + P_i = n(\alpha_e + \alpha_i)E \quad (2.42)$$

2.15.3 Dipolar Polarization

This type of polarization is observed for polar dielectrics, having permanent electric dipoles (*e.g. HCl, NO₂ etc.*). Generally in absence of external electric field, these dipoles are randomly oriented to cancel the effects of each other, to have zero electric dipole moment. But when an external electric field is applied, these dipoles are oriented along the direction of the electric field, to produce a net dipole moment and hence a net torque on it (Fig. 2.8). The polarization thus developed is known as the orientational polarization. This type of polarization varies inversely with the temperature. The torque acting on the polar molecules of the dipole when subjected to an electric field of intensity E is given by,

$$\tau = p_0 E \sin \theta \quad (2.43)$$

where p_0 is the orientational dipole moment. When the dipole is rotated through 180° , maximum work is done and the maximum orientational dipole energy is given by,

$$E_{\max} = \int_0^\pi p_0 E \sin \theta = 2 p_0 E \quad (2.44)$$

and when the dipole is along the E -field, there is no work done. So, the average orientational dipole energy is given by

$$E_{av} = \frac{2 p_0 E}{2} = p_0 E \quad (2.45)$$

Now, the orientational polarization will be effective when the ratio of average orientational dipole energy and average thermal energy is greater than unity. Thus, the average orientational polarization is

given by,
$$P_o \propto \frac{p_0 E_{av}}{E_{th(av)}}$$

here,
$$E_{th(av)} = \frac{5}{2} kT$$

So,
$$P_o \propto \frac{p_0^2 E}{(5/2)kT}$$

Using M-B statistics, we can obtain the average orientational polarization as,

$$P_o = \frac{n}{3} \frac{p_0^2 E}{kT} = n \alpha_o E \quad (2.46)$$

where n is the number of dipoles per unit volume of the crystal. The orientational polarizability is

thus,
$$\alpha_o = \frac{p_0^2}{3kT} \quad (2.47)$$

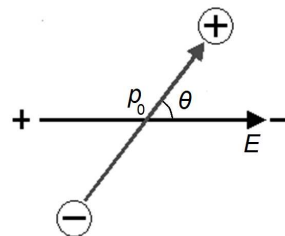


Fig. 2.8: Dipolar orientation in presence of electric field

2.15.4 Interfacial Polarization

This type of polarization occurs in multi-phase dielectric materials possessing different resistivity for different phases. In absence of electric field there is no separation between positive and negative charges within the material. But relatively high temperatures in the presence of electric field, due to the sudden change in conductivity across the boundary, the positive and negative charges are being accumulated on the opposite faces of the dielectric materials [Fig. 2.9 (b)]. Due to the displacement of ions, an induced dipole moment is produced and the polarization thus developed is termed as the interfacial or space charge polarization. This type of polarization of the dielectric material strongly depends on the temperature.

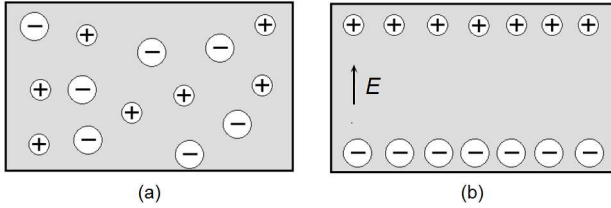


Fig. 2.9: Interfacial polarization with (a) no field ($E = 0$), (b) applied field ($E \neq 0$)

Of the different types of polarization, the electronic polarization is the fastest and typically persists at frequencies between $\sim 10^{13}$ - 10^{15} Hz. On the other hand, ionic polarization is slower and typically occurs at frequencies between $\sim 10^9$ - 10^{13} Hz, while dipolar or oriental polarization involving movement of molecules happens below 10^9 Hz and interface or space charge polarization occurs at frequencies below 10 Hz [Fig. 2.10].

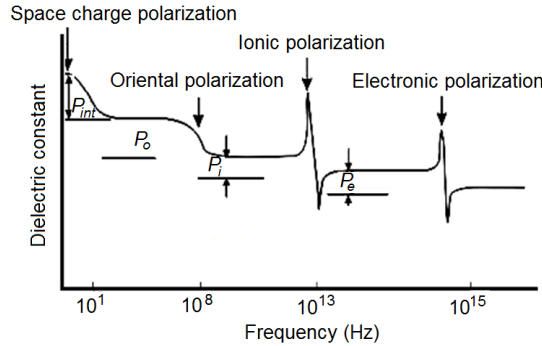


Fig. 2.10: Variation of dielectric constant with frequency

2.16 POLARIZATION OF MONOATOMIC GAS

If an atom with atomic number Z is subjected to an external electric field (E), the center of positive charge is displaced along the applied field direction while the center of the negative charge is displaced in opposite direction (Refer to Fig. 2.6). Thus electric dipole is produced. Electronic polarization can persist to extremely high field frequencies because electronic standing waves within atoms have higher natural frequencies. When the atom is placed into an external electric field E , the nucleus and electron cloud move in the opposite direction. It is important to mention that the equilibrium can be shifted with the nucleus. It is shifted slightly relative to centre of the electron by a distance x proportional to the applied electric field which can be expressed as,

$$x = \frac{4\pi\epsilon_0 r^3 E}{Ze}$$

As a consequence, the induced dipole moment which is proportional to the applied field can be written as,

$$p = Zex = 4\pi\epsilon_0 r^3 E$$

Again the electric polarizability is given by,

$$\alpha = \frac{p}{E} = 4\pi\epsilon_0 r^3$$

If the number of atoms per unit volume be n , the electric polarization will be,

$$P = np = n\alpha E = 4\pi\epsilon_0 nr^3 E$$

2.17 POLARIZATION OF POLYATOMIC GAS

For common dielectric materials, space charge polarization is not very important as it is observed only at the interfaces of a dielectric material and is so small to be neglected. Hence the total polarization of a dielectric material may be assumed as the combined effect of atomic, ionic and orientational polarizations. So, for a polyatomic gas having n molecules per unit volume considering the effect of atomic, ionic and orientational polarizations the total polarization is given by,

$$P = P_e + P_i + P_o = n\alpha_e E + n\alpha_i E + n\alpha_o E = n(\alpha_e + \alpha_i + \alpha_o)E \quad (2.48)$$

or,

$$\epsilon_0 \chi E = \epsilon_0 (\epsilon_r - 1) E = n \left[\alpha_e + \alpha_i + \frac{p_0^2}{3kT} \right] E$$

or,

$$\epsilon_0 (\epsilon_r - 1) = n \left[\alpha_e + \alpha_i + \frac{p_0^2}{3kT} \right] = n\alpha_T \quad (2.49)$$

where, the total polarizability can be obtained as

$$\alpha_T = \alpha_e + \alpha_i + \frac{p_0^2}{3kT} \quad (2.50)$$

2.18 CLAUSIUS MOSSOTTI RELATION

The electric field which is responsible for polarizing molecule of dielectric is called molecular field and in this section we will be denoting it by \vec{E}_m . It is a macroscopic quantity which differs from the external field and is therefore termed as the local field. To obtain an expression for it, a small piece of dielectric is drawn leaving a spherical cavity of radius r , surrounding the point under consideration.

Now, let us put the dielectric back into the cavity, molecule by molecule except for the molecule where we wish to find \vec{E}_m . These molecules may be treated not as a continuum but as individual dipole. Let, O be the centre of the spherical cavity in the dielectric and its radius is large compared with the inter molecular distance but small compared with the dimension of the whole dielectric. Let, the dielectric is placed between two parallel plates.

Now, the electric field experienced by a molecule of dielectric assumed to be placed at the centre of the cavity is given by,

$$\vec{E}_m = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

here, \vec{E}_1 is the field between two parallel charged plates with no dielectric, \vec{E}_2 is the field due to polarized charges on the outer surfaces of the dielectric, \vec{E}_3 is the field due to polarized charges on the inner surfaces of the spherical cavity and \vec{E}_4 is the field due to all dipoles inside the spherical cavity.

$$\text{In this case, } \vec{E}_1 = \frac{\sigma}{\epsilon_0}$$

where σ is the surface charge density,

$$\vec{E}_2 = \frac{\vec{P}}{\epsilon_0}$$

where \vec{P} is the polarization vector due to the polarization of the dielectric when placed within the external field \vec{E}_1 .

$$\therefore \vec{E}_m = \vec{E}_1 - \frac{\vec{P}}{\epsilon_0} + \vec{E}_3 + \vec{E}_4 \quad (2.51)$$

If the dipoles in the cavity are located at the regular atomic position of a cubic crystal, $\vec{E}_4 = 0$ but in case of anisotropic medium $\vec{E}_4 \neq 0$.

Let, \vec{E} be the electric field in the dielectric when it is placed in an external field \vec{E}_1 . From the consideration that the normal component of electric displacement \vec{D} is continuous across the vacuum-dielectric interface, as \vec{D} is perpendicular to the surface, hence \vec{D} in the vacuum just outside the dielectric should be equal to that in the dielectric *i.e.*,

$$\epsilon_0 \vec{E}_1 = \epsilon_0 \vec{E} + \vec{P}$$

Combining the equations and using $\vec{E}_4 = 0$, we get,

$$\vec{E}_m = \vec{E} + \frac{\vec{P}}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0} + \vec{E}_3 + 0 = \vec{E} + \vec{E}_3 \quad (2.52)$$

The electric field \vec{E}_3 generated by the polarization is equivalent to the field produced by the charge distribution associated with the polarization. If this value throughout the volume of the dielectric is constant, then

$$\rho_p = -\vec{\nabla} \cdot \vec{P} = 0 \text{ and } \sigma_p = \vec{P} \cdot \hat{r} = P \cos \theta.$$

Thus, the electric field at the centre O due to charge over the surface area S of the surface at point A with polar coordinates (r, θ) is given by,

$$dE' = \frac{1}{4\pi\epsilon_0} \frac{\sigma_p dS}{r^2} \hat{r}$$

$$\text{Now, } dS = 2\pi r \sin \theta \cdot r d\theta = 2\pi r^2 \sin \theta d\theta$$

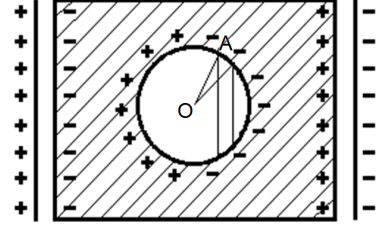


Fig. 2.11: Spherical cavity

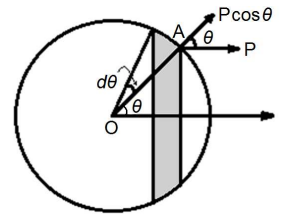


Fig. 2.12: Concept of polar coordinates

$$\therefore dE' = \frac{1}{4\pi\epsilon_0} \frac{P \cos \theta \cdot 2\pi r^2 \sin \theta d\theta}{r^2} \hat{r} = \frac{P \cos \theta \sin \theta d\theta}{2\epsilon_0} \hat{r}$$

From symmetry it is clear that only the component of E' along the direction of P will contribute to the total field E_3

$$\therefore dE_3 = dE' \cos \theta = \frac{P \cos^2 \theta \sin \theta d\theta}{2\epsilon_0}$$

$$\therefore E_3 = \frac{P}{2\epsilon_0} \int_0^\pi \cos^2 \theta \sin \theta d\theta = \frac{P}{3\epsilon_0}$$

$$\text{or, } \vec{E}_3 = \frac{\vec{P}}{3\epsilon_0} \quad (2.53)$$

$$\therefore \vec{E}_m = \vec{E} + \frac{\vec{P}}{3\epsilon_0} \quad (2.54)$$

Now, for ideal dielectric $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ and $\vec{D} = \epsilon \vec{E}$

$$\therefore \epsilon \vec{E} = \epsilon_0 \vec{E} + \vec{P}$$

$$\text{or, } \vec{P} = (\epsilon - \epsilon_0) \vec{E} = \epsilon_0 \left(\frac{\epsilon}{\epsilon_0} - 1 \right) \vec{E} = \epsilon_0 (k - 1) \vec{E}$$

$$\text{Hence, } \vec{E}_m = \vec{E} + \frac{\epsilon_0 (k - 1) \vec{E}}{3\epsilon_0} = \frac{(k + 2) \vec{E}}{3} \quad (2.55)$$

$$\therefore \vec{E} = \frac{3\vec{E}_m}{(k + 2)} \quad (2.56)$$

Substituting in Eq. (2.54) we get,

$$\vec{E}_m = \vec{E} + \frac{\vec{P}}{3\epsilon_0} = \frac{3\vec{E}_m}{(k + 2)} + \frac{\vec{P}}{3\epsilon_0}$$

$$\text{or, } \vec{P} = 3\epsilon_0 \vec{E}_m \left(\frac{k - 1}{k + 2} \right) \quad (2.57)$$

The dipole moment of a molecule per unit polarizing field is called molecular polarizability *i.e.*,

$$\alpha = \frac{\vec{p}_m}{\vec{E}_m}$$

If there is n number of molecules per unit volume then polarization

$$\vec{P} = n \vec{p}_m = n \alpha \vec{E}_m$$

$$\therefore n\alpha \bar{E}_m = 3\varepsilon_0 \bar{E}_m \left(\frac{k-1}{k+2} \right)$$

$$\text{or, } \frac{n\alpha}{3\varepsilon_0} = \frac{k-1}{k+2} \quad (2.58)$$

This equation is known as the Clausius-Mossotti relation. The function is known as the volume polarizability. At optical frequencies $k = \mu^2$ where μ is the refractive index of the medium.

$$\therefore \frac{n\alpha}{3\varepsilon_0} = \frac{\mu^2 - 1}{\mu^2 + 2} \quad (2.59)$$

This equation is known as the Lorentz- Lorentz relation.

2.19 DIELECTRIC SPHERE IN UNIFORM ELECTRIC FIELD

When a dielectric sphere is placed in a uniform electric field, the field will be distorted in the close proximity of the sphere.

Let a dielectric sphere of radius a and permittivity ε_1 is placed in a uniform electric field intensity \bar{E}_0 which acts in the Z-direction as shown in Fig. 2.13. Since the induced charge is symmetric about the axis therefore a potential at any point can be expressed in terms of zonal harmonics. Let us consider a point P outside the sphere whose polar co-ordinates are (r, θ) with respect to the origin considered to be situated at the centre of the sphere.

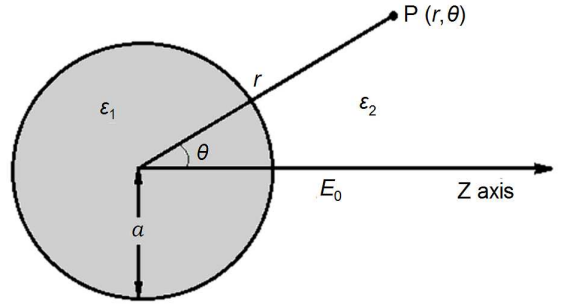


Fig. 2.13: Dielectric sphere placed in a uniform electric field

The potential at point P may be written as,

$$\phi(r, \theta) = \sum_{n=0}^{\infty} [A_n r^n + B_n r^{-(n+1)}] p_n(\cos \theta) \quad (2.60)$$

where, the coefficients A_n and B_n can be determined by using the proper boundary conditions. $p_n(\cos \theta)$ in Eq. (2.60) is the Legendre polynomial (see Annexure IV for details).

The boundary conditions which can be used to evaluate the coefficients A_n and B_n are,

- i) Laplace's equation will be satisfied at every points inside and outside the sphere *i.e.*, $\nabla^2 \phi_1 = 0$ and $\nabla^2 \phi_2 = 0$ at every points inside and outside the sphere.
- ii) ϕ_1 is finite at every points inside the sphere *i.e.*, for $r < a$.
- iii) As $r \rightarrow a$, $\phi_2 \rightarrow -E_0 Z = -E_0 r \cos \theta$.
- iv) Tangential component of electric field intensity is continuous across the interface of two media, *i.e.*, $\phi_1 = \phi_2$ at $r = a$.

v) Since there is no free charge available in the vicinity of the sphere, therefore the normal component of electric displacement is continuous across the interface of two media *i.e.*, $\epsilon_1 \frac{\partial \phi_1}{\partial r} = \epsilon_2 \frac{\partial \phi_2}{\partial r}$.

From second boundary condition, inside potential ϕ_1 may be written as,

$$\phi_1 = \sum_{n=0}^{\infty} [A_n r^n p_n(\cos \theta)] \quad (2.61)$$

and outside potential ϕ_2 may be written as,

$$\phi_2 = \sum_{n=0}^{\infty} [C_n r^n + B_n r^{-(n+1)}] p_n(\cos \theta) \quad (2.62)$$

Since there is no net charge therefore there will be no term in ϕ_2 which contains $\frac{1}{r}$.

$$\therefore \phi_2 = \sum_{n=0}^{\infty} C_n r^n p_n(\cos \theta) + \sum_{n=1}^{\infty} D_n r^{-(n+1)} p_n(\cos \theta) \quad (2.63)$$

From third boundary condition, $-E_0 r \cos \theta = \sum_{n=0}^{\infty} C_n r^n p_n(\cos \theta)$

$$\text{or, } -E_0 r \cos \theta = C_0 + C_1 r \cos \theta + \dots$$

Comparing both sides we get, $C_0 = 0$ and $C_1 = -E_0$ and $C_n = 0$ for $n \geq 2$

$$\therefore \phi_2 = -E_0 r \cos \theta + \sum_{n=1}^{\infty} D_n r^{-(n+1)} p_n(\cos \theta) \quad (2.64)$$

For simultaneous satisfaction of 4th and 5th boundary conditions we may write, $A_n = 0, D_n = 0$ for $n \geq 2$. Now, equations (2.61) and (2.63) may be written as,

$$\phi_1 = A_0 + A_1 r \cos \theta \quad (2.65)$$

$$\text{and } \phi_2 = -E_0 r \cos \theta + \frac{D_1}{r^2} \cos \theta \quad (2.66)$$

Now, from fourth boundary condition, $A_0 + A_1 a \cos \theta = -E_0 a \cos \theta + \frac{D_1}{a^2} \cos \theta$

Comparing both sides, we get $A_0 = 0$ and $A_1 = \frac{D_1}{a^3} - E_0$

$$\text{So, } \frac{\partial \phi_1}{\partial r} = A_1 \cos \theta$$

$$\text{and } \frac{\partial \phi_2}{\partial r} = -E_0 \cos \theta - \frac{2D_1}{r^3} \cos \theta$$

Thus, from fifth boundary condition $\epsilon_1 A_1 \cos \theta = \epsilon_2 (-E_0 \cos \theta - \frac{2D_1}{a^3} \cos \theta)$

Comparing both sides, we get, $\varepsilon_1 A_1 = -\varepsilon_2 \left(E_0 + \frac{2D_1}{a^3} \right)$

or,
$$\varepsilon_1 \left(\frac{D_1}{a^3} - E_0 \right) = -\varepsilon_2 \left(E_0 + \frac{2D_1}{a^3} \right)$$

or,
$$-\varepsilon_1 E_0 + \varepsilon_2 E_0 = -\varepsilon_2 \frac{2D_1}{a^3} - \varepsilon_1 \frac{D_1}{a^3} = -\frac{1}{a^3} (2\varepsilon_2 + \varepsilon_1) D_1$$

$$\therefore D_1 = -\frac{(-\varepsilon_1 + \varepsilon_2) E_0 a^3}{2\varepsilon_2 + \varepsilon_1} = \frac{(\varepsilon_1 - \varepsilon_2) E_0 a^3}{2\varepsilon_2 + \varepsilon_1}$$

$$\therefore A_1 = \frac{(\varepsilon_1 - \varepsilon_2) E_0}{2\varepsilon_2 + \varepsilon_1} - E_0 = \frac{(\varepsilon_1 - \varepsilon_2 - 2\varepsilon_2 - \varepsilon_1) E_0}{2\varepsilon_2 + \varepsilon_1} = -\frac{3\varepsilon_2 E_0}{2\varepsilon_2 + \varepsilon_1}$$

$$\therefore \phi_1 = -\frac{3\varepsilon_2 E_0 r \cos \theta}{2\varepsilon_2 + \varepsilon_1} \quad (2.67)$$

and
$$\phi_2 = -E_0 r \cos \theta + \frac{(\varepsilon_1 - \varepsilon_2) E_0 a^3}{2\varepsilon_2 + \varepsilon_1} \frac{\cos \theta}{r^2} \quad (2.68)$$

So, the field inside the dielectric sphere will be,

$$\vec{E}_1 = -\frac{\partial \phi_1}{\partial z} \hat{z}$$

where, $z = r \cos \theta$

$$\therefore \vec{E}_1 = \frac{\partial}{\partial z} \left[\frac{3\varepsilon_2 E_0 z}{2\varepsilon_2 + \varepsilon_1} \right] \hat{z} = \frac{3\varepsilon_2 E_0}{2\varepsilon_2 + \varepsilon_1} \hat{z} \quad (2.69)$$

again,
$$\vec{E}_2 = E_{2z} \hat{z} + E_{2r} \hat{r} + E_{2\theta} \hat{\theta}$$

where,
$$E_{2z} = -\frac{\partial}{\partial z} (-E_0 r \cos \theta) = E_0$$

$$E_{2r} = -\frac{\partial}{\partial r} \left[\frac{(\varepsilon_1 - \varepsilon_2) E_0 a^3 \cos \theta}{2\varepsilon_2 + \varepsilon_1} \frac{1}{r^2} \right] = \frac{2E_0 a^3 (\varepsilon_1 - \varepsilon_2) \cos \theta}{2\varepsilon_2 + \varepsilon_1} \frac{1}{r^3}$$

and,
$$E_{2\theta} = -\frac{1}{r} \frac{\partial}{\partial \theta} \left[\frac{(\varepsilon_1 - \varepsilon_2) E_0 a^3 \cos \theta}{2\varepsilon_2 + \varepsilon_1} \frac{1}{r^2} \right] = \frac{E_0 a^3 (\varepsilon_1 - \varepsilon_2) \sin \theta}{2\varepsilon_2 + \varepsilon_1} \frac{1}{r^3}$$

Thus,
$$\vec{E}_2 = E_0 \hat{z} + \frac{2E_0 a^3 (\varepsilon_1 - \varepsilon_2) \cos \theta}{2\varepsilon_2 + \varepsilon_1} \frac{1}{r^3} \hat{r} + \frac{E_0 a^3 (\varepsilon_1 - \varepsilon_2) \sin \theta}{2\varepsilon_2 + \varepsilon_1} \frac{1}{r^3} \hat{\theta}$$

or,
$$\vec{E}_2 = E_0 \hat{z} + \frac{E_0 a^3 (\varepsilon_1 - \varepsilon_2)}{(2\varepsilon_2 + \varepsilon_1) r^3} [2 \cos \theta \hat{r} + \sin \theta \hat{\theta}] \quad (2.70)$$

Also we know that the radial component of field intensity is,

$$E_{2r} = \frac{2E_0 a^3 (\epsilon_1 - \epsilon_2) \cos \theta}{2\epsilon_2 + \epsilon_1} \frac{1}{r^3} = \frac{p \cos \theta}{2\pi \epsilon_2 r^3} \quad [\text{where } p \text{ is the induced dipole moment}]$$

or,

$$\vec{p} = \frac{4\pi E_0 \epsilon_2 a^3 (\epsilon_1 - \epsilon_2)}{2\epsilon_2 + \epsilon_1} \hat{z} \quad (2.71)$$

So, the polarization is

$$\vec{P} = \frac{\vec{p}}{\frac{4}{3}\pi a^3} = \frac{3\pi E_0 \epsilon_2 (\epsilon_1 - \epsilon_2)}{2\epsilon_2 + \epsilon_1} \hat{z} \quad (2.72)$$

2.20 DIELECTRIC STRENGTH AND BREAKDOWN

When the applied electric field on a dielectric material is sufficiently large, it can pull the electrons out of the molecule to produce conductivity and then a *dielectric breakdown* is said to occur within the dielectric material. The maximum value of the electric field that can be tolerated without a breakdown is called as the *dielectric strength* of the dielectric material.

When a dielectric substance losses its resistivity and allow a large current to flow through it, dielectric breakdown is noticed. Generally, high electric field is responsible for such a dielectric breakdown. Other important factors are temperature and impurity. High-voltage transformers contain oil as their insulating dielectric. When a critical electric field is exceeded, conduction path grow at μs speeds through the oil in the form of branched trees, called streamers. These can lead to destructive breakdown.



2.21 APPLICATIONS OF DIELECTRICS

Based on the properties like the dependency in temperature, range of permittivity, dielectric strength, insulation etc. dielectric materials are used in various industrial purposes for the manufacturing of different electrical devices. Most common uses of these materials are in capacitor, power transformer, cables, spark generators, transducers, electric wiring, electric heating, electric distribution lines *etc.* Specifically, in transformer, the dielectric material are used as insulator as well as cooling agents and in capacitor dielectric material is used to store electrical energy.

UNIT SUMMARY

- **Dielectrics**

$$\text{Relative permittivity } k = \epsilon_r = \frac{\epsilon}{\epsilon_0} = \frac{C}{C_0}$$

- **Polarization in Dielectrics**

Polar dielectrics

have permanent dipole moment even in the absence of electric field.

Non Polar dielectrics

don't have permanent dipole moment in absence of electric field

- **Electric dipole moment**

$$\vec{p} = q\vec{d}$$

- **Polarization Vector**

$$\vec{P} = n\vec{p}$$

$$\vec{P} = \lim_{\Delta V \rightarrow 0} \frac{\Delta \vec{p}}{\Delta V} = \frac{d\vec{p}}{dV}$$

- **Electric susceptibility**

$$\chi = \frac{P}{\epsilon_0 E}$$

- **Atomic polarizability**

$$\alpha = \frac{p}{E}$$

- **Behaviour of dielectric substance in an electric field**

$$\vec{E}' = \vec{E} - \vec{E}_p$$

- **Dielectric loss: Behavior of dielectric under alternating field**

$$P_a = 2\pi\omega\epsilon\omega E^2 \tan \delta$$

- **Homogeneity, Linearity and Isotropy**

Homogeneous medium

Physical characteristic remains same at all points in a given direction

Inhomogeneous medium

Physical characteristic changes with the change in positions

Linear medium

Permittivity of a medium does not change with the applied electric field

Non-linear medium

Permittivity of a medium changes with the applied electric field

Isotropic medium

Physical characteristic remains same in every directions

Anisotropic medium

Physical characteristic changes with the change in directions

- **Relationship between flux density, field intensity and polarization**

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

- **Electric Flux density and Electric Flux**

$$\phi = \int_S \vec{D} \cdot d\vec{S}$$

- **Gauss law in Dielectrics**

$$\phi = \int_S \vec{D} \cdot d\vec{S} = q$$

- **Relationship between different physical parameters: Constitutive relations**

$$\frac{\vec{E}'}{\vec{E}} = 1 + \chi = k$$

- **Types of polarization**

Induced (Electronic) polarization: $p = Zex = 4\pi\epsilon_0 r^3 E$

Atomic (Ionic) polarization: $P_i = np_i = n\alpha_i E$

Dipolar (orientational) polarization: $P_o = \frac{n}{3} \frac{p_o^2 E}{kT} = n\alpha_o E$

- **Polarization of monoatomic gas**

$$p = Zex = 4\pi\epsilon_0 r^3 E$$

- **Polarization of polyatomic gas**

$$P = P_e + P_i + P_o \\ = n(\alpha_e + \alpha_i + \alpha_o)E$$

- **Spherical Cavity: Clausius Mossotti Relation**

$$\frac{n\alpha}{3\epsilon_0} = \frac{k-1}{k+2}$$

- **A dielectric sphere placed in a uniform electric field**

$$\vec{P} = \frac{\vec{p}}{V} = \frac{3\pi E_0 \epsilon_2 (\epsilon_1 - \epsilon_2)}{2\epsilon_2 + \epsilon_1} \hat{z}$$

- **Dielectric strength and dielectric breakdown**

Dielectric strength

Maximum electric field intensity that can be tolerated without breakdown

Dielectric breakdown

Sufficiently large electric field pull electrons out of the molecule

- **Applications of dielectrics**

Uses of these materials are in capacitor, power transformer, cables, spark generators, transducers, electric wiring, electric heating, electric distribution lines etc

EXERCISES

Multiple Choice Questions

- 2.1 If \vec{P} represents the polarization vector and $\vec{\nabla} \cdot \vec{P} = \rho$ then ρ is the density of
 (a) free charge (b) bound charge
 (c) free charge at the boundary of the dielectric
 (d) sum of free and bound charges
- 2.2 The dielectric constant for an ideal conductor is
 (a) 0 (b) 1 (c) -1 (d) infinity
- 2.3 The electric dipole moment of a particle (atom or molecule) per unit polarizing electric field is termed as
 (a) polarization (b) polarizability (c) net dipole moment (d) susceptibility
- 2.4 The electronic polarizability (α_e) of an atom is related to its radius (R) as
 (a) $\alpha_e \propto R^3$ (b) $\alpha_e \propto R^2$ (c) $\alpha_e \propto R$ (d) $\alpha_e \propto R^0$
- 2.5 Dielectrics are substances which are
 (a) semiconductor (b) conductor (c) insulator (d) both (b) and (c)
- 2.6 The electrical conductivity of an insulator is zero due to the absence of
 (a) bound electrons (b) free electrons (c) protons (d) neutrons
- 2.7 The ionic polarizability is
 (a) dependent on temperature (b) independent on temperature
 (c) dependent on current density (d) dependent on the concentration of ions
- 2.8 If the applied electric field is \vec{E} , the net electric field \vec{E}' within the dielectric is
 (a) $\vec{E}' = \vec{E} - \epsilon_0 \vec{P}$ (b) $\vec{E}' = \vec{E} + \epsilon_0 \vec{P}$ (c) $\vec{E}' = \vec{E} + \frac{\vec{P}}{\epsilon_0}$ (d) $\vec{E}' = \vec{E} - \frac{\vec{P}}{\epsilon_0}$
- 2.9 The unit of polarizability is
 (a) Nm^2/C^2 (b) debye (c) Fm^2 (d) V/m
- 2.10 For polar dielectrics, orientational polarizability α_o is given by
 (a) $\alpha_o = 3\mu_p kT$ (b) $\alpha_o = \frac{\mu_p}{3kT}$ (c) $\alpha_o = \frac{\mu_p^2}{3kT}$ (d) $\alpha_o = \frac{\mu_p^2}{kT}$
- 2.11 At lower frequency ($\sim 100\text{Hz}$) there exist
 (a) only orientational polarization (b) only ionic polarization
 (c) only electronic polarization (d) all these types of polarization

2.12 Electrical susceptibility χ_e is

(a) $\chi_e = \frac{P}{\epsilon_0 E}$ (b) $\chi_e = \frac{P}{3\epsilon_0 E}$ (c) $\chi_e = \epsilon_0 EP$ (d) $\chi_e = \frac{3\epsilon_0 E}{P}$

2.13 The electronic polarizability of a monoatomic gas atom is

(a) $4\pi\epsilon_0$ (b) $4\pi\epsilon_0 R$ (c) $4\pi\epsilon_0 R^3$ (d) $4\pi\epsilon_0 R^2$

2.14 The SI unit of polarizability is

(a) Fm^{-2} (b) Fm (c) Fm^{-1} (d) Fm^2

2.15 The electric susceptibility of the dielectric medium is given by,

(a) $\chi = \frac{n\alpha}{\epsilon_0}$ (b) $\chi = \frac{n\alpha}{3\epsilon_0}$ (c) $\chi = \frac{\alpha}{n\epsilon_0}$ (d) $\chi = \frac{n\epsilon_0}{\alpha}$

2.16 The relationship between the electric displacement vector and polarization vector is,

(a) $\vec{P} = \epsilon_0 \vec{E} + \vec{D}$ (b) $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ (c) $\vec{D} = \vec{E} + \epsilon_0 \vec{P}$ (d) $\epsilon_0 \vec{D} = \vec{E} + \vec{P}$

2.17 The relationship between electric susceptibility and dielectric constant of the dielectric medium is

(a) $k = 1 - \chi$ (b) $k = 1 + \chi$ (c) $k = \chi - 1$ (d) $\frac{1}{k} = \chi - 1$

2.18 The relationship between electric susceptibility of the dielectric medium and atomic polarizability is,

(a) $k = 1 + \frac{\alpha}{n\epsilon_0}$ (b) $k = 1 + \frac{n}{\alpha\epsilon_0}$ (c) $k = 1 + \frac{n\alpha}{\epsilon_0}$ (d) $k = 1 + \frac{\epsilon_0 n}{\alpha}$

2.19 In vacuum electric susceptibility is

(a) > 1 (b) 0 (c) < 1 (d) negative

2.20 Orientational polarizability is related to orientational dipole moment as

(a) $\alpha_o = \frac{p_0^2}{kT}$ (b) $\alpha_o = \frac{p_0^2}{2kT}$ (c) $\alpha_o = \frac{p_0^2}{3kT}$ (d) $\alpha_o = \frac{3p_0^2}{kT}$

2.21 Relationship between dielectric constant and atomic radius is

(a) $k = 1 + 4\pi na^3$ (b) $k = 1 + 4\pi na^2$ (c) $k = 1 - \frac{4\pi a^3}{n}$ (d) $k = 1 - 4\pi a^3$

2.22 The dielectric constant of the medium is the ratio of the

- (a) net electric field \vec{E}' and the applied electric field \vec{E} within the dielectric
 (b) applied electric field \vec{E} and the net electric field \vec{E}' within the dielectric
 (c) difference of applied electric field \vec{E} and the net electric field \vec{E}' within the dielectric
 (d) none of these

2.23 The electric displacement vector in vacuum is given by,

(a) $\vec{D} = \epsilon \vec{E}$ (b) $\vec{D} = \epsilon_0 \vec{P}$ (c) $\vec{D} = \epsilon_0 \vec{E}$ (d) $\vec{D} = \frac{\vec{E}}{\epsilon_0}$

Answers of Multiple Choice Questions

2.1 (b), 2.2 (d), 2.3 (b), 2.4 (a), 2.5 (c), 2.6 (b), 2.7 (b), 2.8 (d), 2.9 (c), 2.10 (c), 2.11 (a), 2.12 (a), 2.13 (c), 2.14 (d), 2.15 (a), 2.16 (b), 2.17 (b), 2.18 (c), 2.19 (d), 2.20 (c), 2.21 (a), 2.22 (b), 2.23 (c)

Short and Long Answer Type Questions

Category I

- 2.1 Explain the flux density, field intensity and polarization of a dielectric medium.
- 2.2 Rewrite the differential form of Coulomb's law in terms of displacement vector.
- 2.3 Give an idea we get from electronic polarizability. How can you polarize the monoatomic gases?
- 2.4 Explain the term atomic polarizability. Show the interrelation between polarization and atomic polarizability.
- 2.5 Show the relation between electronic polarizability to the volume of the atom.
- 2.6 What is the use of getting idea of dielectric polarizability?
- 2.7 Mention the important characteristics of dielectric materials.
- 2.8 Explain electric displacement vector.
- 2.9 Mention the considerations required to show $k = 1 + \chi_e$, where k and χ_e are the dielectric constant and dielectric susceptibility of the medium respectively.
- 2.10 "An isotropic dielectric medium displacement vector is given by $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$, where \vec{P} is the polarization vector". Define the terms in the expression.
- 2.11 Explain the role of polarization in a dielectric medium.
- 2.12 Discuss the role of the Gauss' law in dielectrics.
- 2.13 Dielectric constant related to electrical susceptibility. Establish the relation.
- 2.14 Explain how the concept of polarization has got so importance.
- 2.15 Differentiate between polar and non-polar dielectric.
- 2.16 Orientational polarizability can be expressed by using a standard relation as given below $\alpha_o = \frac{p_0^2}{3kT}$. Identify the different notations in the expression.
- 2.17 One can express $k = 1 + \frac{n\alpha}{\epsilon_0}$, where the symbols have their usual significance. Mention the terms in the expression.
- 2.18 Find the utility of Clausius-Mossotti equation.

- 2.19 Obtain the differences between polar and non-polar molecules.
- 2.20 Explain the use of electronic polarizability of non-polar dielectric gases.
- 2.21 Discuss critically about the electric breakdown of a dielectric material.
- 2.22 In the expression, $\chi_e = \epsilon_r - 1$ define the different symbols used.
- 2.23 Explain polarization in monatomic gases? Show the relation between electronic polarizability and volume of the atom.
- 2.24 Classify the terms: a) dielectric strength, b) dielectric absorption, and c) dipole moment.

Category II

- 2.25 “When a dielectric is placed in an electric field, the field within the dielectric becomes weaker than the original field”. Why?
- 2.26 “When a dielectric is placed in an electric field, the field within the dielectric becomes weaker/stronger than the original field”. Choose the correct statement and justify.
- 2.27 Explain dielectric loss and find the reasons for this loss.
- 2.28 “The dielectric constant of He at NTP is 1.0000684”. What do you mean by this statement?
- 2.29 Find the roles of flux and flux density in the electric displacement.
- 2.30 Mention the need of artificial dielectric.
- 2.31 Can an isolated atom have a permanent electric dipole moment? Why?
- 2.32 Explain how is the capacity of a capacitor increased when a dielectric is inserted between the plates of the capacitor.

Numerical Problems

- 2.1 An air capacitor is given a charge of 60 μC raising its potential to 200 V. If on inserting a dielectric medium its potential falls to 50 V, what is the dielectric constant of the medium?
[Ans: 4]
- 2.2 Calculate the polarizability of a gas if its susceptibility is 10^{-3} . Given the density and molecular weight of the gas are 2 kg/m^3 and 43.98.
[Ans: $3.23 \times 10^{-37} \text{ Fm}^2$]
- 2.3 If the dielectric constant of a gas at NTP is 1.000456, find its electric susceptibility.
[Ans: 4.56×10^{-5}]
- 2.4 The polarizability of an inert gas is $1.5 \times 10^{-40} \text{ Fm}^2$. Calculate its dielectric constant of the atom if the gas contains $2 \times 10^{25} \text{ atoms/m}^3$.
[Ans: 1.00034]
- 2.5 The dielectric constant of a gas at NTP is 1.006. Calculate the electronic polarisability of the atoms if the gas contains $4 \times 10^{25} \text{ atoms/m}^3$.
[Ans: $1.33 \times 10^{-39} \text{ Fm}^2$]
- 2.6 A parallel plate capacitor of area 0.0005 m^2 and a plate separation of 5 mm has a charge of 0.5 nC it when a voltage of 50 V is applied between the plates. Calculate the dielectric constant of the material used.

- 2.7 The dielectric constant of helium measured at 0°C and 1 atmosphere is 1.0000684. Under these conditions, the gas contains 2.7×10^{25} atoms/ m^3 . Calculate the radius of electron cloud (atomic radius) and the displacement x when the helium atom is subjected to a field of 10^6 V/m.
[Ans: 2.72×10^{-12} m, 0.7×10^{-22} m]
- 2.8 Calculate the induced dipole moment per unit volume of He gas if it is placed in an electric field of 6000 Vcm^{-1} . Given, the atomic polarizability of He is $0.18 \times 10^{-40} \text{ Fm}^2$ and the density of He is 2.6×10^{25} atoms/ m^3 .
[Ans: $2.81 \times 10^{-10} \text{ C/m}^2$]
- 2.9 Assuming that the electric polarizability of an Argon atom is $1.43 \times 10^{-40} \text{ F.m}^2$, find the dielectric constant of solid Argon. Given density of Argon is 1.8 g.cm^{-3} and atomic mass of Argon is 39.95 g.mol^{-1} .
[Ans: 1.000436]
- 2.10 If the diameter of an argon atom is $3.84 \times 10^{-10} \text{ m}$, find its polarizability at STP. Given dielectric constant of argon is = 1.00044.

PRACTICAL

Determination of dielectric constant of a given dielectric material

Apparatus

Digital voltmeter, Glass medium, Gold-plated disc, CRO, Audio oscillator

Theory

The oscillator for the audio part is placed within the instrument properly. If C_{SC} and C_{DC} represent the capacitances of the standard capacitor and dielectric cell respectively and if V_{SC} and V_{DC} are the voltages across SC and DC then

$$\frac{V_{SC}}{I} = \frac{1}{\omega C_{SC}}$$

$$\text{or,} \quad I = \omega C_{SC} V_{SC} \quad (i)$$

The same current passes through the dielectric cell *i.e.*,

$$\frac{V_{DC}}{I} = \frac{1}{\omega C_{DC}}$$

$$\begin{aligned} \text{or,} \quad C_{DC} &= \frac{I}{\omega V_{DC}} \\ &= \frac{\omega C_{SC} V_{SC}}{\omega V_{DC}} \end{aligned}$$

or,
$$C_{DC} = \left(\frac{V_{SC}}{V_{DC}} \right) C_{SC} \quad (ii)$$

Now the capacitance of standard parallel plate capacitor with air as dielectric between the plates is given by,

$$\begin{aligned} C_0 &= \frac{\epsilon_0 A}{d} \\ &= \frac{\epsilon_0 \pi r^2}{d} \\ &= \frac{r^2}{36d} \times 10^{-9} \text{ F} \\ &= \frac{r^2}{36d} \text{ nF} \end{aligned} \quad (iii)$$

Here, r is the radius of the capacitor plate and d is the separation between the plates (in m).

Also we can express the dielectric constant of the material in the following way

$$\epsilon_r = \frac{C_{DC}}{C_0} \quad (iv)$$

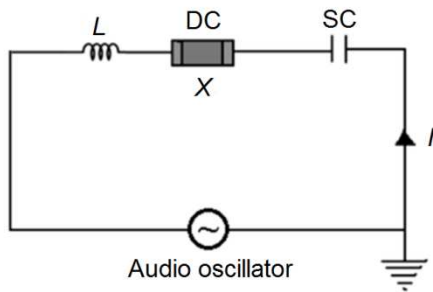


Fig. (i)



Fig. (ii)

Procedure

1. Measure the diameter and hence radius of the plates of the standard capacitor, which will give you radius r .
2. Measure the thickness of the dielectric slab. This will give you d .
3. Put the material whose dielectric constant is to be measured in the dielectric cell.
4. Connect the standard capacitor, dielectric cell and the resistance in series with the oscillator.
5. Turn on the oscillator, set a frequency and an input voltage. Note down the voltage across the standard capacitor and the dielectric cell.
6. Change the input voltage to few other values, note down the voltage across the standard capacitor and the dielectric cell. Draw a curve of V_{SC} vs. V_{DC} .

7. Change the input frequency, repeat step 5.

8. V_{SC} may be plotted against V_{DC} and the slope may be used to calculate C_{DC} .

Experimental Data

Table 1: Thickness of the Dielectric Block

No. of Obs.	Thickness			Mean Thickness d (in cm)	d (in m)
	Linear scale	Circular scale	Total		

Table 2: Diameter of the standard Capacitor

No. of Obs.	Diameter			Mean diameter D (in cm)	$r = D/2$ (meter)
	M.S.R	V.S.R	Total		

Table 3: Voltage across C_{DC} and C_{SC}

Given $C_{SC} = \dots\dots\dots$

Input Frequency (Hz)	Input Voltage (V)	Voltage across		$C_{DC} = (V_{SC}/V_{DC}) C_{SC}$	Mean C_{DC} (pF)
		C_{DC}	C_{SC}		

Calculation

$$\epsilon_r = C_{DC}/C_0 = \dots\dots\dots \text{pF}.$$

Precautions

1. Dielectric samples like glass/PZT are brittle and may be damaged if not properly placed between the discs. So, special care should be taken during the placement of such dielectric samples.
2. Don't put a sample in between the gold plated discs having diameter lesser than the discs.

KNOW MORE

Dielectric material is an electrical insulator that can be polarized by an applied electric field. If a dielectric material is kept in an electric field, electric charges do not flow through the material as they do in an electrical conductor but slightly shift from their mean equilibrium positions causing dielectric polarization.

Activity

A dielectric resonator oscillator is an electronic device to produce resonance of the polarization response for a narrow range of frequencies, generally in the microwave band. It consists of a "puck" of ceramic having a large dielectric constant and a low dissipation factor.

This type of resonator is frequently used to provide a frequency reference in an oscillator circuit. An unshielded dielectric resonator can be used as a dielectric resonator antenna for electronics and communication engineering.

Interesting facts

There are notable interesting areas covered under the dielectric materials including dielectric sphere in an electric field. Details can be found from the following:

1. Dielectric Sphere in an Electric Field
2. DoITPoMS Teaching and Learning Package "Dielectric Materials"
3. Texts on Wikisource:
 - a. *"Dielectric". Encyclopedia Americana. 1920.*
 - b. *"Dielectric". Encyclopædia Britannica (11th ed.). 1911.*

Analogy

The Army Research Laboratory (ARL) conducted research on thin film technology from 2002 to 2004. Barium strontium titanate, a ferroelectric thin film, was used to investigate for the fabrication of radio frequency and microwave components, for voltage-controlled oscillators, tunable filters and phase shifters.

The study was part of an effort to provide the Army with highly-tunable microwave-compatible materials for broadband electric-field tunable devices that operate consistently in extreme temperatures. This work improved tunability of bulk barium strontium titanate, a thin film enabler for electronics components.

ARL in 2004 explored how low concentration of acceptor dopants can modify the properties of ferroelectric materials.

History

In late eighteenth century, scientists developed highly sensitive instruments to detect "electrification." The phenomenon of electrification by contact, known as contact electrification or contact tension was discovered. When two objects were touched together, sometimes they became spontaneously charged one developed a net negative charge and the other an equal and opposite positive charge.

Timelines

1750 - 1774: With famous kite experiment, *Benjamin Franklin* advances knowledge of electricity, inspiring his English friend Joseph Priestley for studying in the same field.

1800 - 1819: *Alessandro Volta* invented the first primitive battery, discovering that electricity can be generated through chemical processes; scientists soon seize on the new tool to invent electric lighting. Further, a profound insight into the relationship between electricity and magnetism goes largely unnoticed.

1840 - 1849: The legendary *Faraday* with his prolific research largely contributed and the telegraph reaches a milestone when a message is sent between Washington, DC, and Baltimore, MD.

1850-1869: The Industrial Revolution is in full force, *Gramme* invented his dynamo and *James Clerk Maxwell* formulated his series of equations on electrodynamics.

1870-1879: The telephone and first practical incandescent light bulb were invented while the word "electron" enters the scientific lexicon.

1880-1889: *Nikola Tesla* and *Thomas Edison* developed the way to transmit electricity and Heinrich Hertz was the first person to broadcast and receive radio waves.

1890-1899: Scientists discovered X-rays and radioactivity, while inventors competed to build the first radio.

1900-1909: *Albert Einstein* published his special theory of relativity and his theory on the quantum nature of light, which he identified as both a particle and a wave. With ever new appliances, electricity begins to transform everyday life.

1910-1929: Understanding of the structure of the atom and of its component particles developed, the phone and radio become common and the modern television was born.

1930-1939: New tools like special microscopes and the cyclotron researched to higher levels, while average citizens enjoyed novel amenities like the FM radio.

1940-1959: Defense-related research leads to the computer, the world entered the atomic age and TV conquers America.

1960-1979: Computers evolved into PCs, researchers discovered one new subatomic particle after another and the space age gave science a new context.

1980-2003: Scientists explored new energy sources, the World Wide Web produced a vast network and nanotechnology started its journey.

Applications (Real Life / Industrial)

Based on different properties like insulation, permittivity, temperature dependency, dielectric strength, and dielectric material are used as various industrial materials for manufacturing electrical devices. Most common and largely uses of these materials are in power transformer, capacitor, cables, spark generators, transducers etc.

Case Study (Environmental / Sustainability / Social / Ethical Issues)

Dielectric materials can be solids, liquids or gases. Solid dielectrics are the most commonly used dielectrics in electrical and electronics engineering. Many solids like porcelain, glass plastics are very good quality insulators. Air, nitrogen and sulfur hexafluoride are the three most commonly used gaseous dielectrics. It is important to note that industrial coatings such as Parylene can give a dielectric barrier between the substrate and its environment. Mineral oil is largely used inside electrical transformers as a fluid dielectric and to assist in cooling.

Dielectric fluids with higher dielectric constants, like electrical grade castor oil, are often used in high voltage capacitors to help prevent corona discharge and increase capacitance. As dielectrics resist the flow of electricity, the surface of a dielectric may retain stranded excess electrical charges which may occur when the dielectric is rubbed, the tribo-electric effect. This can be useful as in a Van de Graaff generator or electrophorus or it can be potentially destructive as in the case of electrostatic discharge.

Inquisitiveness and Curiosity Topics

Specially processed dielectrics, known as electrets may retain excess internal charge or "frozen in" polarization. Electrets have a semi-permanent electric field and are the electrostatic equivalent to magnets. Electrets have many practical applications in the home and industry.

Some dielectrics can generate a potential difference owing to mechanical stress or change of the physical shape when an external voltage is applied across the material which is known as piezoelectricity property. Some ionic crystals and polymer dielectrics show a spontaneous dipole moment that can be reversed by an externally applied electric field. This behavior is termed as the ferroelectric effect. These materials are analogous to the ferromagnetic materials and have very high dielectric constants, making useful for capacitors.

REFERENCES AND SUGGESTED READINGS

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9. https://uomustansiriyah.edu.iq/media/lectures/9/9_2018_12_19!12_27_30_AM.pdf
10. <https://nptel.ac.in/content/storage2/courses/115101005/downloads/lectures-doc/Lecture-22.pdf>

3

Magnetostatics

UNIT SPECIFICS

The following topics are considered in this unit:

- Electric current, drift velocity, current density;
- Continuity equation;
- Lorentz force and force on a small current element placed in a magnetic field;
- Divergence of magnetic field;
- Magnetic scalar potential;
- Magnetic vector potential;
- Application of Biot-Savart's law to solve magnetostatic problems;
- Application of Ampere's circuital law to solve magnetostatic problems.

The practical applications of the topics are discussed for generating further curiosity and creativity as well as improving problem solving capacity.

Besides giving a large number of multiple choice questions as well as questions of short and long answer types marked in two categories following lower and higher order of Bloom's taxonomy, assignments through a number of numerical problems, a list of references and suggested readings are given in the unit so that one can go through them for practice. It is important to note that for getting more information on various topics of interest some QR codes have been provided in different sections which can be scanned for relevant supportive knowledge.

After the related practical, based on the content, there is a "Know More" section. This section has been judiciously designed so that the supplementary information provided in this part becomes beneficial for the users of the book. This section mainly highlights the initial activity, examples of some interesting facts, analogy, history of the development of the subject focusing the salient observations and finding, timelines starting from the development of the concerned topics up to the recent time, applications of the subject matter for our day-to-day real life or/and industrial applications on variety of aspects, case study related to environmental, sustainability, social and ethical issues whichever applicable, and finally inquisitiveness and curiosity topics of the unit.

RATIONALE

This fundamental unit on magnetostatics will help students to get a primary idea about the electric current, drift velocity, current density and then on the basis of this knowledge they will be able to understand the continuity equation and Lorentz force as well as force on a small current element when placed in a magnetic field. Some other basic ideas like divergence of magnetic field, magnetic vector and scalar potential will also be gathered in addition to applications of some primary laws like Biot-Savart's law, Ampere's circuital law useful for solving many magnetostatic problems.

In contrast to electrostatics that essentially deals with static electric charges magnetostatics considers stationary electric currents popularly known as Amperian approach. Oersted in 1820 made a remarkable discovery showing the relationship between electric and magnetic fields wherein it was pointed out that the moving charges are surrounded by a magnetic field. As a matter of fact, always the current carrying conductor is surrounded by a magnetic field. When a steady or time invariant current passes through a conductor, a steady magnetic field is developed surrounding the conductor. The concerned steady current so developed is nothing but a direct current (DC).

PRE-REQUISITES

Mathematics: Vector Calculus (Class XII)

Physics: Electromagnetism (Class XII)

UNIT OUTCOMES

List of outcomes of this unit is as follows:

U3-O1: Describe electric current, drift velocity, current density

U3-O2: Explain continuity equation

U3-O3: Explain Lorentz force and force on a small current element placed in a magnetic field

U3-O4: Describe divergence of magnetic field and magnetic vector and scalar potential

U3-O5: Apply Biot-Savart's law, Ampere's circuital law to solve magnetostatic problems

Unit-3 Outcomes	EXPECTED MAPPING WITH COURSE OUTCOMES (1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation)					
	CO-1	CO-2	CO-3	CO-4	CO-5	CO-6
U3-O1	3	1	-	1	-	-
U3-O2	1	2	-	1	-	1
U3-O3	2	1	-	-	-	-
U3-O4	1	2	-	1	1	1
U3-O5	1	1	-	1	-	1

3.1 INTRODUCTION

Experiments show that the origin of magnetostatic field is steady electric current. Actually the magnetic field produced by a permanent magnet can also be explained representing it by a current distribution and thus the magnetic effect of steady electric current develops the study of *magnetostatics*. Moving charges *i.e.*, the flow of electric current can develop a steady magnetic field as the static charges can develop a steady electric field. The magnitude and direction of the produced magnetic field can be determined with the help of Biot-Savart's law or Ampere's circuital law. In this unit we will discuss some important laws of magnetism and subsequent applications in detail.

3.2 CURRENT AND CURRENT DENSITY

The amount of electric charge flowing per unit time through a particular section of a conductor may be referred as the electric current flowing through that section of the conductor. To formulate it let us consider charges in uniform motion through a particular section of a conductor. Now as the electric current is the rate at which charges can flow through that particular section of the conductor, it may be

defined as,
$$I = \frac{q}{t} \quad (3.1)$$

where q is the amount of charge flowing in time t through that particular section of the conductor. For infinitesimal charge distribution, the current may be expressed as,

$$I = \lim_{\Delta t \rightarrow 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt} \quad (3.2)$$

The SI unit of electric current is ampere (A).

The amount of electric current flowing through unit area of a conductor is called the *current density* and may be expressed as,

$$J = \frac{I}{S} \quad (3.3)$$

where S is the cross-sectional area of the conductor.

For infinitesimal charge distribution, the current density may be expressed as,

$$J = \lim_{\Delta S \rightarrow 0} \frac{\Delta I}{\Delta S} = \frac{dI}{dS} \quad (3.4)$$

This is a vector quantity and its SI unit is A/m^2 .

Using Eq. (3.4) the total current flowing through the entire surface S of the conductor may be expressed as,
$$I = \int dI = \int_S \vec{J} \cdot d\vec{S} \quad (3.5)$$

This represents another way to define current through a particular section of the conductor, if the current density is known to us.

3.3 ELECTRICAL CONDUCTIVITY

Electrical conductivity is the ability of a current carrying conductor to carry electric current. Physically it can be defined as the current density per unit electric field.

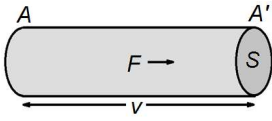


Fig. 3.1: Drift of electron through a conductor

To calculate an expression for it, let us consider a current carrying conductor AA' of cross sectional area S (Fig. 3.1). If due to an applied electric field F , v be the drift velocity of the charge carriers then the number of carriers within the chosen section of the conductor will be, $n_0 v S$ (where n_0 is the number of carriers per unit volume). Thus, the total charge contained in the cylindrical section AA' of the conductor is $n_0 e v S$.

As this charge crosses AA' per second, the electric current flowing through the conductor will be,

$$I = n_0 e v S \quad (3.6)$$

Thus the current density *i.e.*, the current through unit cross sectional area will be,

$$J = \frac{I}{S} = n_0 e v \quad (3.7)$$

Now, the mobility of the carrier *i.e.*, the average drift velocity per unit electric field is

$$\mu = \frac{v}{F}$$

So,
$$v = \mu F \quad (3.8)$$

Thus, using Eq. (3.8) in Eq. (3.7) we get current density

$$J = n_0 e \mu F \quad (3.9)$$

Now, the electrical conductivity is the current density per unit electric field. Thus, from Eq. (3.9) the electrical conductivity can be expressed as,

$$\sigma = \frac{J}{F} = n_0 e \mu \quad (3.10)$$

The SI unit of electrical conductivity is S/m.

3.4 CONTINUITY EQUATION

A continuity equation, alternatively known as the transport equation is actually an equation which describes the transport of some physical quantity. It can be very significantly applied to a conserved quantity. Here, we will establish a relationship between charge density and current density at a point which will effectively express the charge conservation principles. To derive such an equation let us start from the definition of electric current. As we know from our previous discussion using Eq. 3.2 that the current through an arbitrary surface is given by,

$$I = \int_S dI = \int_S \vec{J} \cdot d\vec{S} = \int_S \vec{J} \cdot \hat{n} dS \quad (3.11)$$

where \hat{n} is the unit normal through the arbitrary surface S .

Thus the current entering a volume V bounded by S is given by,

$$I = - \int_S \vec{J} \cdot \hat{n} dS \quad (3.12)$$

The negative sign implies that the current is inward and not outward through S .



Now using divergence theorem in vector calculus we have from Eq. (3.12),

$$I = -\int_S \vec{J} \cdot \hat{n} dS = -\int_V \vec{\nabla} \cdot \vec{J} dV \quad (3.13)$$

This current will increase the total charge contained in the given volume V and the rate of increase of charge in the volume may be represented from Eq. (3.2) as,

$$I = \frac{dq}{dt} = \frac{d}{dt} \int_V \rho dV \quad (3.14)$$

where ρ is the associated volume charge density.

Hence, comparing Eqs. (3.13) and (3.14) we may write,

$$-\int_V \vec{\nabla} \cdot \vec{J} dV = \frac{d}{dt} \int_V \rho dV = \int_V \frac{\partial \rho}{\partial t} dV \quad [\rho \text{ is a function of position coordinates only}]$$

or,

$$\int_V \left(\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} \right) dV = 0$$

The integrand of the above equation must be zero as it holds good for any arbitrary volume element V . Thus,

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \quad (3.15)$$

This is known as the *continuity equation* relating charge density and current density at a point. It signifies that the increase of charge at a point in a given volume of a conductor is due to the inward flow of charge in it and *vice-versa* and thus expresses the principle of conservation of charge in differential form.

3.5 STEADY CURRENT

Steady current means a constant current which is independent of time. When a conductor is carrying a steady current $\frac{\partial \rho}{\partial t} = 0$ and hence the continuity equation becomes,

$$\vec{\nabla} \cdot \vec{J} = 0 \quad (3.16)$$

This equation gives the condition of steady electric current through a conductor.

Using $\vec{J} = \sigma \vec{E}$ from Eq. (3.16) we get,

$$\vec{\nabla} \cdot \vec{E} = 0$$

Now, using $\vec{E} = -\vec{\nabla} \phi$ (where ϕ is the electrostatic potential) we have,

$$\nabla^2 \phi = 0 \quad (3.17)$$

which is the Laplace's equation. So, Laplace's equation must be satisfied to maintain steady electric current through a conductor.

3.6 LORENTZ FORCE

The Lorentz force on a point charge is the sum of electric and magnetic force acting on it when placed in an electromagnetic field. The expression for the force experienced by a charge q moving with velocity \vec{v} in the presence of both the electric field \vec{E} and magnetic field \vec{B} is given by,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad (3.18)$$

where $\vec{F}_e = q\vec{E}$ is the electric Lorentz force and $\vec{F}_m = q\vec{v} \times \vec{B}$ is the magnetic Lorentz force.

Eq. (3.18) effectively gives the total Lorentz force experienced by the charge particle in presence of electric and magnetic fields.

EXAMPLE 3.1

Example 3.1 A $1 \mu\text{C}$ charge is moving with velocity $(2\hat{j} + 3\hat{k}) \text{ m/s}$ in a magnetic field $(2\hat{j} + 3\hat{k}) \text{ Wb/m}^2$. Find the force acting on the charge.

Solution

Given, charge $q = 1 \mu\text{C}$, velocity $\vec{v} = (2\hat{j} + 3\hat{k}) \text{ m/s}$ and

magnetic field $\vec{B} = (2\hat{j} + 3\hat{k}) \text{ Wb/m}^2$.

Thus, force on the charge will be obtained as,

$$\vec{F} = q\vec{v} \times \vec{B} = 1 \times 10^{-6} \times (2\hat{j} + 3\hat{k}) \times (2\hat{j} + 3\hat{k}) = 0.$$

EXAMPLE 3.2

Example 3.2 A test charge having charge 0.4 C is moving with a velocity $4\hat{i} - \hat{j} - 2\hat{k} \text{ m/s}$ through an electric field of intensity $10\hat{i} + 10\hat{k}$ and a magnetic field of induction $2\hat{i} - 6\hat{j} - 6\hat{k}$. Determine the magnitude and direction of the Lorentz force acting on the test charge.

Solution

The expression for the Lorentz force is, $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Here $\vec{E} = 10\hat{i} + 10\hat{k}$, $\vec{v} = 4\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 6\hat{j} - 6\hat{k}$

$$\text{So, } \vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 2 \\ 2 & -6 & -6 \end{vmatrix} = 18\hat{i} + 28\hat{j} - 22\hat{k}$$

$$\begin{aligned} \text{and thus } \vec{F} &= q(\vec{E} + \vec{v} \times \vec{B}) = 0.4 \times (10\hat{i} + 10\hat{k} + 18\hat{i} + 28\hat{j} - 22\hat{k}) \\ &= 11.2\hat{i} + 11.2\hat{j} - 4.8\hat{k}. \end{aligned}$$

So, magnitude of experienced Lorentz force is $|\vec{F}| = \sqrt{11.2^2 + 11.2^2 + 4.8^2} = 16.6 \text{ unit}$.

3.7 BIOT-SAVART LAW

The Biot-Savart law gives a mathematical formulation to calculate the generated magnetic field due to a constant electric current flowing across a conductor. To find a generalized expression of magnetic field at a fixed point in space, let us consider a current carrying conductor carrying current I situated at a distance r from that point P (Fig. 3.2). Now, due to the flow of current, the developed magnetic field (dB) surrounding the point for an elemental length (dl) of the conductor is found to be proportional to the current (I) flowing, proportional to the elemental length (dl) of the conductor, proportional to the *sine* of the angle between the conductor and the point of observation and is inversely proportional to the square of the distance between the conductor and the point of observation. Therefore,

$$\begin{aligned} dB &\propto I \\ &\propto dl \\ &\propto \sin \theta \\ &\propto \frac{1}{r^2} \end{aligned}$$

or,
$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$

or,
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{I} \times d\vec{l}}{r^2} \quad (3.19)$$

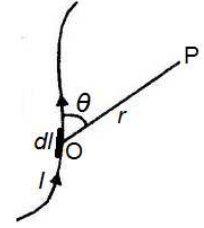


Fig. 3.2: Illustration of Biot-Savart's law

where, μ_0 is the free space permeability and θ is the angle made by the elemental length of the conductor at P.

Now, using the definition of current density we may write,

$$Idl = JSdl = JdV \quad (3.20)$$

Thus,
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Idl}{r^3} \hat{r} = \frac{\mu_0}{4\pi} \frac{JdV}{r^3} \hat{r} \quad (3.21)$$

Hence, according to Biot-Savart's law the total magnetic field at P due to the entire current carrying conductor may be obtained by integrating Eq. (3.21) as,

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Idl}{r^3} \hat{r} = \frac{\mu_0}{4\pi} \int \frac{JdV}{r^3} \hat{r} \quad (3.22)$$

Eq. (3.22) is very useful to determine the magnitude of the magnetic field developed at a fixed distance from a current carrying conductor. In the following section we will discuss a few such applications of the Biot-Savart's law in detail.

3.7.1 Applications

i) Magnetic field due to a long straight current carrying conductor: Biot-Savart's law can be applied to find out the magnetic field at a point due to a long straight current carrying conductor. Let XY be a current carrying conductor, carrying current I . We have to calculate the magnetic field due to it at P at a distance D from it (Fig. 3.3).

Now, the magnetic field at P due to the elemental length dl_0 of the wire can be obtained by using Biot-Savart's law as,

$$dB = \frac{\mu_0}{4\pi} \frac{I dl_0}{l^2} \sin\left(\frac{\pi}{2} + \theta\right) \quad (3.23)$$

From the geometry of Fig. 3.3 we get,

$$\tan \theta = \frac{l_0}{D}$$

and $\sec \theta = \frac{l}{D}$

Thus, $l_0 = D \tan \theta$ and $l = D \sec \theta$ (3.24)

Therefore, $dl_0 = D \sec^2 \theta d\theta$ (3.25)

Hence using Eq. (3.23) we get,

$$dB = \frac{\mu_0}{4\pi} \frac{ID \sec^2 \theta d\theta}{D^2 \sec^2 \theta} \sin\left(\frac{\pi}{2} + \theta\right)$$

or, $dB = \frac{\mu_0 I}{4\pi D} \cos \theta d\theta$ (3.26)

Therefore the total field at P due to the entire current carrying conductor will be,

$$B = \frac{\mu_0 I}{4\pi D} \int_{-\theta_1}^{\theta_2} \cos \theta d\theta = \frac{\mu_0 I}{4\pi D} (\sin \theta_1 + \sin \theta_2) \quad (3.27)$$

For an infinitely long wire, $\theta_1 = \theta_2 = \frac{\pi}{2}$ and thus using Eq. (3.27) the magnetic field will be,

$$B = \frac{\mu_0 I}{2\pi D} \quad (3.28)$$

This gives the magnetic field at a distance D due to an infinitely long straight conductor carrying current I . It is clear from Eq. (3.28) that the magnetic field is directly proportional to I and is inversely proportional to D .

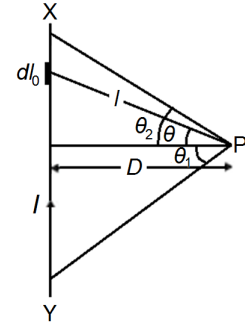


Fig. 3.3: Long straight current carrying conductor

EXAMPLE 3.3

Example 3.3 Find the magnetic field at a distance of 1 cm from a long thin wire, carrying a current of 1 A.

Solution

Here, $I = 1$ A and $D = 1$ cm = 0.01 m.

So, using Biot-Savart's law the magnetic field at the desired point will be

$$B = \frac{\mu_0 I}{2\pi D} = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.01} = 2 \times 10^{-5} \text{ T.}$$

Example 3.4 Find the magnetic field at the centre of a current carrying conductor, in the form of a square loop of side 0.2 m, carrying 1 A current.

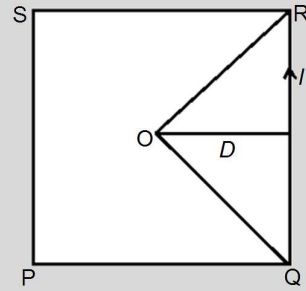
Solution

Using Biot-Savart's law as given by Eq. (3.27) the magnetic field at O is given by,

$$B = \frac{\mu_0 I}{4\pi D} (\sin \theta_1 + \sin \theta_2)$$

Here $\theta_1 = \theta_2 = 45^\circ$, $D = 0.2$ m, $I = 1$ A.

$$\therefore B = \frac{4\pi \times 10^{-7} \times 1}{4\pi \times 0.1} \times \frac{2}{\sqrt{2}} = \sqrt{2} \times 10^{-6} \text{ Wb/m}^2.$$



EXAMPLE 3.4

Example 3.5 A proton moves with a velocity $0.6c$ parallel to a current 1 A at a distance of 10 cm from the straight current carrying conductor. What is the magnetic force on the proton?

Solution

Here, $I = 1$ A and $D = 10$ cm = 0.1 m.

The magnetic field due to the current I can be obtained by using Eq. (3.28) as

$$B = \frac{\mu_0 I}{2\pi D} = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.1} = 2 \times 10^{-6} \text{ Wb/m}^2.$$

The direction of B can be given by Maxwell's screw rule.

The magnetic force acting on the proton is

$$F = qvB \quad [\text{as } \vec{v} \text{ is perpendicular to } \vec{B}]$$

Here $v = 0.6c = 0.6 \times 3 \times 10^8$ m/s.

So magnitude of the force is

$$F = (1.6 \times 10^{-19}) \times (0.6 \times 3 \times 10^8) \times (2 \times 10^{-6}) = 5.78 \text{ N}.$$

and the direction of the force is towards the wire *i.e.* attractive in nature.

EXAMPLE 3.5

Example 3.6 Two parallel wires carry equal current of 10 A along with the same direction and are separated by a distance of 2.0 cm. Find the magnetic field at a point which is 2 cm away from any of these wires.

Solution

As the directions of the current through the two wires are same so we have to add magnetic fields created by two wires.

Using the formula $B = \frac{\mu_0 I}{2\pi D}$ we have the resultant magnetic field at a distance

EXAMPLE 3.6

EXAMPLE 3.6

$$\begin{aligned}
 \text{2 cm from a wire is } B &= \frac{\mu_0 I}{2\pi \times 0.02} + \frac{\mu_0 I}{2\pi \times 0.04} \\
 &= \frac{\mu_0 I}{2\pi} \left[\frac{1}{0.02} + \frac{1}{0.04} \right] \\
 &= \frac{3\mu_0 I}{8\pi} \times 10^2 = \frac{3 \times 4\pi \times 10^{-7} \times 10}{8\pi} \times 10^2 = 1.5 \times 10^{-4} \text{ T.}
 \end{aligned}$$

ii) Magnetic field on the axis of a circular conductor carrying current: To calculate the magnetic field due a circular conductor of radius a in the form of a loop and carrying a current I , let us consider the elemental length dl_0 of the loop as shown in Fig. 3.4. Now, the magnetic field at point P due to this elemental length dl_0 of the loop can be obtained by using Biot-Savart's law as,

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l}_0 \times \vec{r}}{r^3} = \frac{\mu_0 I}{4\pi} \frac{dl_0}{l^2} \quad (3.29)$$

$$[\text{as the angle between } dl_0 \text{ and } l \text{ is } \frac{\pi}{2}]$$

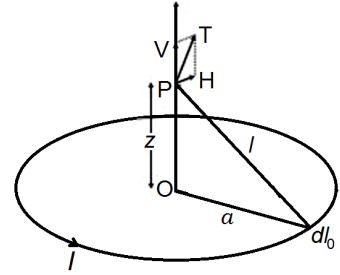


Fig. 3.4: Circular current carrying conductor

This will be acting along PT with the vertical component PV along the axis of the loop and the horizontal component PH perpendicular to the axis of the loop (Fig. 3.4). Due to symmetry PH components will cancel each other and only the components along the axis will contribute. Therefore, the net magnetic field at P due to dl_0 will be,

$$dB \sin \angle HPT = dB \frac{a}{l} = \frac{\mu_0 I a}{4\pi l^3} dl_0 \quad (3.30)$$

Thus the magnetic field due to the entire loop is,

$$B = \frac{\mu_0 I a}{4\pi l^3} \int dl_0 = \frac{\mu_0 I a}{4\pi l^3} \cdot 2\pi a = \frac{\mu_0 I a^2}{2l^3} = \frac{\mu_0 I a^2}{2(z^2 + a^2)^{3/2}} \quad (3.31)$$

For a loop having n turns Eq. (3.31) can be generalized as,

$$B = \frac{\mu_0 n I a^2}{2(z^2 + a^2)^{3/2}} \quad (3.32)$$

Now, the magnetic field at the centre of the loop can be obtained by putting $z = 0$ in Eq. (3.32) and is given by,

$$B_0 = \frac{\mu_0 n I}{2a} \quad (3.33)$$

So, along the axis of the loop the magnetic field increases as the current flowing in the loop increases and it decreases as the radius of the loop increases.

Example 3.7 Calculate the magnetic field at the center of a circular current carrying loop of radius 10 cm. Given that the current flowing through the loop is 2 A.

Solution

Using Biot-Savart's law the magnetic field at any axial point due to a circular current carrying conductor can be written as,

$$B = \frac{\mu_0}{4\pi} \frac{2\pi nr^2 I}{(r^2 + x^2)^{3/2}} \text{ Wb/m}^2$$

At the centre of the coil, $x = 0$.

So,
$$B = \frac{\mu_0}{4\pi} \frac{2\pi ni}{r} = \frac{\mu_0 n I}{2r} \text{ Wb/m}^2.$$

Here $n = 1$, $I = 2$ A and $r = 10$ cm = 0.01 m.

So, using the given data we have,

$$B = \frac{4\pi \times 10^{-7} \times 1 \times 2}{2 \times 0.01} = 4\pi \times 10^{-7} \text{ Wb/m}^2.$$

EXAMPLE 3.7

Example 3.8 A current carrying coil having 50 turns carries a current of 0.1 A. If the radius of the coil is 4 cm, calculate the magnetic field due to it at a distance of 5 cm on the axis.

Solution

For a loop having n turns we have,

$$B = \frac{\mu_0 n I a^2}{2(z^2 + a^2)^{3/2}}$$

Here, $n = 50$, $I = 0.1$ A, $a = 4$ cm and $z = 5$ cm.

$$\therefore B = \frac{\mu_0 n I a^2}{2(z^2 + a^2)^{3/2}} = \frac{4\pi \times 10^{-7} \times 50 \times 0.1 \times (0.04)^2}{2 \times [(0.05)^2 + (0.04)^2]^{3/2}} = 3.14 \text{ T}.$$

EXAMPLE 3.8

Example 3.9 A steel rod of radius 1 cm is bent to form a circular loop and wound with a coil having 200 turns. If the mean radius of the loop is 10 cm, calculate the amount of current that will produce a flux of 0.5 mWb in the core. Given that for steel $\mu_r = 1000$.

Solution

When the steel rod is bent to form a circular loop it will form a toroid. Now, using Biot-Savart's law the magnetic field at the centre of the core of the toroid is,

$$B = \frac{\mu_r n I}{2\pi r}$$

EXAMPLE 3.9

EXAMPLE 3.9

Here $\mu_r = 1000$, $r = 10 \times 10^{-2} \text{ m}$ and $n = 200$.

Again the flux produced at the core of the solenoid is,

$$\pi R^2 B = 0.5 \text{ mWb} = 0.5 \times 10^{-3} \text{ Wb}$$

Given, the radius of the rod $R = 1 \text{ cm} = 1 \times 10^{-2} \text{ m}$.

$$\therefore 0.5 \times 10^{-3} = \frac{\mu_r n I}{2r} R^2 = \frac{1000 \times 200 \times I}{2 \times 10 \times 10^{-2}} \times (1 \times 10^{-2})^2$$

$$\text{or, } I = \frac{2 \times 0.5 \times 10}{1000 \times 200 \times 10^{-2}} = 0.5 \mu\text{A}$$

which gives the amount of current that will produce a flux of 0.5 mWb in the core.

iii) Axial magnetic field of a solenoid: Let, L is the length of a solenoid having n number of turns. Also let a be the radius of the solenoid which is carrying the current I [Fig. 3.5].

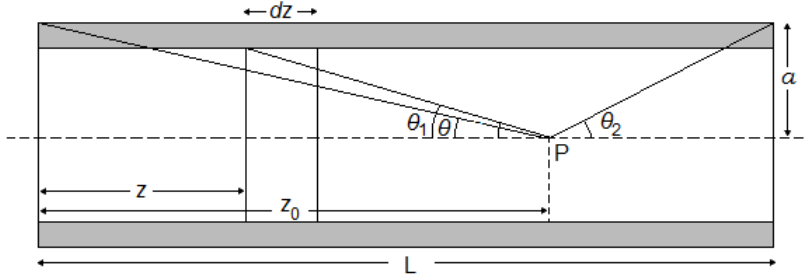


Fig. 3.5: Long solenoid

To calculate the magnetic field at any point P on its axis, let us consider an elemental segment of thickness dz . Number of turns in this segment will be $\left(\frac{n}{L}\right) dz$.

Therefore using Eq. (3.32) we may write the expression for magnetic field at point P due to the current flowing through the elemental segment of thickness dz of the solenoid as,

$$dB = \frac{\mu_0 n I a^2 dz}{2L \left[(z_0 - z)^2 + a^2 \right]^{3/2}} \quad (3.34)$$

Now from Fig. 3.5 we have, $\tan \theta = \frac{a}{z_0 - z}$

$$\text{or, } dz = a \sec^2 \theta d\theta \quad (3.35)$$

$$\text{Again } \sin \theta = \frac{a}{\left[(z_0 - z)^2 + a^2 \right]^{1/2}} \quad (3.36)$$

Thus using Eqs. (3.35) and (3.36) in Eq. (3.34) we get,

$$dB = \frac{\mu_0 n I \sin \theta d\theta}{2L} \quad (3.37)$$

Therefore total magnetic field at P due to the whole solenoid will be,

$$\begin{aligned} B &= \frac{\mu_0 n I}{2L} \int_{\theta_1}^{\pi-\theta_2} \sin \theta d\theta \\ &= \frac{\mu_0 n I}{2L} (\cos \theta_1 + \cos \theta_2) \end{aligned} \quad (3.38)$$

If P is the midpoint of the solenoid then $\theta_1 = \theta_2 = \theta$.

$$\text{Therefore, } B_{mid} = \frac{\mu_0 n I}{L} \cos \theta \quad (3.39)$$

For an infinitely large solenoid $\theta \rightarrow 0$ and $\cos \theta \rightarrow 1$

$$\text{Therefore, } B_{inf} = \frac{\mu_0 n I}{L} \quad (3.40)$$

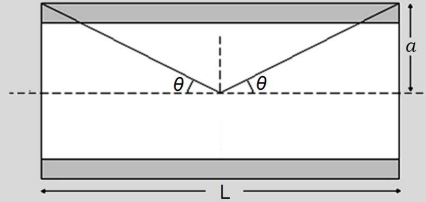
Example 3.10 A long solenoid of length 0.6 m and diameter of 0.4 m is carrying 5 mA of current. If the number of turns is 1000, calculate the magnetic field at the middle of the solenoid.

Solution

$$\text{We have } B = \frac{\mu_0 n I}{2L} (\cos \theta_1 + \cos \theta_2)$$

At midpoint of the solenoid $\theta_1 = \theta_2 = \theta$.

$$\begin{aligned} \text{Therefore, } B_{mid} &= \frac{\mu_0 n I}{L} \cos \theta \\ &= \frac{\mu_0 n I}{L} \frac{L/2}{\sqrt{a^2 + (L/2)^2}} \\ &= \frac{\mu_0 n I}{2} \frac{1}{\sqrt{a^2 + (L/2)^2}} \end{aligned}$$



Here length of the solenoid $L = 0.6$ m and diameter of the solenoid $2a = 0.4$ m.

$$\begin{aligned} B_{mid} &= \frac{4\pi \times 10^{-7} \times \frac{1000}{0.6} \times 5 \times 10^{-3}}{2} \frac{1}{\sqrt{(0.2)^2 + (0.6/2)^2}} \\ &= 1.45 \times 10^{-5} \text{ Wb/m}^2. \end{aligned}$$

Example 3.11 A long solenoid of 40 cm length has 300 turns. If the solenoid carries a 3.5 A current, find the magnetic field at one end of the solenoid.

Solution

Here, $n = 300/0.4 = 750$ turns/m and $I = 3.5$ A.

So, magnetic field at any point on the axis of the solenoid is

$$B = \mu_0 n I = 4\pi \times 10^{-7} \times 750 \times 3.5$$

$$= 3.3 \times 10^{-3} \text{ T.}$$

Thus, magnetic field at one end of the solenoid will be

$$B = \frac{\mu_0 n I}{2} = \frac{4\pi \times 10^{-7} \times 750 \times 3.5}{2}$$

$$= 1.65 \times 10^{-3} \text{ T.}$$

iv) Magnetic field due to a hexagonal current carrying conductor at its center: Let us consider a hexagonal current carrying conductor having side length x , carrying current I . Now, the magnetic field at its center O due to the side PQ can be obtained by using Eq. (3.27) as,

$$B_1 = \frac{\mu_0 I}{4\pi D} (\sin \theta + \sin \theta) = \frac{\mu_0 I}{2\pi D} \sin \theta \quad (3.41)$$

So, total magnetic field at O due to the whole conductor will be,

$$B = 6B_1 = \frac{3\mu_0 I}{\pi D} \sin \theta \quad (3.42)$$

Again from Fig. 3.6,

$$\tan \theta = \frac{x}{2D}$$

or, $\frac{1}{\sqrt{3}} = \frac{x}{2D} \quad (\text{here, } \theta = 30^\circ)$

or, $D = \frac{\sqrt{3}x}{2}$

Thus, $B = \frac{6\mu_0 I}{\pi \sqrt{3}x} \frac{1}{2}$

$$= \frac{\sqrt{3}\mu_0 I}{\pi x} \quad (3.43)$$

which gives the desired magnetic field at the center of the hexagonal loop in terms of the current flowing in the loop and its side length.

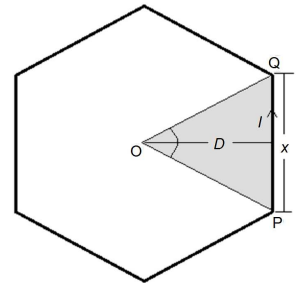


Fig. 3.6: Hexagonal loop



Example 3.12 A conducting wire in the shape of an equilateral triangle of side a carries a current I . Calculate the magnetic field at its centroid.

Solution

In the figure if O be the centroid of the equilateral triangle with side a then

$$ON = r = \frac{a}{2\sqrt{3}}$$

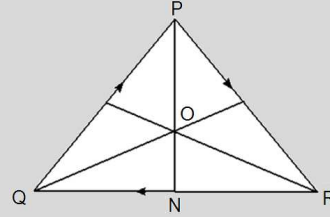
Now, the magnetic field at the point O due to the segment QR will be,

$$\frac{\mu_0 I}{4\pi r} [\sin \theta]_{-\pi/3}^{\pi/3}$$

As the lengths are equal so magnetic field at the point O due to three current carrying wires (PQ , QR and PR) are

$$B = 3 \frac{\mu_0 I}{4\pi r} [\sin \theta]_{-\pi/3}^{\pi/3} = 3 \frac{\mu_0 I}{4\pi \frac{a}{2\sqrt{3}}} 2 \sin \frac{\pi}{3} = 3 \frac{\mu_0 I}{4\pi \frac{a}{2\sqrt{3}}} \times \sqrt{3} = \frac{9\mu_0 I}{2\pi a}.$$

So, the required magnetic field at the point O is $B = \frac{9\mu_0 I}{2\pi a}$.



EXAMPLE 3.12

3.8 CURRENT CARRYING CONDUCTOR

When a current carrying conductor is kept in a magnetic field, it experiences a force. Ampere first demonstrated this effect of experiencing a force by a current carrying wire while placed in a magnetic field. When a current is established in the conductor, it gets displaced. In this way, one can verify the existence of a force on the conductor.

The direction of this force developed can be determined by using the Fleming's left-hand rule. When the magnetic field acts along the perpendicular direction of the current flowing, the force acting on the conductor should be perpendicular to both the magnetic field and the current. Let us examine it critically.

To determine the expression for such a force, we consider a current carrying conductor AB carrying current I [Fig. 3.7] anywhere in space. The magnetic field at P due to an elemental length dl of the conductor will be given by using Biot-Savart's law as,

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$

So, the force exerted on a magnetic pole strength m situated at P will be,

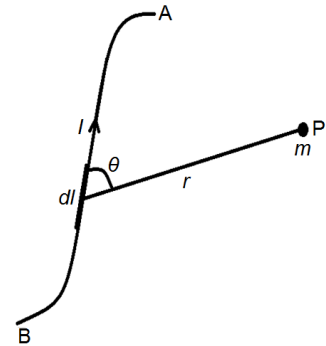


Fig. 3.7: Force on a current carrying conductor

$$dF = mdB = \frac{\mu_0}{4\pi} \frac{mIdl \sin \theta}{r^2} \quad (3.44)$$

The magnetic pole will also exert the same force on dl . Now, the magnetic field at a distance r due to a magnetic pole of pole strength m is,

$$B = \frac{\mu_0}{4\pi} \frac{m}{r^2} \quad (3.45)$$

Therefore, $dF = BIdl \sin \theta$ (3.46) (a)

or, in vector form we may write

$$d\vec{F} = Id\vec{l} \times \vec{B} \quad (3.46) (b)$$

So, the force acting on the conductor is found to be perpendicular to both the magnetic field and the current flowing in the conductor.

3.9 MOVING CHARGE IN A MAGNETIC FIELD

When some magnetic force is applied on a current carrying conductor, it effectively forces a charged particle to move along a circular path or along a spiral. We know the force on a current carrying conductor of length dl carrying current I in a magnetic field B is,

$$dF = BIdl \sin \theta$$

Now, $I = neAv$

where, A is the area of cross section of the conductor, e is the charge of a carrier, v is the drift velocity of carrier and n is the number of charge carrier present per unit volume of the conductor.

So, $dF = B(neAv)dl \sin \theta = e(nAdl)nB \sin \theta = eNvB \sin \theta$ (3.47)

where $N = nAdl$ = total no. of charge inside the conductor.

So, the force experienced by a moving charge carrier in a magnetic field will be

$$\begin{aligned} F_m &= \frac{eNvB \sin \theta}{N} \\ &= evB \sin \theta \end{aligned} \quad (3.48)$$

or, in vector form it can be expressed as

$$\vec{F}_m = e\vec{v} \times \vec{B} \quad (3.49)$$

We call it as the *magnetic Lorentz force* and the total Lorentz force experienced by a charge carrier in an electromagnetic field can be obtained by adding up the electric force or the *electric Lorentz force* with the magnetic Lorentz force and is given by

$$\begin{aligned} \vec{F} &= \vec{F}_e + \vec{F}_m \\ &= e(\vec{E} + \vec{v} \times \vec{B}) \end{aligned} \quad (3.50)$$

where $\vec{F}_e = e\vec{E}$ is the electric Lorentz force.



3.10 AMPERE'S CIRCUITAL LAW

In magnetostatics, Ampere's circuital law relates the integrated magnetic field around a closed path to the electric current passing through that path. It states that the line integration of the tangential component of a given magnetic field around a closed path is equal to the enclosed current by the path multiplied by the permeability of free space. Mathematically it can be expressed as,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \quad (3.51)$$

It is analogous to the Gauss's law in electrostatics. The law can be derived easily by using Biot-Savart's law.

To do that let us consider a long straight current carrying conductor carrying a current I through it. The magnetic field due to it at a distance r from it is given by using Biot-Savart's law as,

$$B = \frac{\mu_0 I}{2\pi r}$$

showing that it is constant at all radial points. Hence the line integral of around the closed circular loop C with radius r is,

$$\begin{aligned} \oint_C \vec{B} \cdot d\vec{l} &= \oint_C B dl \\ &= B \oint_C dl = B \cdot 2\pi r = \frac{\mu_0 I}{2\pi r} \cdot 2\pi r \quad [\because B = \text{constant, at all radial points}] \end{aligned}$$

or,
$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

This is Ampere's circuital law in integral form.

3.10.1 Ampere's Circuital Law in Differential Form

The integral form of Ampere's circuital law as given in Eq. (3.51) can be redefined in a differential manner. As, the total current flowing through a surface of a conductor may be expressed as,

$$I = \int_S \vec{J} \cdot d\vec{S}$$

thus the Ampere's circuital law may be expressed as,

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \int_S \vec{J} \cdot d\vec{S} \quad (3.52)$$

Again using Stoke's theorem in vector calculus when applied to magnetic field we can write,

$$\oint_C \vec{B} \cdot d\vec{l} = \int_S \vec{\nabla} \times \vec{B} \cdot d\vec{S} \quad (3.53)$$

Thus comparing Eq. (3.52) and Eq. (3.53) we get,

$$\int_S \vec{\nabla} \times \vec{B} \cdot d\vec{S} = \int_S \mu_0 \vec{J} \cdot d\vec{S}$$

As the equation is true for any arbitrary surface, so equating the integrands on both side we get,

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad (3.54)$$

This is the differential form of Ampere's circuital law. From this equation it is clear that the magnetic field is not a conservative one.

3.10.2 Applications of Ampere's Law

Ampere's circuital law is also very useful to determine the magnitude of the magnetic field developed at a fixed distance from a current carrying conductor. In this section we will consider few such examples.

i) Magnetic field due to a long straight current carrying conductor: Consider an infinitely long straight wire carrying current I . If a be the radius of the wire then the current density will be,

$$J = \frac{I}{\pi a^2} \quad (3.55)$$

Now the magnetic field B is a function of distance r only. Applying Gauss's law to the pillbox shown in Fig. 3.8 we have,

$$B_r \cdot 2\pi r l = 0$$

Now using Ampere's circuital law to the loop $PQRS$ in Fig. 3.8 we have,

$$\oint \vec{B} \cdot d\vec{l} = [B_z(r_2) - B_z(r_1)]z = 0$$

or, $B_z(r_2) = B_z(r_1) \quad (3.56)$

Thus B_z is a constant and let us take this to be equal to zero. We draw a circle of radius r , concentric with the wire in a plane perpendicular to it in the magnetic field. This will be constant at every point and tangential to it. Therefore,

$$\oint \vec{B} \cdot d\vec{l} = \int_0^{2\pi} B_r d\theta = 2\pi r B = \mu_0 I_e \quad (3.57)$$

where, I_e is the enclosed current.

If $r > a$, $I_e = I$. Thus from Eq. (3.57) we have

$$B = \frac{\mu_0 I}{2\pi r} \quad [\text{for } r \geq a] \quad (3.58)$$

If $r < a$ then $I_e = \pi r^2 J = \frac{r^2}{a^2} I$

Thus from Eq. (3.57) we have,

$$B = \frac{\mu_0 I r}{2\pi a^2} \quad [\text{for } r \leq a] \quad (3.59)$$

Variation of magnetic field due to a long straight current carrying conductor with distances along its axis is shown in Fig. 3.9.

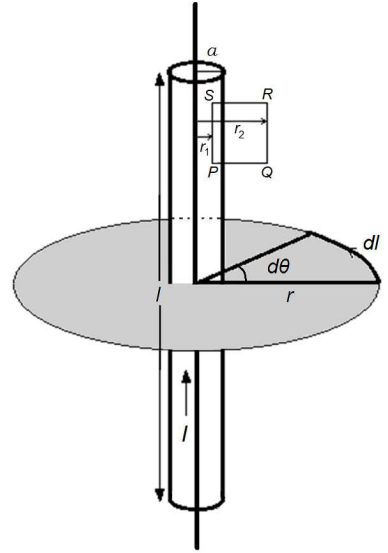


Fig. 3.8: Amperian loop: Long straight conductor

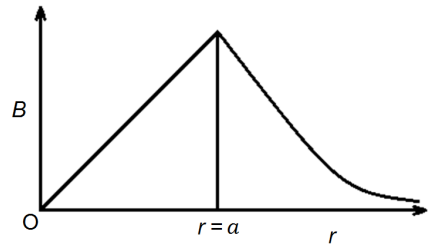


Fig. 3.9: Variation of B due to long straight current carrying conductor

Example 3.13 Calculate the magnetic field just outside and inside a hollow cylinder of radius 4 cm if it carries 50 A current.

Solution

(a) For a point outside cylinder, according to Ampere's circuital law we can write

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

or,
$$\oint_C \vec{H} \cdot d\vec{l} = I$$

or,
$$H \cdot 2\pi r = I$$

or,
$$H = \frac{I}{2\pi r} = \frac{50}{2\pi \times 0.04} = 198.04 \text{ A/m.}$$

So, the magnetic field of induction is

$$\begin{aligned} B &= \mu_0 H = 4\pi \times 10^{-7} \times \frac{50}{2\pi \times 0.04} \\ &= 2.5 \times 10^{-4} \text{ T.} \end{aligned}$$

(b) Inside the cylinder, $B = 0$, since the current exists only on the surface.

EXAMPLE 3.13

ii) Magnetic field between two infinitely long current carrying conductors: Let I_1 and I_2 be the currents in the two wires X and Y separated by a distance d in free space (Fig. 3.10). Now, the magnetic field produced by the current I_2 in Y at any point in X is given by,

$$B = \frac{\mu_0 I_2}{2\pi d}$$

Therefore, the force per unit length exerted on the wire will be,

$$\vec{F} = I_1 \hat{z} \times \vec{B}$$

or,
$$\begin{aligned} \vec{F} &= \frac{\mu_0 I_1 I_2}{2\pi d} \hat{z} \times \hat{x} \\ &= -\frac{\mu_0 I_1 I_2}{2\pi d} \hat{y} \end{aligned} \quad (3.60)$$

where \hat{z} is a unit vector in the direction of I_1 and \hat{x} is a unit vector in the direction of \vec{B} (Fig. 3.10). The force \vec{F} is directed towards Y, if I_1 and I_2 flow in the same direction and it is directed away from Y if they are in the opposite direction.

Thus, the wires which carry parallel current will attract each other while the wires which carry anti-parallel current will repel each other. The magnitude of force per unit length is given by,

$$B = \frac{\mu_0 nI}{2\pi r} \quad (3.61)$$

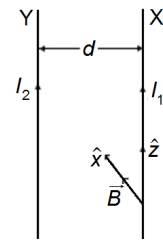


Fig. 3.10: Two long current carrying conductors separated by a distance

EXAMPLE 3.14

Example 3.14 Two straight long parallel wires carrying 1 A current each, are separated by a distance of 2 cm. Calculate the force experienced by either of them.

Solution

Here $I_1 = I_2 = 1 \text{ A}$ and $r = 2 \text{ cm} = 0.02 \text{ m}$.

Hence the magnitude of the force experienced by either of the wires will be,

$$F = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{4\pi \times 10^{-7} \times 1 \times 1}{2\pi \times 0.02} = \frac{2 \times 10^{-7}}{0.02} = 10^{-5} \text{ N}.$$

EXAMPLE 3.15

Example 3.15 A long horizontal wire carries a current of 100 A. Another wire carrying 20 A current is parallel to it and weighs 0.03 N/m. If the second wire is to be supported on the first by magnetic repulsion, calculate their intermediate distance.

Solution

Let, the intermediate distance between the wires carrying currents I_1 and I_2 is r .

According to the problem,

$$\frac{\mu_0 I_1 I_2}{2\pi r} = 0.03$$

or,

$$\frac{4\pi \times 10^{-7} \times 100 \times 20}{2\pi r} = 0.03$$

or,

$$r = \frac{4 \times 10^{-7} \times 10^3}{0.03} = 1.33 \times 10^{-2} \text{ m}.$$

which is the required intermediate distance.

iii) Magnetic field for a long coaxial cable: Consider a coaxial cable containing a central cylindrical conductor of radius a and a coaxial outer cylindrical conductor of radius b carrying same current I in opposite direction. Due to symmetry the magnetic field at a distance r ($a < r < b$), should be a function of r only. Using Ampere's circuital law for the closed loop C of radius r (Fig. 3.11) we have,

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

or,

$$B \cdot 2\pi r = \mu_0 I$$

or,

$$B = \frac{\mu_0 I}{2\pi r} \quad [\text{where } a < r < b] \quad (3.62)$$

When $r > b$ then, from Ampere's circuital law we have,

$$B \cdot 2\pi r = \mu_0 (I - I) = 0$$

or,

$$B = 0$$

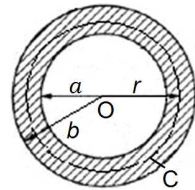


Fig. 3.11: Coaxial cable

(3.63)

iv) Magnetic field due to an infinitely long solenoid: Consider a solenoid of length L . Also let us consider a closed loop $abcd$ in it, as shown in Fig. 3.12. Ampere's circuital law when applied to this loop we get,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \times 0 = 0$$

Thus, $\vec{B} = 0$ as $d\vec{l} \neq 0$.

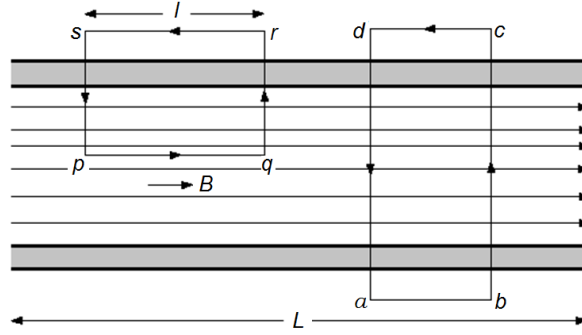


Fig. 3.12: Amperian loop: Long solenoid

This means that the magnetic field outside the solenoid is zero. To find the magnetic field inside the solenoid let us consider another closed loop $pqrs$ in it (Fig. 3.12). Ampere's circuital law when applied to this loop we get,

$$\oint \vec{B} \cdot d\vec{l} = \int_{pq} \vec{B} \cdot d\vec{l} + \int_{qr} \vec{B} \cdot d\vec{l} + \int_{rs} \vec{B} \cdot d\vec{l} + \int_{sp} \vec{B} \cdot d\vec{l} = \mu_0 \times \text{current enclosed by } pqrs \quad (3.64)$$

For the path pq , \vec{B} and $d\vec{l}$ are in the same direction. Thus,

$$\int_{pq} \vec{B} \cdot d\vec{l} = \int_{pq} B dl = Bl$$

For the paths qr and sp , $d\vec{l}$ is in the opposite direction. Thus,

$$\int_{qr} \vec{B} \cdot d\vec{l} = - \int_{sp} \vec{B} \cdot d\vec{l}$$

or,

$$\int_{qr} \vec{B} \cdot d\vec{l} + \int_{sp} \vec{B} \cdot d\vec{l} = 0$$

For the path rs (since it is outside the solenoid),

$$\int_{rs} \vec{B} \cdot d\vec{l} = 0$$

So, from Eq. (3.64) we get after using Ampere's circuital law,

$$\oint \vec{B} \cdot d\vec{l} = \int_{pq} \vec{B} \cdot d\vec{l} = Bl = \mu_0 nIl, \text{ where } n \text{ is the no. of turns/unit length}$$

or,

$$B = \mu_0 nI \quad (3.65)$$

EXAMPLE 3.16

Example 3.16 A long solenoid of length 2 m and number of turns 3000 is carrying a current of 1 A. Find the magnetic field at any point along its axis.

Solution

Here, number of turns per unit length is $n = 3000/2 = 1500$ turns/m and $I = 1$ A.

So, magnetic induction at any point along the axis of the solenoid is

$$B = \mu_0 n I = 4\pi \times 10^{-7} \times 1500 \times 1 = 1.9 \times 10^{-3} \text{ T.}$$

EXAMPLE 3.17

Example 3.17 A long solenoid of length 1.5 m and number of turns 3000 is carrying a current of 4 A. Find the magnetic field at any point i) inside it, ii) at the end of the solenoid and iii) outside it.

Solution

Here, number of turns per unit length is $n = 3000/1.5 = 2000$ turns/m and $I = 4$ A.

i) So, magnetic induction at any point inside the solenoid is

$$B_{in} = \mu_0 n I = 4\pi \times 10^{-7} \times 2000 \times 4 = 1.005 \times 10^{-2} \text{ T.}$$

ii) Magnetic induction at the end of the solenoid is

$$B_{end} = \frac{\mu_0 n I}{2} = \frac{1.005 \times 10^{-2}}{2} \text{ T} = 5.025 \times 10^{-3} \text{ T.}$$

iii) Magnetic induction at any point outside the solenoid is $B_{out} = 0$ [since the current exists only on the surface].

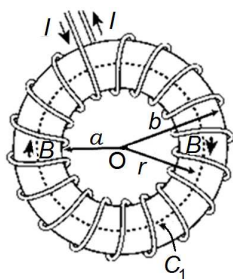


Fig. 3.13: Toroid

v) Magnetic field due to a toroid: Consider a closely spaced winding of a wire wound around a ring. It is termed as a toroid and can be regarded as an endless solenoid. Let I amount of current is flowing through the windings along C_1 [Fig. 3.13] and N be the total number of turns in the toroid. Then using Ampere's circuital law for the path C_1 we have,

$$\oint_{C_1} \vec{B} \cdot d\vec{l} = \mu_0 N I$$

or,

$$B \cdot 2\pi r = \mu_0 N I$$

or,

$$B = \frac{\mu_0 N I}{2\pi r} = \mu_0 n I \quad (3.66)$$

where $n = \frac{N}{2\pi r}$ is the number of turns per unit length of the toroid.

3.11 CURL OF MAGNETIC FIELD

The simplest statement which can relate the magnetic field and moving charges can be addressed by taking the curl of a magnetic field at any point. It practically gives the current density at that point. To justify this let us begin with the Ampere's circuital law, from which we have



$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I = \mu_0 \iint_S \vec{J} \cdot d\vec{S} \quad (3.67)$$

Now, using Stokes' theorem we have,

$$\oint_C \vec{B} \cdot d\vec{l} = \iint_S \vec{\nabla} \times \vec{B} \cdot d\vec{S} \quad (3.68)$$

Thus,
$$\iint_S \vec{\nabla} \times \vec{B} \cdot d\vec{S} = \mu_0 \iint_S \vec{J} \cdot d\vec{S}$$

or,
$$\iint_S (\vec{\nabla} \times \vec{B} - \mu_0 \vec{J}) \cdot d\vec{S}$$

Here the surface S is arbitrary. Thus,

$$\vec{\nabla} \times \vec{B} - \mu_0 \vec{J} = 0$$

or,
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad (3.69)$$

From which we can define the current density at a point in a region of space.

If in a region of space there is no current density *i.e.*, if $\vec{J} = 0$ then,

$$\vec{\nabla} \times \vec{B} = 0 \quad (3.70)$$

3.12 GAUSS'S LAW IN MAGNETOSTATICS

Gauss's law for magnetostatics is one of the four Maxwell's equations (will be discussed in detail in unit 7) which states that the magnetic field has a zero divergence. To establish this we will apply Biot-Savart's law to a current carrying conductor. Let the conductor carrying current I is placed at a distance r from a point P (Fig. 3.14). Due to the flow of current the magnetic field developed surrounding the point will be,

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \vec{r}}{r^3}$$

Thus the magnetic field due to the entire path is,

$$\vec{B} = \frac{\mu_0}{4\pi} \oint \frac{Id\vec{l} \times \vec{r}}{r^3}$$

Thus,
$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \vec{\nabla} \cdot \oint \frac{Id\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0 I}{4\pi} \oint \vec{\nabla} \cdot \frac{d\vec{l} \times \vec{r}}{r^3} \quad (3.71)$$

Now, using the vector identity,

$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot \vec{\nabla} \times \vec{A} - \vec{A} \cdot \vec{\nabla} \times \vec{B} \quad (3.72)$$

we have,
$$\vec{\nabla} \cdot \left(d\vec{l} \times \frac{\vec{r}}{r^3} \right) = \frac{\vec{r}}{r^3} \cdot \vec{\nabla} \times d\vec{l} - d\vec{l} \cdot \vec{\nabla} \times \frac{\vec{r}}{r^3}$$

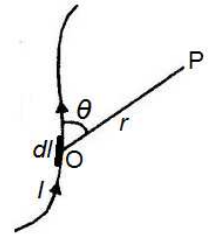


Fig. 3.14: Current carrying conductor

Thus,
$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0 I}{4\pi} \oint \left[\frac{\vec{r}}{r^3} \cdot \vec{\nabla} \times d\vec{l} - d\vec{l} \cdot \vec{\nabla} \times \frac{\vec{r}}{r^3} \right] \quad (3.73)$$

But, $\vec{\nabla} \times d\vec{l} = 0$ since $d\vec{l}$ is not a function of coordinate (x, y, z) of the field point P where we wish to find $\vec{\nabla} \times \vec{B}$.

Also $\vec{\nabla} \times \frac{\vec{r}}{r^3} = 0$ and hence from Eq. (3.73) we get,

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (3.74)$$

Thus we can conclude that the divergence of magnetic field is zero, which implies that the magnetic field is solenoidal. Eq. (3.74) may be regarded as the differential form of Gauss's law in magnetostatics.

Now, from divergence theorem we may write,

$$\int_V \vec{\nabla} \cdot \vec{B} dV = \iint_S \vec{B} \cdot d\vec{S}$$

Thus, magnetic flux, $\phi = \iint_S \vec{B} \cdot d\vec{S} = 0 \quad (3.75)$

Eq. (3.75) may be regarded as the integral form of Gauss's law in magnetostatics. It signifies that the net flux is zero *i.e.*, the net flux entering and leaving a particular volume are the same. So, there is no source or sink of magnetic flux. Comparing Eq. (3.75) with the Gauss's law in electrostatics $[\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0]$ we have for magnetic fields,

$$\rho_m = 0$$

which signifies the non-existence of magnetic monopoles.

Example 3.18 Prove that $\oint_C \vec{B} \cdot d\vec{S} = 0$, when \vec{B} is the magnetic field and \vec{S} is a closed surface.

Solution

Using $\vec{\nabla} \cdot \vec{B} = 0$ we have after taking volume integral on both sides $\int_V \vec{\nabla} \cdot \vec{B} dV = 0$.

Now, using Gauss's divergence theorem we can write

$$\int_V \vec{\nabla} \cdot \vec{B} dV = \oint_C \vec{B} \cdot d\vec{S} = 0.$$

3.13 MAGNETIC SCALAR POTENTIAL

Magnetic scalar potential in magnetostatics is analogous to electric potential in electrostatics. In a similar manner as we use the electric potential to find the electric field in electrostatics, magnetic scalar potential can be used to specify the magnetic field in cases when there are no free currents.

In a region like that in a current carrying conductor, $\vec{\nabla} \times \vec{B} \neq 0$ however if we have a current carrying conductor surrounded by empty space, then outside it $\vec{\nabla} \times \vec{B} = 0$ and we can write \vec{B} as a negative gradient of a scalar function as,

$$\vec{B} = -\vec{\nabla}\varphi_m \quad (3.76)$$

where φ_m is called as the magnetic scalar potential. It may be treated as a potential whose negative gradient at any point in a magnetic field gives the magnetic field at that point due to a closed current carrying loop. To find it, let us consider a current loop carrying a current I as shown in Fig. 3.15. According to Biot-Savart's law, the magnetic induction at A at a distance r relative to the current carrying element $d\vec{l}$ is given by,

$$\vec{B} = \frac{\mu_0}{4\pi} \oint \frac{I d\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l} \times \vec{r}}{r^3} \quad (3.77)$$

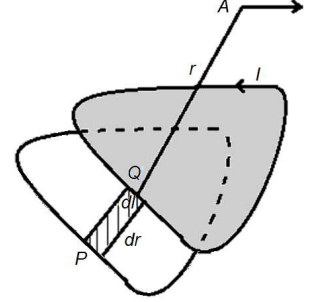


Fig. 3.15: Magnetic scalar potential

If the point of observation is now moved through a distance $d\vec{r}$ to the point Q then,

$$\vec{B}.d\vec{r} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l} \times \vec{r}}{r^3} \cdot d\vec{r} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l} \times \vec{r}}{r^3} \cdot d\vec{r} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{r} \cdot (d\vec{l} \times \vec{r})}{r^3}$$

$$\begin{aligned} \text{or, } \vec{B}.d\vec{r} &= \frac{\mu_0 I}{4\pi} \oint \frac{(d\vec{r} \times d\vec{l}) \cdot \vec{r}}{r^3} \\ &= -\frac{\mu_0 I}{4\pi} \oint \frac{(d\vec{r} \times -d\vec{l}) \cdot \vec{r}}{r^3} \end{aligned} \quad (3.78)$$

Obviously, when P is shifted to Q , let the solid angle subtended by the loop at the point of observation be changed by an amount $d\Omega$ and the area traced out by the current element $d\vec{l}$ during the displacement $-d\vec{r}$ is $d\vec{a} = d\vec{r} \times -d\vec{l}$.

$$\text{and hence, } \vec{B}.d\vec{r} = -\frac{\mu_0 I}{4\pi} \oint \frac{d\vec{a} \cdot \vec{r}}{r^3} = -\frac{\mu_0 I}{4\pi} d\Omega \quad (3.79)$$

$$\text{where, } d\Omega = \oint \frac{d\vec{a} \cdot \vec{r}}{r^3} \quad (3.80)$$

$$\text{or, } \vec{B}.d\vec{r} = -\frac{\mu_0 I}{4\pi} \vec{\nabla}\Omega \cdot d\vec{r}$$

$$\text{or, } \vec{B} = -\frac{\mu_0 I}{4\pi} \vec{\nabla}\Omega = -\vec{\nabla} \left(\frac{\mu_0 I}{4\pi} \Omega \right) \quad (3.81)$$

Comparing Eq. (3.76) with Eq. (3.81) we get, the magnetic scalar potential as,

$$\varphi_m = \frac{\mu_0 I}{4\pi} \Omega \quad (3.82)$$

3.14 MAGNETIC VECTOR POTENTIAL

The magnetic vector potential is a vector field that serves as the potential for the magnetic field. As we already know from Gauss's law in magnetostatics that, $\vec{\nabla} \cdot \vec{B} = 0$ and also from vector calculus we have the divergence of curl of a vector is zero, thus \vec{B} can be expressed as,

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad (3.83)$$

where \vec{A} can be regarded as a potential vector and is called as the magnetic vector potential.

Now we have,

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\text{Thus, } \vec{\nabla} \times \vec{\nabla} \times \vec{A} = \mu_0 \vec{J} \quad (3.84)$$

Now, from vector identity we have,

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$$

$$\text{If we choose } \vec{\nabla} \cdot \vec{A} = 0$$

$$\text{we get } \nabla^2 \vec{A} = -\mu_0 \vec{J} \quad (3.85)$$

In Cartesian component the above equation may be written as,

$$\nabla^2 A_x = -\mu_0 J_x, \nabla^2 A_y = -\mu_0 J_y \text{ and } \nabla^2 A_z = -\mu_0 J_z \quad (3.86)$$

Each of these equations is similar to Poisson's equation $\nabla^2 \phi = -\frac{\rho}{\epsilon_0}$ in electrostatics having the solution of the form,

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}_0) dV_0}{|\vec{r} - \vec{r}_0|} \quad (3.87)$$

Thus in a similar manner the solution of Eq. (3.86) may be written as,

$$A(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{J(\vec{r}_0) dV_0}{|\vec{r} - \vec{r}_0|} \quad (3.88)$$

Now for a line current I flowing through a wire of length element dl_0 , we can replace JdV_0 by $I dl_0$ and thus the above equation may be re-written as,

$$A(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I(\vec{r}_0) dl_0}{l} \quad (3.89)$$

where $l = |\vec{r} - \vec{r}_0|$ is distance of the length element from the point of observation.

Eq. (3.89) is giving the expression for magnetic vector potential at a distance l from a current carrying conductor carrying a current I .

Example 3.19 If at position (x, y, z) in a given magnetic region, the vector potential is $\vec{A} = (x^2 + y^2 - z^2)\hat{j}$, find the magnetic field at $(1, 1, 1)$.

Solution

We have, $\vec{B} = \vec{\nabla} \times \vec{A}$

here $\vec{A} = (x^2 + y^2 - z^2)\hat{j}$

$$\text{So, magnetic field } \vec{B} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & y^2 & -z^2 \end{vmatrix} = 0$$

Thus, at point $(1, 1, 1)$ the magnetic field is zero.

EXAMPLE 3.19

Example 3.20 If the vector potential $\vec{A} = (10x^2 + y^2 - z^2)\hat{j}$ is given any position in a magnetic region, then find the magnetic field vector at the point $(1, 1, 2)$.

Solution

We have, $\vec{B} = \vec{\nabla} \times \vec{A}$

here $\vec{A} = (10x^2 + y^2 - z^2)\hat{j}$

So, magnetic field

$$\begin{aligned} \vec{B} = \vec{\nabla} \times \vec{A} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 10x^2 + y^2 - z^2 & 0 \end{vmatrix} \\ &= 2\hat{i} + 20\hat{k} \end{aligned}$$

Hence, at point $(1, 1, 2)$ the magnetic field is $= 2\hat{i} + 20\hat{k}$.

EXAMPLE 3.20

Example 3.21 If magnetic vector potential at a point is $\vec{A} = \vec{a} \times \vec{r}$, where \vec{a} is representing a constant vector; find the magnetic induction vector at that point.

Solution

We have, $\vec{B} = \vec{\nabla} \times \vec{A}$

$$\text{where } \vec{A} = \vec{a} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ x & y & z \end{vmatrix}$$

$$= \hat{i}(a_y z - a_z y) + \hat{j}(a_z x - a_x z) + \hat{k}(a_x y - a_y x)$$

EXAMPLE 3.21

EXAMPLE 3.21

So,

$$\vec{B} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (a_y z - a_z y) & (a_z x - a_x z) & (a_x y - a_y x) \end{vmatrix}$$

$$= 2a_x \hat{i} + 2a_y \hat{j} + 2a_z \hat{k} = 2\vec{a}$$

which is the required magnetic induction vector at that point.

3.15 COMPARISON OF ELECTRIC AND MAGNETIC FIELDS

Electric field	Magnetic field
i) It is developed due to static charges <i>i.e.</i> , charges at rest.	i) It is developed due to steady electric current or a moving charge.
ii) Electrostatic field can be expressed as $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$ where ρ is the volume density of charge. This implies that, the electrostatic lines of force will originate from positive charge and terminate to negative charge, satisfying the existence of electric monopole.	ii) Magnetic field can be expressed as $\vec{\nabla} \cdot \vec{B} = 0$. This implies that the magnetic flux over a closed surface is zero <i>i.e.</i> magnetic monopole cannot exist due to the non existence of any source or sink of magnetic lines of force.
iii) Electrostatic field is conservative field <i>i.e.</i> , $\vec{\nabla} \times \vec{E} = 0$. So electric field can be expressed as a negative gradient of scalar potential (V) at any point <i>i.e.</i> , $\vec{E} = -\vec{\nabla}V$.	iii) Magnetic field is not conservative as $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ (\vec{J} is the electric current density). So magnetic field cannot be expressed as a negative gradient of any scalar potential at a point.

UNIT SUMMARY

- **Electric current and current density**

$$I = \int dI = \int_S \vec{J} \cdot d\vec{S}$$

- **Electrical conductivity**

$$\sigma = \frac{J}{E} = n_0 e \mu$$

- **Continuity equation**

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

- **Steady current**

$$\nabla^2 \phi = 0$$

- **Lorentz force**

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

- **Biot-Savart's law**

$$\vec{B} = \frac{\mu_0}{4\pi} \int_l \frac{Id\vec{l}}{r^3} \hat{r} = \frac{\mu_0}{4\pi} \int_V \frac{JdV}{r^3} \hat{r}$$

- **Force on a current carrying conductor placed in a magnetic field**

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

- **Force on a moving charge placed in a magnetic field**

$$\vec{F}_m = e\vec{v} \times \vec{B}$$

- **Ampere's circuital law**

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

- **Curl of magnetic field**

$$\vec{\nabla} \times \vec{B} = 0$$

- **Divergence of B: Gauss's law in magnetostatics**

$$\vec{\nabla} \cdot \vec{B} = 0$$

- **Magnetic scalar potential**

$$\vec{B} = -\vec{\nabla} \phi_m$$

- **Magnetic vector potential**

$$A(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I(\vec{r}_0) dl_0}{l}$$

• **Comparison between electrostatic field and magnetic field**

Electrostatic field

It is developed due to static charges

Electrostatic field is conservative field

Magnetic field

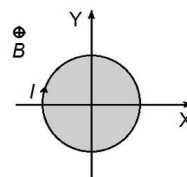
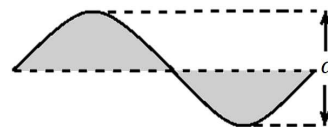
It is developed due to steady electric current

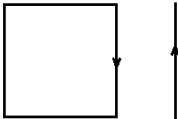
Magnetic field is not conservative.

EXERCISES

Multiple Choice Questions

- 3.1 If $\vec{B} = \vec{\nabla} \times \vec{A}$, \vec{B} and \vec{A} are any vectors then
 (a) $\vec{\nabla} \cdot \vec{B} = 0$ (b) $\vec{\nabla} \cdot \vec{B} = 1$ (c) $\vec{\nabla} \cdot \vec{B} = -1$ (d) $\vec{\nabla} \cdot \vec{B} = A$
- 3.2 Amongst the following statement which one is not characteristic of a static magnetic field?
 (a) it is solenoidal (b) it is conservative
 (c) magnetic flux lines are always closed (d) it has no sink or source
- 3.3 At a point just outside a current carrying conducting wire
 (a) electric field is zero while the magnetic field is non-zero
 (b) magnetic field is zero while the electric field is not
 (c) both electric and magnetic fields are zero
 (d) neither the electric field is zero nor the magnetic field is zero
- 3.4 A copper wire is bent in the form of a sine wave of wavelength λ and peak to peak value a as shown in figure. A magnetic field of flux density B tesla acts perpendicular to the plane of the figure in the entire region. If the wire carries a steady current I ampere, the magnetic force on the wire is
 (a) $I\sqrt{(a^2 + \lambda^2)}B$ (b) IaB (c) $I(a + \lambda)B$ (d) $I\lambda B$
- 3.5 A conducting loop carrying current I is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have tendency to
 (a) contract (b) expand
 (c) move towards the positive X axis
 (d) move towards the negative X axis
- 3.6 The divergence of magnetic flux density is
 (a) 0 (b) 1 (c) -1 (d) none of these



- 3.7 The direction of magnetic induction due to a straight infinitely long current carrying wire is
 (a) perpendicular to the wire (b) parallel to the wire
 (c) at an inclination of 30° to the wire (d) none of these
- 3.8 A current carrying straight wire cannot move, but a current carrying square loop adjacent to it can move under the influence of magnetic force. The square loop will
 (a) remain stationary (b) move towards the wire
 (c) move away from the wire (d) none of these
- 
- 3.9 The physical interpretation of $\vec{\nabla} \cdot \vec{B} = 0$ is (\vec{B} is the magnetic field)
 (a) magnetic monopole cannot exist (b) magnetic field is irrotational
 (c) magnetic field is conservative (d) magnetic lines of force are open curves
- 3.10 A proton travelling vertically downwards experiences a southward force due to a magnetic field directed at right angles to its path. An electron travelling northward in the same magnetic field will experience a magnetic force directed
 (a) upwards (b) downwards (c) towards east (d) towards west
- 3.11 Ampere's circuital law is applicable when the current density is
 (a) constant over space (b) time independent (c) solenoidal (d) irrotational
- 3.12 The vector potential corresponding to a constant magnetic field \vec{B} along Z-axis can be represented by
 (a) $-Bz\hat{k}$ (b) $\frac{B}{2}(x\hat{i} - y\hat{j})$ (c) $B(x\hat{j} - y\hat{i})$ (d) $\frac{B}{2}(x\hat{j} - y\hat{i})$
- 3.13 The force experienced by a charged particle in a magnetic field is independent of
 (a) velocity of the particle (b) strength of the field
 (c) charge of the particle (d) mass of the particle
- 3.14 The equation of continuity essentially represents
 (a) conservation of mass (b) conservation of charge
 (c) conservation of potential (d) conservation of force
- 3.15 The continuity equation for steady current is
 (a) $\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$ (b) $\vec{\nabla} \cdot \vec{J} = 0$ (c) $\frac{\partial \rho}{\partial t} = 0$ (d) $\vec{\nabla} \times \vec{J} = 0$
- 3.16 The energy associated with magnetic field \vec{H} is
 (a) $\frac{1}{2}H^2$ (b) $\mu_0 H^2$ (c) $\frac{1}{2}\mu_0 H^2$ (d) $\frac{1}{2\mu_0}H^2$
- 3.17 A moving charge produces
 (a) electric field only (b) magnetic field only (c) both of them (d) none of them

- 3.18 The magnetic flux linked with a coil at any instant t is given by $\phi = 10t^2 - 100t + 50$, the *emf* induced in the coil at $t = 4$ seconds is
 (a) 20V (b) 10V (c) 100V (d) 200V
- 3.19 The magnetic field of induction \vec{B} at the centre of a circular coil of radius r and total no. of turns N , carrying current I is
 (a) $\frac{\mu_0 NI}{2\pi r}$ (b) $\frac{\mu_0 I}{2r}$ (c) $\frac{\mu_0 NI}{2r}$ (d) $\frac{\mu_0 N^2 I^2}{2r}$
- 3.20 A wire of length L carrying a current I is bent into a circle. The magnitude of the magnetic field at the centre of the circle is
 (a) $\frac{\pi\mu_0 I}{L}$ (b) $\frac{\mu_0 I}{2\pi L}$ (c) $\frac{\mu_0 I}{2L}$ (d) $\frac{2\pi\mu_0 I}{L}$
- 3.21 The work done by the Lorentz force \vec{F} on a charged particle is
 (a) $\vec{F} \cdot d\vec{r}$ (b) zero (c) $\frac{q}{\epsilon_0}$ (d) qF
- 3.22 If total charge in a system is conserved, then
 (a) $\vec{\nabla} \cdot \vec{J} = 0$ (b) $\vec{\nabla} \cdot \vec{J} = \frac{\partial \rho}{\partial t}$ (c) $\vec{\nabla} \cdot \vec{J} = -\frac{\partial^2 \rho}{\partial t^2}$ (d) $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$
- 3.23 The relation between scalar potential ϕ and vector potential \vec{A} is
 (a) $\vec{\nabla} \times \vec{A} = \mu_0 \epsilon_0 \frac{\partial \phi}{\partial t}$ (b) $\vec{\nabla} \times \vec{A} = -\mu_0 \epsilon_0 \frac{\partial \phi}{\partial t}$ (c) $\vec{\nabla} \cdot \vec{A} = \mu_0 \epsilon_0 \frac{\partial \phi}{\partial t}$ (d) $\vec{\nabla} \cdot \vec{A} = -\mu_0 \epsilon_0 \frac{\partial \phi}{\partial t}$
- 3.24 If the current density $\vec{J} = k\hat{r}$ where \hat{r} is a unit vector along $x\hat{i} + y\hat{j}$, the current through the surface $x^2 + y^2 = a^2$, bounded by $z = 0$ and $z = h$ is
 (a) $\pi a^2 h k$ (b) zero (c) $2\pi a h k$ (d) $\frac{a^3 k}{\pi h}$
- 3.25 Two infinitely long conductors carrying currents I_1 and I_2 in the same direction are placed parallel to each other at a distance r apart. The current in one conductor due to the other will be,
 (a) $\frac{\mu_0 I_1 I_2}{2\pi r}$ N away from the first conductor (b) $\frac{\mu_0 I_1 I_2}{2\pi r}$ N towards the first conductor
 (c) $\frac{\mu_0 I_1 I_2}{2\pi r}$ N away from the second conductor (d) none of these
- 3.26 A toroid with inner and outer radius r and R is having N turns and a current I is flowing through it. The magnetic field in the midway from r to R is
 (a) $\frac{N\mu I}{\pi r}$ (b) $\frac{N\mu I}{\pi R}$ (c) $\frac{N\mu I}{\pi(R+r)}$ (d) $\frac{N\mu I}{\pi(R-r)}$

- 3.27 Vector potential of a moving charge depends on its
 (a) charge (b) velocity (c) both (a) and (b) (d) none of these
- 3.28 A wire of radius R is carrying a current I . The magnetic induction inside the wire at a distance r ($< R$) is
 (a) proportional to $1/R$ (b) proportional to r (c) proportional to $1/r$ (d) none of these
- 3.29 When a charge particle is moving perpendicular to a magnetic field
 (a) its energy and momentum remains constant
 (b) its energy and momentum will vary
 (c) its momentum vary but energy remains constant
 (d) none of these
- 3.30 The magnetic field at a point on the axis and the end face of a solenoid having n turns and carrying a current I is
 (a) $\mu_0 NI$ (b) $\frac{\mu_0 NI}{2}$ (c) $\frac{\mu_0 NI}{3}$ (d) $\frac{\mu_0 NI}{4}$
- 3.31 The magnetic vector potential satisfies
 (a) Poisson's equation (b) Laplace's equation (c) both (a) and (b) (d) none of these
- 3.32 The magnetic induction at the centre of a circular loop of radius R carrying current I is given by
 (a) $\frac{\mu_0 I}{4\pi R}$ (b) $\frac{\mu_0 I}{2\pi R}$ (c) $\frac{\mu_0 I}{2R}$ (d) $\frac{\mu_0 I}{2R^2}$

Answers of Multiple Choice Questions

3.1 (a), 3.2 (b), 3.3 (a), 3.4 (d), 3.5 (b), 3.6 (a), 3.7 (a), 3.8 (c), 3.9 (a), 3.10 (a), 3.11 (c), 3.12 (d), 3.13 (d), 3.14 (b), 3.15 (b), 3.16 (c), 3.17 (c), 3.18 (a), 3.19 (c), 3.20 (a), 3.21 (b), 3.22 (d), 3.23 (d), 3.24 (c), 3.25 (b), 3.26 (c), 3.27 (c), 3.28 (c), 3.29 (c), 3.30 (b), 3.31 (a), 3.32 (c)

Short and Long Answer Type Questions

Category I

- 3.1 Outline the method of measurement of the magnetic field due to a long straight current carrying wire using Biot-Savart's law.
- 3.2 Using Biot-Savart's law write down the expression of the magnetic field at the centre of a circular coil carrying current I .
- 3.3 Find the differences between the Ampere's law in magnetostatics in integral form and in differential form.
- 3.4 A conducting wire in the shape of an equilateral triangle is given to you where each side carries a current I . What will be the magnetic field at its centroid?
- 3.5 Mention the importance of Biot-Savart's law in electromagnetism.

- 3.6 Express the mathematical form of the magnetic field with direction at a distance r from a straight infinite wire carrying a current i .
- 3.7 Find the magnetic induction \vec{B} , at a point on the axis of an infinitely long solenoid carrying current I assuming the number of turns per unit length of the solenoid being n .
- 3.8 Obtain the magnetic field of a circular loop carrying current I on a point on the axis of the loop.
- 3.9 Differentiate clearly between electrostatic field and magnetic field.
- 3.10 What do you mean by magnetic vector potential? Why is it called so?
- 3.11 Starting from the definition of current density derive the relation $\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$, where the symbols have their usual meaning. What is the condition of steady current?
- 3.12 Applying Ampere's law of magnetostatics, deduce an expression of magnetic field B due to a straight conductor of infinite length carrying current I .
- 3.13 Find the magnetic field at a point $P(z)$ on the axis of a circular current carrying conductor. Hence find the magnetic field at the centre of the circular ring.
- 3.14 Apply Ampere's law to find the magnetic field at an external point due to a toroid.
- 3.15 Find the force experienced by a current element placed in a magnetic field.
- 3.16 Find the differences between electric and magnetic Lorentz force.
- 3.17 Applying Biot-Savart's law find the magnetic induction due to a circular conducting loop, carrying current I , on the axial point.
- 3.18 Using Biot-Savart's law obtain an expression for the magnetic flux intensity at the centre of a long current carrying solenoid. Hence show that the field at the end of such a solenoid is half of that at the centre.
- 3.19 Obtain the magnetic field induction \vec{B} at a point on the axis of a current carrying circular conductor (loop) with n turns.

Category II

- 3.20 Give the significance of vector potential and write its importance.
- 3.21 "Magnetic force does not perform any work". Explain mathematically.
- 3.22 Define ampere from the consideration of force between two parallel current carrying conductors.
- 3.23 A proton moves with a velocity $0.8c$ parallel to a straight current 1 A at a distance of 1 m from the current. What is the magnetic force on the proton? State the law you used in solving the above problem.
- 3.24 Express Ampere's circuital law in terms of magnetic vector potential.
- 3.25 Show how the Ampere's law implied that the current is in the steady state.
- 3.26 Express Biot-Savart's law in terms of current density and hence justify why that the magnetic field is solenoidal.

Numerical Problems

- 3.1 A test charge having charge 0.4 C is moving with a velocity $4\hat{i} - \hat{j} - 2\hat{k} \text{ m/s}$ through an electric field of intensity $10\hat{i} + 10\hat{k}$ and a magnetic field of induction $2\hat{i} - 6\hat{j} - 6\hat{k}$. Determine the magnitude and direction of the Lorentz force acting on the test charge.
- 3.2 Apply Biot-Savart's law to calculate field at any axial point due to a circular current carrying conductor carrying current 2 A and calculate B at the centre of the circle with radius $a = 10 \text{ cm}$.
- 3.3 Calculate the magnetic field intensity just outside and inside of a hollow cylinder of radius 4 cm carrying 50 A current.
- 3.4 A charge $q = (1.6 \times 10^{-19} \text{ C})$ is moving in an electromagnetic field of $\vec{E} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = 2\hat{i} + 5\hat{j}$ with velocity $\vec{v} = \hat{i} + 2\hat{j} + 2\hat{k} \text{ m/s}$. Find out the force.
- 3.5 Find magnetic field at a point $(1, 1, 1)$ if vector potential at that position is $\vec{A} = (10x^2 + y^2 - z^2)\hat{j}$.
- 3.6 A proton moves with a velocity 0.6 c parallel to a current of 1 A at a distance of 10 cm from the current carrying conductor. What is the magnetic force on the proton?
- 3.7 Two parallel wires carry equal current of 10 A along with the same direction and are separated by a distance of 2.0 cm . Find the magnetic field at a point which is 2 cm away from any of these wires.
- 3.8 A long solenoid of 40 cm length has 300 turns. If the solenoid carries a current of 3.5 A , find the magnetic field at one end of the solenoid.
- 3.9 Find the magnetic field at the centre of a semicircular conducting loop of radius 3.14 cm carrying current $1/\pi \text{ amp}$.
- 3.10 A circular ring having diameter 0.2 m , cross-section 0.02 m^2 , carrying a current of 1 A and having 200 number of turns. Find the magnetic flux in the ring.
- 3.11 Find the force per unit length of two straight parallel wires 0.02 m apart and carrying parallel current 1 A in each.
- 3.12 A steel rod of radius 1 cm is bent to form a circular loop and wound with a coil having 200 turns. If the mean radius of the loop is 10 cm , calculate the amount of current that will produce a flux of 0.5 mWb in the core. Given that for steel $\mu_r = 1000$.

PRACTICALS

1. Study of the variation of magnetic field using Helmholtz coils

Aim

1. To study the magnetic field variation with position of current carrying paired coils along their axis
2. To determine the radius of the coil
3. To investigate the principle of superposition of magnetic field

Apparatus

Two coils, constant current source, Gauss meter, movable magnetic field sensor.

Theory

Magnetic field at a point at a distance r from an elementary current carrying conductor can be obtained by using Biot-Savart's law and in vector form it can be expressed as:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3} \quad (i)$$

Applying the law to a circular current carrying coil of radius a and number of turns n , the magnetic field at a distance r from centre of coil is given by:

$$B = \frac{\mu_0}{4\pi} \frac{2\pi n I a^2}{(a^2 + r^2)^{3/2}} \quad (ii)$$

where I = current flowing in the coil and μ_0 represents the free space permeability, B is expressed in tesla or Wb/m^2 . The value of the magnetic field is maximum at the centre of the coil and is given by

$$B = \frac{\mu_0 n I}{2a} = \frac{4\pi \times 10^{-7} n I}{2a} \text{ T} \quad (iii) (a)$$

or,
$$B = \frac{4\pi \times 10^{-3} n I}{2a} \text{ G} \quad [\text{as } 1\text{ T} = 10^4 \text{ G}] \quad (iii) (b)$$

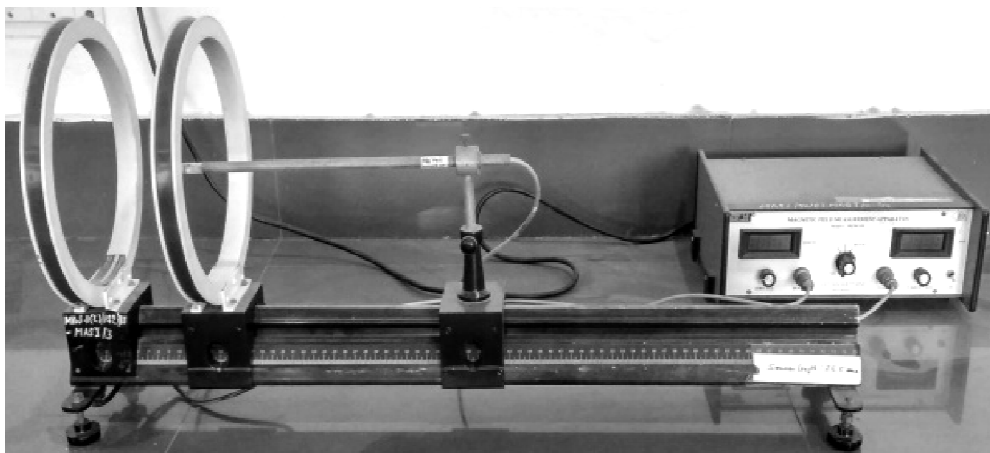


Fig. (i)

Experimental set up is shown in Fig. (i). If we move away from the centre of the coil either towards the right or towards the left, the magnetic field intensity decreases. If two such identical coils are placed coaxially, then depending on the relative sense of flow of current through them, the individual fields are added or subtracted to obtain the resultant magnetic field at any point on the axis.

Such an arrangement of a pair of identical coaxial coils separated by a distance equal to the radius of either coil is the Helmholtz coils [Fig. (ii)]. In Helmholtz coils, the individual coil produces uniform magnetic field between them.



Fig. (ii)

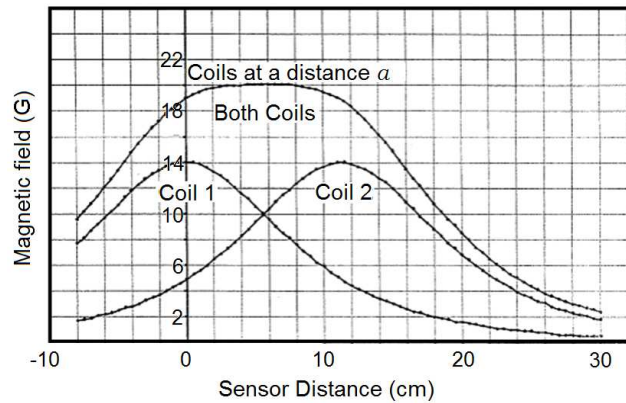


Fig. (iii)

For the same current flowing through them in the same sense two fields added up and the resultant magnetic field at an axial point at a distance x from 1st coil is given by

$$B_r = \frac{\mu_0 n I a^2}{2} \left[\frac{1}{(a^2 + x^2)^{3/2}} + \frac{1}{\{a^2 + (x - a)^2\}^{3/2}} \right] \quad (iv)$$

Fig. (iii) shows a typical variation of the resultant field for Helmholtz coils when current through them is same and flowing in the same sense through the individual coil.

Procedure

1. After switching the power ON, the current is to be reduced to zero by rotating the current adjusting knob towards the anti-clockwise direction up to the minimum position.
2. The position of the 2nd coil is to be fixed at a distance equal to the radius of the coils, from the 1st coil.
3. Keep the sensor at about 60 mm away from 1st coil and adjust the zero of Gauss meter with ZERO ADJ knob keeping current 0.0 mA.
4. Put the COIL knob to position 1 so that the 1st coil is connected to the current source. Adjust the current to say 500 mA.
5. Note down the magnetic field at about 60 mm along the axis of the 1st coil. Now put the COIL knob to position 2, so that 2nd coil is connected to current source and note down the magnetic field.
6. Put the coil knob to position 3, i.e. BOTH, so that 1st and 2nd coil will be connected to the source. Now again note down the readings.
7. Keep the current same and note down the magnetic field for about 270 mm range at an interval of 5 mm for all the positions i.e. 1st coil, 2nd coil and both of them.
8. Draw the graphs between distance and magnetic field due to 1st coil, 2nd coil and both of them along the axis of coils as shown in Fig. (ii).

Observations

No of turns = ; Position of coil 1 = ; Position of coil 2 =

Table 1: Determination of magnetic fields for Coil 1, Coil 2 and both of them

No of Obs.	Position of the Sensor (in cm)	Magnetic Field (in gauss)			
		Coil 1 (B_1)	Coil 2 (B_2)	Both Coils (B)	$B' = B_1 + B_2$
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Results

Draw a graph to realize the change of magnetic field with the position along the circular coils carrying current. You can check that the magnetic field at any point when the current is flowing through both the coils is the sum of the magnetic field due to the individual coil. Also you can check that the magnetic field is uniform over a large region when the distance between the individual coils is equal to their radius. This property can be widely used for scientific as well as industrial applications.

By using the relation $B = \frac{4\pi \times 10^{-3}}{2a} nI$, the radius of coil can be calculated as:

$$a = \frac{2\pi nI \times 10^{-3}}{B} \text{ m} \quad (v)$$

Using the values of n , I and B radius of coil a can be determined.

Precautions

1. While performing the experiment care should be taken so that there is no stray magnetic field or ferromagnetic material like keys, screwdriver or similar things near the experimental set up.
2. Each time before starting the experiment, the Gauss meter zero should be adjusted and after completion of the experiment it is to be verified again by decreasing the current to 0 in both the coils.
3. To avoid the interference of individual magnetic field the axis of coils should be placed along the east-west direction.
4. The radius of the coil is to be calculated from the centre of winding.

2. Measurement of Hall co-efficient and carrier concentration of a semi-conducting sample using Hall Effect experiment

Aim

- i) To determine the Hall co-efficient R_H
- ii) To determine the carrier concentration of a given semiconducting sample

Apparatus

Gauss meter, Hall probe, semiconducting (*p*-type / *n*-type Ge single crystal) sample with PCB mounting, power supply (0-16 V, 5A), constant current power supply (0-50 mA)

Theory

The Hall Effect is due to the nature of the current in a conductor. Current has the role for the movement of large number of charge carriers, *e.g.*, electrons, holes, ions or all of the three. If there is a magnetic field, charges will get a force, known as the Lorentz force. If this type of magnetic field disappears, the charges move nearly straight, 'line of sight' paths between collisions with phonons, impurities, *etc.* But if a magnetic field with a perpendicular component can be applied, then the paths between collisions are noted curve. As a result charges under motion accumulate only on one face of the material. This leaves same and opposite charges to expose on the other face, where mobile charges have scarcity. This causes an asymmetric distribution of charge density across the Hall element that arises due to a force which is perpendicular to both the 'line of sight' path and the magnetic field applied. The charge separation produces an electric field which opposes the migration of new charge. Consequently, a steady electrical potential is created during the period of flow of charge. When electric current goes through a conductor within a magnetic field, it exerts a transverse force on the carriers of moving charge that pushed them to one of the sides of the conductor. This can be largely seen in case of a thin flat conductor as we have illustrated.

Charge development at the conductors' sides balances the effect of the magnetic field, creating a voltage that can be measured between the two sides of the conductor. This measurable transverse voltage is termed as the Hall Effect after the name of E. H. Hall who did this contribution in 1879. It may be pointed out that the current I is the conventional current, maintain the direction, so that the electrons motion becomes opposite in the direction. This effect has importance for finding:

- i) Type of charge carriers *e.g.*, *n*-type or *p*-type,
- ii) Concentration of the carrier or charge carrier's number density and
- iii) Charge carrier's mobility.

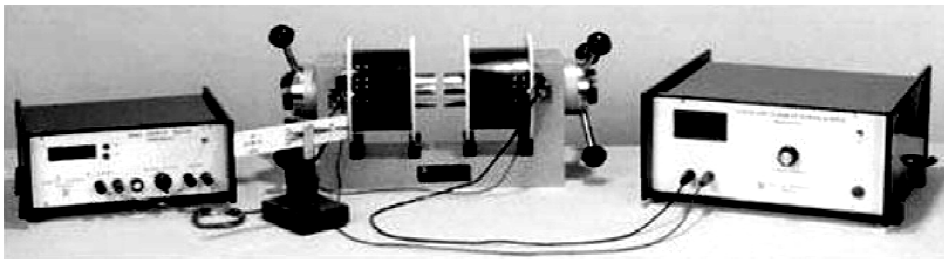


Fig. (i)

Procedure

1. Switch on the power supply of electromagnet and measure the magnetic flux density in between the pole faces using Gauss probe.
2. Place the specimen in between the pole faces such that the magnetic field is perpendicular to the strip.
3. Connect one pair of contacts of specimen on the opposite faces to the current source and other pair to the multi-meter of Hall Effect setup.
4. Using current source allow the current (of mA order) to flow in the specimen and thereby determine the Hall voltage in the multi-meter.
5. Increase the current through the specimen gradually and measure the corresponding Hall voltages.

Observations

Table 1: Determination of Hall current and Hall voltage

Constant current supply: 1.5 A

Sl No	Current I (mA)	Hall Voltage V_H (mV)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Precautions

1. Care should be taken to check whether the Gauss meter is initially showing zero value or not. This can be done by placing the probe at somewhere and adjusting offset zero knob until zero is displayed.
2. Place the specimen at the centre of the pole pieces of the electromagnet and ensure that it is situated exactly perpendicular to the magnetic field.
3. For the measurement of the magnetic flux density, Hall probe is to be kept at centre position of the pole pieces.
4. Identify carefully direction of coils of the electromagnet for generating highest magnetic field. It is further verified by keeping the soft iron close to the magnetic field generated. It is then observed to note whether it becomes strongly attracted or weakly attracted to the related magnetic poles.

Results

The expression for the Hall co-efficient is:

$$R_H = \frac{V_H}{I} \times \frac{t}{B} \quad (i)$$

where V_H = Hall voltage (in mV),

I = current (in mA) through the specimen,

t = thickness of specimen in m,

and B = magnetic flux density in T.

Typically $t = 0.5$ mm and $\frac{V_H}{I}$ can be calculated from the slope of the curve in Fig. (ii).

So, Hall co-efficient $R_H = \dots\dots\dots \Omega\text{-m/T}$.

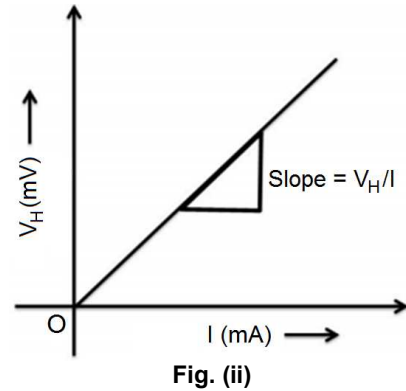
The expression for the carrier concentration is:

$$n = \frac{1}{qR_H} \quad (ii)$$

where q = charge of electrons/holes,

and R_H = Hall co-efficient.

So, carrier concentration $n = \dots\dots\dots /m^3$.



KNOW MORE

Magnetostatics may be considered as the subfield of electromagnetics focusing a static magnetic field, like the one generated by a steady electric current or a permanent magnet. People of ancient Greece and China found that a lodestone align itself at all the times in a longitudinal direction when allowed to rotate freely. Lodestone was the first known magnet which is a type of iron ore known as magnetite (Fe_3O_4).

Activity

The essential source of any magnetostatic field is the permanent magnets and constant electric currents. The widely used laws of the magnetostatic fields are the Biot-Savart's law as well as the Ampere's law. The property of lodestone allowed for making the compass in two thousand years ago. This was the first known use of the magnet.

Interesting facts

Pierre de Maricourt in 1263 mapped the magnetic field of a lodestone with the help of a compass when he discovered the two poles of the magnet. William Gilbert, a physician of Queen Elizabeth I, through some experiments came to the conclusion in the 1600's that Earth itself was a huge magnet.

A permanent magnet may be accepted as a natural source of magnetostatics field. An accelerated charge is responsible to produce changing electric and magnetic field.

This further gives rise for propagating electromagnetic field and hence this does not produce a static magnetic field.

Analogy

Danish physicist Hans Christian Oersted, in 1820, discovered that an electric current flowing through a wire can deflect a compass needle. This also concludes that magnetism and electricity are related.

History

Ancient Greeks were first familiar to have used this mineral, which they termed as a magnet due to its ability to attract other pieces of the same material and iron.

The Englishman William Gilbert (1540-1603) first investigated the properties of magnetism systematically following scientific methods.

Timelines

600 BC : *Thales* of Miletus discovered attraction of lodestone's to iron

1200 AD: Chinese started to use lodestone compass for navigation

1259 AD: *Petrus Peregrinus* of Italy discovered the same thing with further improvement

1600 AD: *William Gilbert* discovered that the earth has a giant magnetic field

1742 AD: *Thomas LeSeur* established inverse cube law for magnets

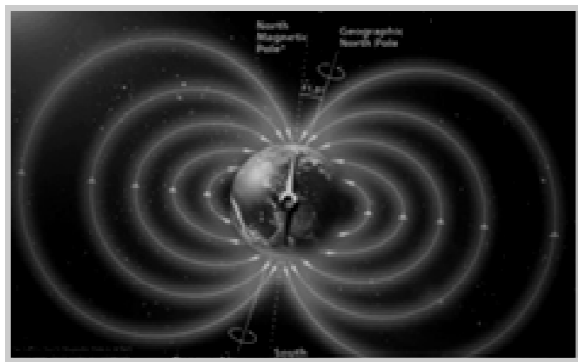
1820 AD: *Hans Christian Oersted* discovered that current twists magnets

Andre Marie Ampere showed that parallel currents attract/repel

Jean-Baptiste Biot and *Felix Savart* established inverse square law

Applications (Real Life / Industrial)

Magnetostatics is used widely in different applications of micromagnetics like models of magnetic storage devices related to computer memory. Magnetostatic focusing can be obtained either by a permanent magnet or by sending current through a coil of wire whose axis is in coincident with the beam axis.



Pierre de Maricourt, in 1269, from a series of observations found that the direction of a compass near a magnet formed curved lines, which are passed through two points diametrically opposed to each other. These points were termed by him as poles.

In 1600 AD William Gilbert published some interesting findings of his experiments with magnetism. He proved that earth itself has a large permanent magnet.

Case Study (Environmental / Sustainability / Social / Ethical Issues)

The term magnet came from the ancient Greek city of magnesia, at which many natural magnets were found. As early as 121 AD, it was known by the Chinese that an iron rod when brought close to the natural magnet would acquire and retain the magnetic property. Also when this iron rod suspended from a string they observed that it aligned itself in a north-south direction.

It is important to note that while the magnetic flux through a closed surface becomes always zero but the magnetic flux through an open surface need not be zero. This is considered as an important quantity in electromagnetism, particularly when the electromagnetic wave propagates through the atmosphere.

Inquisitiveness and Curiosity Topics

A usual technique is there to solve a series of magnetostatic problems at incremental time steps for the purpose of using these solutions for reintroducing Faraday's law. Plugging the result so obtained one can implement those into Faraday's law to find a value which had previously not considered. Though the technique applied is not a true solution of Maxwell's equations but it can provide a very good approximation for slowly varying fields.

The famous Indian surgeon Sushruta was the first to make use of the magnet for sophisticated surgical purposes.

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4

Magnetostatics in a Linear Dielectric Medium

UNIT SPECIFICS

We have discussed the following topics in this unit:

- Magnetization and the relation between magnetic intensity, magnetic intensity and magnetization;
- Diamagnetism and Larmor frequency;
- Susceptibility and Curie law;
- Weiss molecular field theory;
- Curie-Weiss law;
- Anti-ferromagnetism;
- Ferromagnetism;
- Different types of ferrites.

The practical applications of the topics are discussed for generating further curiosity and creativity as well as improving problem solving capacity.

Besides giving a large number of multiple choice questions as well as questions of short and long answer types marked in two categories following lower and higher order of Bloom's taxonomy, assignments through a number of numerical problems, a list of references and suggested readings are given in the unit so that one can go through them for practice. It is important to note that for getting more information on various topics of interest some QR codes have been provided in different sections which can be scanned for relevant supportive knowledge.

After the related practical, based on the content, there is a "Know More" section. This section has been judiciously designed so that the supplementary information provided in this part becomes beneficial for the users of the book. This section mainly highlights the initial activity, examples of some interesting facts, analogy, history of the development of the subject focusing the salient observations and finding, timelines starting from the development of the concerned topics up to the recent time, applications of the subject matter for our day-to-day real life or/and industrial applications on variety of aspects, case study related to environmental, sustainability, social and ethical issues whichever applicable, and finally inquisitiveness and curiosity topics of the unit.

RATIONALE

This unit on Magnetostatics in a Linear Dielectric Medium will help our students to get a clear idea about magnetization M and the relation between B , H and M . It will help to develop some important ideas on diamagnetism and Larmor frequency as well as susceptibility and Curie law. The explanation on Weiss molecular field theory and Curie-Weiss law are topics whose understanding help for further development of the subject. Anti-ferromagnetism and ferromagnetism are also outlined as supporting topics with large area of applications.

It is important to mention that the magnetization need not be static. The equations developed by the scientists on magnetostatics can be successfully utilized for predicting fast magnetic switching events which occur on nanoseconds or less time scales. Interestingly, magnetostatics is a very good approximation under the situation when the currents are not static. It has a wide use in various emerging applications of micromagnetics, *e.g.*, models of magnetic storage devices in computer memory.

PRE-REQUISITES

Mathematics: Vector Calculus (Class XII)

Physics: Electromagnetism (Class XII)

UNIT OUTCOMES

List of outcomes of this unit is as follows:

- U4-O1: Describe magnetization and the relation between magnetic intensity, magnetic intensity and magnetization
- U4-O2: Describe diamagnetism and Larmor frequency
- U4-O3: Explain susceptibility and Curie law
- U4-O4: Explain Weiss molecular field theory and Curie-Weiss law
- U4-O5: Discuss ferromagnetism, antiferromagnetism and ferrites

Unit-4 Outcomes	EXPECTED MAPPING WITH COURSE OUTCOMES (1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation)					
	CO-1	CO-2	CO-3	CO-4	CO-5	CO-6
U4-O1	3	2	-	1	1	1
U4-O2	-	-	-	-	3	-
U4-O3	-	-	-	-	3	-
U4-O4	-	-	-	-	3	-
U4-O5	-	-	-	-	3	-

4.1 INTRODUCTION

The substances which can be magnetized when placed in an external magnetic field are known as the *magnetic materials* and the corresponding magnetic field is sometimes called the *magnetizing field*. Depending on the exhibited magnetic properties, substances can be grouped under the following:

- i) *diamagnetic*,
- ii) *paramagnetic*,
- iii) *ferromagnetic*,
- iv) *antiferromagnetic* and
- v) *ferrimagnetic*.

Some of these substances do not show any permanent magnetic dipole moment, whereas some other exhibits permanent magnetic dipole moment even after the removal of the external magnetic field. Based on this feature, in this unit we will discuss some important magnetic properties of materials.

4.2 MAGNETIC INDUCTION AND INTENSITY

Magnetic induction may be defined as the total quantity of magnetic flux in any area which is taken as perpendicular to the direction of the magnetic flux.

Its SI unit is Wb/m^2 or alternately tesla (T) while the CGS unit is gauss (G). The relation between the two unit is $1\text{T} = 10^4\text{G}$.

At any point in a magnetic field, the magnetic field intensity is defined as the force experienced by a unit magnetic north pole which is kept at that said point in the concerned magnetic field.

Alternatively, magnetic field intensity is the amount of magnetic flux in a unit area perpendicular to the direction of magnetic field. Its SI unit is A/m and CGS unit is Oe.

The relation between the magnetic induction B and the magnetic field intensity H in vacuum can be expressed as,

$$B = \mu_0 H \quad (4.1)$$

In Eq. (4.1), μ_0 represents the permeability in free space. Its value in SI unit is $4\pi \times 10^{-7}\text{H/m}$.

4.3 MAGNETIZATION

In an external magnetic field, if a magnetic material is kept, it gets magnetized due to the process of *magnetic induction*. The *intensity of magnetization* is defined as the magnetic dipole moment produced per unit volume of the substance. Hence,

$$\text{Magnetization } (M) = \frac{\text{Magnetic moment } (\mu)}{\text{Volume } (V)}$$

$$\text{or, } M = \frac{\mu}{V} \quad (4.2)$$

Actually, it is the measure of the developed magnetism in a magnetic substance when placed in a magnetic field. For a magnet of length l , with the strength of the pole m and cross-sectional area α , the magnetization will be,

$$M = \frac{\mu}{V} = \frac{lm}{l\alpha} = \frac{m}{\alpha} \quad (4.3)$$

Thus the intensity of magnetization is the strength of the pole per unit area of the magnetic substance. In a solenoid of n turns per unit length, the magnetization will be,

$$M = \frac{nI\alpha}{\alpha} = nI \quad (4.4)$$

Its SI unit is A/m and CGS unit is Oe (same as magnetic field intensity).

4.4 MAGNETIC SUSCEPTIBILITY

The magnetic susceptibility is the measurement of how much a material will be magnetized in any applied magnetic field. It can be defined as the ratio of magnetization \vec{M} , *i.e.*, the magnetic moment per unit volume to the magnetizing field intensity \vec{H} applied there.

For isotropic material, \vec{M} and \vec{H} are parallel and the susceptibility is expressed as,

$$\vec{M} = \chi \vec{H}$$

or,
$$\chi = \frac{M}{H} \quad (4.5)$$

χ is then a scalar quantity, known as the *magnetic susceptibility* and is essentially a unit less number (the ratio of two similar quantities). It is the property of a magnetic substance which determines how easily a magnetic substance can be magnetized. In case of a gram-molecule one may introduce the molar susceptibility χ_m .

EXAMPLE 4.1

Example 4.1 If a magnetic field inside of a solenoid is 10^{-3} T when it is empty and 2 T when it is filled with iron, calculate the relative permeability of iron.

Solution

Here, $M = 2$ T and $H = 10^{-3}$ T.

So, susceptibility of iron is $\chi = \frac{M}{H} = \frac{2}{10^{-3}} = 2000$.

Thus, the relative permeability of iron is $\mu_r = 1 + \chi = 1 + 2000 = 2001$.

EXAMPLE 4.2

Example 4.2 If the susceptibility of a medium is 3.7, find the absolute and relative permeability.

Solution

The susceptibility of the medium is $\chi = 3.7$.

Thus, the relative permeability of the medium is

$$\mu_r = 1 + \chi = 1 + 3.7 = 4.7.$$

and the absolute permeability of the medium is

$$\mu = \mu_r \mu_0 = 4.7 \times 4\pi \times 10^{-7} = 5.9 \times 10^{-6} \text{ N/A}^2.$$

Example 4.3 If the maximum permeability of a material is $0.15 \times 10^{-2} \text{ N/A}^2$, calculate the relative permeability and the susceptibility of the material.

Solution

Here, $\mu = 0.15 \times 10^{-2} \text{ N/A}^2$ and $\mu_0 = 4\pi \times 10^{-7}$.

Thus, the relative permeability of the material is

$$\begin{aligned}\mu_r &= \frac{\mu}{\mu_0} = \frac{0.15 \times 10^{-2}}{4\pi \times 10^{-7}} \\ &= 11.94 \times 10^2\end{aligned}$$

and the susceptibility of the material is $\chi = \mu_r - 1 = 11.94 \times 10^2 - 1 = 1193$.

EXAMPLE 4.3

Example 4.4 For a bar of a metal alloy the magnetization is $1.2 \times 10^6 \text{ A/m}$ when the magnetic field is 200 A/m . Determine the magnetic susceptibility and magnetic induction within the alloy.

Solution

Here $M = 1.2 \times 10^6 \text{ A/m}$ and $H = 200 \text{ A/m}$.

Thus, the magnetic susceptibility is $\chi = \frac{M}{H} = \frac{1.2 \times 10^6}{200} = 6000$.

and the magnetic induction within the alloy is

$$\begin{aligned}B &= \mu H = \mu_0 (1 + \chi) H \\ &= 4\pi \times 10^{-7} \times (1 + 6000) \times 200 = 1.507 \text{ T}.\end{aligned}$$

EXAMPLE 4.4

Example 4.5 A magnetizing field of 1000 A/m produces a magnetic flux 10^5 Wb in a bar of iron of cross sectional area 0.2 cm^2 . Find the susceptibility and permeability of the bar.

Solution

Given, $H = 1000 \text{ A/m}$, $\phi = 10^5 \text{ Wb}$ and $A = 0.2 \text{ cm}^2 = 0.00002 \text{ m}^2$.

Thus, $B = \frac{\phi}{A} = \frac{10^{-5}}{0.00002} = 0.5 \text{ Wb/m}^2$.

So, permeability of the bar is $\mu = \frac{B}{H} = \frac{0.5}{1000} = 5 \times 10^{-4} \text{ Wb/m}^2$.

and the susceptibility of the bar is $\chi = \frac{\mu}{\mu_0} - 1 = \frac{5 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 397.1$.

EXAMPLE 4.5

Example 4.6 A sample of iron having length 5 cm cross sectional area $2 \times 10^{-4} \text{ m}^2$ develops a magnetic moment of 1000 A/m^2 . Find the intensity of magnetization and magnetic induction if the magnetic field intensity is 10^6 A/m .

Solution

Given, magnetic moment of the specimen (μ) = 1000 A/m^2 , length of the specimen (l) = 5 cm, cross sectional area of the specimen (A) = $2 \times 10^{-4} \text{ m}^2$ and the magnetizing field (H) = 10^6 A/m .

Thus, the volume of the sample is $V = lA = 5 \times 10^{-2} \times 2 \times 10^{-4} = 10^{-5} \text{ m}^3$.

So, the intensity of magnetization is, $M = \frac{\mu}{V} = \frac{1000}{10^{-5}} = 10^8 \text{ A/m}$,

the susceptibility of the sample is $\chi = \frac{M}{H} = \frac{10^8}{10^6} = 10^2 \text{ A/m}$

and the permeability of the sample is

$$\mu = \mu_0(1 + \chi) = 4\pi \times 10^{-7}(1 + 10^2) = 1.27 \times 10^{-4}.$$

Thus, the magnetic induction is $B = \mu H = 1.27 \times 10^{-4} \times 10^6 = 0.0127 \text{ T}$.

4.5 RELATION AMONG B, H AND M

For any medium with permeability $\mu (= \mu_0 \mu_r)$, the magnetic induction B and the intensity of the magnetic field H are related as:

$$B = \mu H = \mu_0 \mu_r H = \mu_0 \mu_r H + \mu_0 H - \mu_0 H \quad [\text{where } \mu_r \text{ is the relative permeability}]$$

or,

$$B = \mu_0 H + \mu_0 H (\mu_r - 1) = \mu_0 H + \mu_0 \chi H$$

$$= \mu_0 H + \mu_0 M \quad [\text{where } M = \chi H \text{ is the magnetization}]$$

or,

$$\vec{B} = \mu_0 (\vec{M} + \vec{H}) \quad (4.6) (a)$$

where,

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \quad (4.6) (b)$$

Example 4.7 If the intensity of the magnetic field and the susceptibility are 10^6 A/m and 10^{-3} respectively, calculate the magnetization field and flux density.

Solution

Here, $H = 10^6 \text{ A/m}$ and $\chi = 10^{-3}$.

So, the magnetization field is $M = \chi H = 10^{-3} \times 10^6 = 10^3 \text{ A/m}$.

and the flux density is $B = \mu_0(M + H) = 4\pi \times 10^{-7} \times (10^3 + 10^6) = 1.26 \text{ T}$.

Example 4.8 Calculate the magnetic induction of a specimen, if the magnetic field intensity is 10^6 A/m and the susceptibility of the material of the specimen is 10^{-4} .

Solution

Given, magnetic field intensity is 10^6 A/m and susceptibility of the specimen material is 10^{-4} .

Thus, the intensity of magnetization is, $M = \chi H = 10^{-4} \times 10^6 = 10^2$ A/m.

and the magnetic induction is $B = \mu_0(H + M) = 4\pi \times 10^{-7} \times (10^6 + 10^2) = 1.256$ T.

EXAMPLE 4.8

4.6 CLASSIFICATIONS OF MAGNETIC MATERIALS

All substances made up of electrons and protons show some kind of magnetic behavior. This magnetic origin of the materials is due to the orbital motion and spin of the electrons and for their mutual interactions. The individual or the group of atoms effectively behaves as a magnetic dipole aligned with the applied magnetic field. The net effect of all such dipoles develops the magnetic behavior of the materials. The magnetic properties of materials can be classified into the following five categories:

- i) *diamagnetism*,
- ii) *paramagnetism*,
- iii) *ferromagnetism*,
- iv) *antiferromagnetism* and
- v) *ferrimagnetism*.

Materials in the first two categories do not show any collective interactions. They are not ordered magnetically. On the other hand, below a certain temperature the materials in the last three categories are ordered magnetically. This temperature is known as the critical temperature. Usually ferromagnetic and ferrimagnetic materials are considered as *magnetic* (like iron). Remaining three categories are so poorly magnetic that they are commonly termed as *non-magnetic*.

4.6.1 Diamagnetism

The diamagnetic materials are composed of atoms having net magnetic moments zero. This happens as in all the orbital shells, there are no unpaired electrons. But with the application of a field a negative magnetization will be developed there making the susceptibility negative. Fig. 4.1 (a) shows the plot of M vs H , indicating that when the field is zero the magnetization is zero. Another important characteristic of diamagnetic materials is that the susceptibility is temperature independent [Fig. 4.1 (b)], e.g., quartz (SiO_2), calcite (CaCO_3), water *etc.*

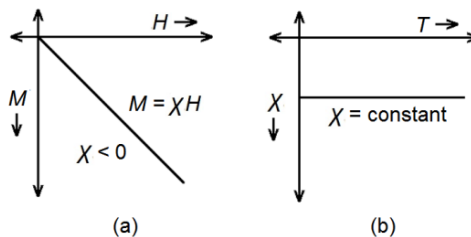


Fig. 4.1: (a) M - H and (b) χ - T curve for diamagnetic substances

Properties of diamagnetic materials: Some of the important properties of diamagnetic materials are given below.

- i) Relative permeability of this type of material is slightly less than unity.
- ii) Magnetic susceptibility of this type of material is negative and is independent of the applied magnetic field strength.
- iii) When kept in any magnetic field, the field lines are repelled.
- iv) Permanent dipoles are absent in this type of material.

4.6.2 Paramagnetism

In this category of atoms or ions there is a resultant magnetic moment owing to unpaired electrons in partially filled shells. But the individual magnetic moments have no capability to interact magnetically and when the field is removed the magnetization becomes zero. In the presence of a field, there is a partial alignment of the atomic magnetic moments in the direction of the field, resulting in a net positive magnetization and positive susceptibility (Fig. 4.2).

Moreover in order to align the moments, the effect of the field is opposed owing to the effects of the temperature. Using the Curie law of magnetostatics, we can express the temperature dependence of susceptibility.

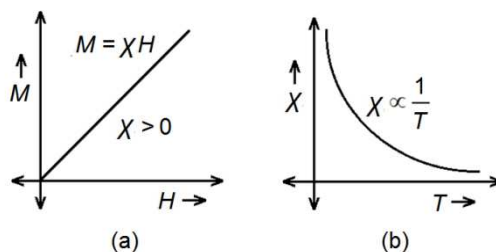


Fig 4.2: (a) M - H and (b) χ - T curve for paramagnetic substances

Properties of paramagnetic materials: Some of the important properties of paramagnetic materials are-

- i) Magnetic susceptibility of this type of material is positive and is independent of the field strength applied but depends on temperature.
- ii) Relative permeability of this type of material is slightly higher than 1.
- iii) If kept in any magnetic field, an attraction will occur among the field lines.
- iv) This type of material possesses permanent dipoles.
- v) In absence of external magnetic field, the dipoles are randomly oriented.
- vi) Spin alignment of these types of material is also random.

4.6.3 Ferromagnetism

These types of materials exhibit parallel alignment of moments causing huge net magnetization even when the magnetic field is not present. Two distinct characteristics of ferromagnetic materials are: i) spontaneous magnetization and ii) the existence of magnetic ordering temperature.

Fig. 4.3 shows the plot of M vs H and χ vs T curves for a typical ferromagnetic substance.

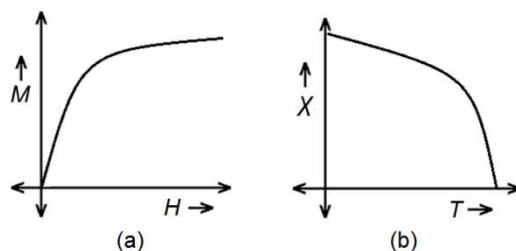


Fig. 4.3: (a) M - H and (b) χ - T curve for ferromagnetic substances

Properties of ferromagnetic materials: Some of the important properties of ferromagnetic materials are-

- i) Magnetic susceptibility of this type of material is highly positive and they behave as paramagnetic material above Curie temperature.
- ii) Relative permeability of this type of material is very much greater than unity.
- iii) A strong attraction is observed when kept in a magnetic field.
- iv) This type of material possesses hysteresis.
- v) This type of material consists of a number of spontaneously magnetized domains.
- vi) Spin alignment of this type of material is parallel.



4.6.4 Ferrimagnetism

Ferrimagnetic materials have high resistivity. They also have anisotropic properties. In practice, the anisotropy is induced due to an external applied field. At the time when the applied magnetic field is aligned with the magnetic dipoles, there develops a net magnetic dipole moment. This causes the dipoles to precess at some frequency governed by the field applied. This frequency is known as the *Larmor precession frequency* (will be discussed later). They exhibit spontaneous magnetization below Curie temperature and show no magnetic ordering (paramagnetic behavior) above this temperature.

However, under certain conditions a temperature below the Curie temperature is attained when the two opposing moments become same, causing a net zero magnetic moment. This is known as the point of *magnetization compensation*. Ferrimagnetisms as revealed by ferrites occur within the molecular magnets. Here we can mention the name of the oldest known magnetic substance magnetite (Fe_3O_4), which is a ferrimagnet in behavior.

Properties of ferrimagnetic materials: Some of the important properties of ferrimagnetic materials are-

- i) Magnetic susceptibility of this type of material is highly positive and they behave as paramagnetic material below *Neel temperature*.
- ii) Above Curie temperature they become paramagnetic.
- ii) This type of material possesses a net magnetic moment.
- iv) Spin alignment of this type of material is anti-parallel with different magnitudes.

4.6.5 Anti-ferromagnetism

The magnetic moments of atoms or molecules for this material are generally related to the spins of electrons. In a regular pattern the atoms are aligned with nearby spins on various sub-lattices which are pointed in reverse directions. Usually, anti-ferromagnetic order may be found to exist at considerable low temperatures and disappearing at certain temperature called the *Neel temperature*. For an anti-ferromagnetic material the susceptibility shows a maximum at this temperature. When we apply a temperature greater than this temperature the material is typically paramagnetic.

In absence of any external applied field anti-ferromagnetic substances have a zero net magnetization. In presence of an applied external magnetic field, a type of ferrimagnetic property may be demonstrated in the anti-ferromagnetic phase, with the absolute value of one of the sub-lattice magnetizations differing from that of the other sub-lattice, resulting in a non-zero net magnetization. It is important to note that the net magnetization should be zero at absolute zero. Hematite is a good example of a typical anti-ferromagnetic material.

Properties of anti-ferromagnetic materials: Some of the important properties of anti-ferromagnetic materials are-

- i) Magnetic susceptibility of this type of material is small positive and they depend largely on temperature.
- ii) Below Curie temperature the susceptibility increases with temperature and above Curie temperature it decreases with temperature.
- iii) Spin alignment of this type of material is anti-parallel with same magnitudes.

4.7 PERMANENT MAGNETIC DIPOLES

Some properties of magnetic materials can be determined by the presence of elementary magnets called permanent magnetic dipoles. Also if a charged particle has an angular momentum, it can behave like a permanent magnetic dipole.

Total magnetic dipole moment of an atom is the resultant of *i)* orbital angular momentum, *ii)* spin angular momentum and *iii)* nuclear spin angular momentum of the electrons in the atom.

i) Orbital angular momentum: For any electron of an atom the motion is quantized and can be described by:

a) Principal quantum number (n): It can take integer values (1, 2, 3, ...). Corresponding electronic shells are designated as K, L, M,

b) Angular momentum quantum number (l): For a given n , it is restricted to $l = 0, 1, 2, \dots, (n - 1)$ and the electrons are respectively designated as the s, p, d, f, g, \dots electrons. The total angular momentum associated with a given l is $\hbar[l(l + 1)]^{1/2}$.

c) Magnetic quantum number (m_l): The possible component of angular momentum along any direction of the magnetic field is obtained by this quantum number. For a given l , it is restricted to the values $0, \pm 1, \pm 2, \pm 3, \dots, \pm l$. The total angular momentum associated with a given m_l is $m_l \hbar$.

For p -electron, possible component of angular momentum along the magnetic field direction are: $-\frac{eh}{2m}, 0, \frac{eh}{2m}$. The quantity $\frac{eh}{2m} = 9.27 \times 10^{-24} \text{ Am}^2$ is known as the *Bohr magneton*. It is used as an atomic unit of magnetic moment and is denoted by μ_B .

ii) Spin angular momentum: The possible components of angular momentum of the spin of electron along the direction of the applied field are $\pm \frac{\hbar}{2}$ giving rise to two spin quantum numbers, $m_s = \pm \frac{1}{2}$.

The component of magnetic moment along the direction of magnetic field is

$$\mu = g \left(\frac{e}{2m} \right) \frac{\hbar}{2}$$

where g is the spectroscopic splitting factor and is equal to 2.0023 for electronic spin. So, for electronic spin, the magnetic dipole moment (μ) is nearly equal to the Bohr magneton along or opposite to the direction of the magnetic field.

Orbital and spin angular momentum can be vectorically combined to obtain the total angular momentum, determined by the quantum number J . For a given l , it can be taken as the values $J = l \pm \frac{1}{2}$. For an atom having a number of electrons, individual orbital angular momentum and spin angular momentum can be combined to the resultant as L and S . For such a system general expression for g is given by,

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \quad (4.7)$$

Example 4.9 Find the Lande g factor for an ion in $2P_{3/2}$ state.

Solution

For, $2P_{3/2}$ state $L = 1$, $S = \frac{1}{2}$, $J = \frac{3}{2}$

$$\begin{aligned} \text{Thus,} \quad g &= 1 + \frac{\frac{3}{2} \left(\frac{3}{2} + 1 \right) + \frac{1}{2} \left(\frac{1}{2} + 1 \right) - 1(1+1)}{2 \frac{3}{2} \left(\frac{3}{2} + 1 \right)} \\ &= 1 + \frac{\frac{3}{2} \left(\frac{3}{2} + 1 \right) + \frac{1}{2} \left(\frac{1}{2} + 1 \right) - 1(1+1)}{2 \frac{3}{2} \left(\frac{3}{2} + 1 \right)} \end{aligned}$$

$$\text{or,} \quad g = 1 + \frac{\frac{9}{2} - 2}{\frac{15}{2}} = \frac{4}{3}.$$

EXAMPLE 4.9

iii) Nuclear spin angular momentum: The spinning of the nucleus also contribute to the permanent dipole moment of the atom and is given by

$$\mu_n = \frac{e\hbar}{2M} \quad (4.8)$$

where M is the nuclear mass. As the mass of the nucleus is about 10^3 times the mass of the electron, the electronic magnetic moment is 10^3 times larger than the nuclear magnetic moment.

4.8 HUND'S RULE

All filled electronic shells are not contributing to the magnetic moment of the atom and it arises only due to the partially filled electronic shells. Keeping Pauli's exclusion principle in mind that no two electrons have all four quantum numbers identical, Hund's rule for the incomplete shells of such atoms states that:

- i) the orbital moments combine to give a maximum value of L ,
- ii) combined maximum value of S , of the electron spins consistent with exclusion principle and
- iii) with a shell less than half occupied $J = L - S$ and for a shell more than half occupied $J = L + S$.

4.9 LANGEVIN'S THEORY OF DIAMAGNETISM

Magnetic dipoles have their origin in the flow of electric currents. From the theory of electricity it is well known that a stationary loop current flowing in a plane creates a magnetic field which at large distances may be described as resulting from a dipole of moment,

$$\vec{\mu} = I\vec{S} \quad (4.9)$$

where I is the current and \vec{S} is the area of the loop. The direction of the magnetic dipole is perpendicular to the plane of the loop. Employing this, let us consider the magnetic dipole moment associated with an electron described in a circular orbit of radius r (Fig. 4.4). Let, the angular velocity of the electron being ω_0 . Thus the loop current in this case will be

$$I = -\frac{e\omega_0}{2\pi} = -\frac{e}{T} \quad (4.10)$$

where T is the time period. Thus the magnetic dipole moment of the related electron orbit is,

$$\vec{\mu} = -\frac{e}{T}\pi r^2 \hat{n} \quad (4.11)$$

where πr^2 is the loop area.

Now, we have the angular momentum of the electron,

$$\vec{L} = I\vec{\omega}_0 = mr^2 \frac{2\pi}{T} \hat{n} \quad (4.12)$$

Relating the magnetic dipole moment to the angular momentum of the electron, we get

$$\vec{\mu} = -\frac{e\vec{L}}{2m} \quad \left[\because \frac{\pi r^2}{T} \hat{n} = \frac{\vec{L}}{2m} \right]$$

$$\text{or,} \quad \vec{\mu} = g\vec{L} \quad (4.13)$$

where $g = -\frac{e}{2m}$ is known as the *gyromagnetic ratio*.

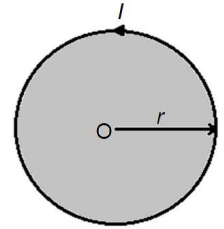


Fig. 4.4: Electronic orbit of radius r

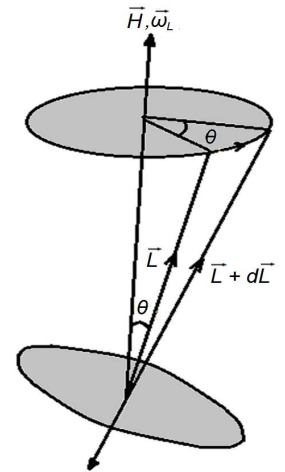


Fig. 4.5: L precessing about H with Larmor frequency

Let us now consider the influence of magnetic field on the motion of an electron in an atom quantitatively. Let us consider an arbitrary direction for the angular momentum vector \vec{L} relative to the magnetic field \vec{B} . We can express the magnetic moment as,

$$\vec{\mu} = -\frac{e\vec{L}}{2m} \quad (4.14)$$

This magnetic moment will produce a torque $\vec{\mu} \times \vec{B}$ on the dipole, so that according to Newtonian mechanics we can write,

$$\frac{d\vec{L}}{dt} = \vec{\mu} \times \vec{B} = -\frac{e}{2m} \vec{L} \times \vec{B} = -\vec{L} \times \frac{e}{2m} \vec{B} = \vec{\omega}_L \times \vec{L} \quad (4.15)$$

This is the equation of motion of a vector \vec{L} , precessing about \vec{B} (Fig. 4.5) with an angular frequency,

$$\vec{\omega}_L = \frac{e}{2m} \vec{B} \quad (4.16)$$

where $\vec{\omega}_L$ is called *Larmor frequency*. For a precession like that said above, from Fig. 4.5 we have,

$$dL = L \sin \theta d\theta = L \sin \theta \omega_L dt$$

$$\therefore \frac{dL}{dt} = \omega_L L \sin \theta = \left| \vec{\omega}_L \times \vec{L} \right| \quad (4.17)$$

In an external field, the plane of the orbit is not stationary, but precesses about \vec{B} . As a result of the charge of the electron, the precession produces an induced magnetic moment with a component opposite to that of \vec{B} .

The induced magnetic moment is given by,

$$\vec{\mu}_{in} = -\frac{e}{2m} \vec{L}_{in} = -\frac{e}{2m} I \vec{\omega}_L = -\frac{e}{2m} m \bar{\rho}^2 \frac{e}{2m} \vec{B}$$

$$\text{or, } \vec{\mu}_{in} = -\frac{\mu_0 e^2}{4m} \bar{\rho}^2 \vec{H} \quad (4.18)$$

where $\bar{\rho}^2$ represents the mean square radius of the projection of the orbit on a plane perpendicular to $\vec{H} (= \vec{B} / \mu_0)$.

For a spherical charge distribution,

$$\bar{x}^2 = \bar{y}^2 = \bar{z}^2$$

Furthermore, $\bar{r}^2 = \bar{x}^2 + \bar{y}^2 + \bar{z}^2$ and $\bar{\rho}^2 = \bar{x}^2 + \bar{y}^2$

$$\therefore \bar{r}^2 = \frac{3\bar{\rho}^2}{2} \text{ (for spherical symmetry)} \quad (4.19)$$

Hence the induced magnetic moment is,

$$\overline{\mu}_{in} = -\frac{\mu_0 e^2}{4m} \frac{2\overline{r}^2}{3} \overline{H} = -\frac{\mu_0 e^2 \overline{r}^2}{6m} \overline{H} \quad (4.20)$$

Thus the induced diamagnetic moment is found to be proportional to \overline{H} and is opposite to that of \overline{H} . When this treatment is extended to solid containing N atoms per m^3 , each atom containing Z electrons, we get the diamagnetic susceptibility *i.e.*, the induced moment per unit volume (m^3) per unit magnetic field as,

$$\chi_{dia} = -\frac{N\mu_0 Z e^2}{6m} \overline{r}^2 \quad (4.21)$$

For an electron, with $\overline{r} = 0.1 \text{ nm}$ and $N = 5 \times 10^{28} / \text{m}^3$, $\chi_{dia} = -3 \times 10^{-6}$.

Eq. (4.21) is known as the *Langevin equation* for diamagnetism. From this equation it is clear that,

- i) the outer electrons of atoms make the largest contribution to χ_{dia} as $\chi_{dia} \propto \overline{r}^2$,
- ii) χ_{dia} increases with the number of atoms,
- iii) χ_{dia} is always negative and is independent of the temperature and
- iv) χ_{dia} is independent of the intensity of the external magnetic field.

EXAMPLE 4.10

Example 4.10 If a circular loop of radius 4 cm carries a current of 100 mA, calculate the magnetic moment associated with it.

Solution

Here, $r = 4 \text{ cm}$ and $I = 100 \text{ mA}$.

Thus, the magnetic moment associated with the loop is

$$\mu = IA = \pi r^2 I = 3.14 \times (0.04)^2 \times 100 \times 10^{-3} = 5.024 \times 10^{-4} \text{ Am}^2.$$

EXAMPLE 4.11

Example 4.11 Assuming $\overline{r} = 5.3 \times 10^{-11} \text{ m}$ and $N = 5 \times 10^{28} / \text{m}^3$, determine the susceptibility of hydrogen atom.

Solution

Here $N = 5 \times 10^{28} / \text{m}^3$, $\overline{r} = 5.3 \times 10^{-11} \text{ m}$ and $Z = 1$.

Thus the diamagnetic susceptibility of hydrogen atom is given by

$$\begin{aligned} \chi_H &= -\frac{N\mu_0 Z e^2}{6m} \overline{r}^2 \\ &= -\frac{5 \times 10^{28} \times 4\pi \times 10^{-7} \times 1 \times (1.6 \times 10^{-19})^2}{6 \times 9.1 \times 10^{-31}} \times (5.3 \times 10^{-11})^2 \\ &= -8.27 \times 10^{-7}. \end{aligned}$$

4.10 LANGEVIN'S THEORY OF PARAMAGNETISM

According to quantum theory of paramagnetism as proposed by Langevin in 1905, the permanent magnetic moment of a given atom or ion is not freely rotating but restricted to a finite set of orientation relative to the applied field. Let us consider a medium containing N atoms per unit volume, the total angular momentum quantum number of each atom being J . Accordingly the magnetic moment will be,

$$\mu = g\mu_B M_J \quad (4.22)$$

where $M_J = J, (J-1), (J-2), \dots, -J$.

Here M_J is the magnetic quantum number associated with J .

The potential energy of a magnetic dipole with a component $\mu = g\mu_B M_J$ along H is $= -g\mu_B M_J H$.

So, according to statistical mechanics, the magnetization is given by,

$$M = \sum_J N P_J \mu_J \quad (4.23)$$

where N is the total number of particles and P_J is the probability distribution given by,

$$P_J = \frac{e^{-E_J/kT}}{\sum_J e^{-E_J/kT}} \quad (4.24)$$

Thus we have the magnetization per unit volume as,

$$M = Ng\mu_B \frac{\sum_{-J}^{+J} M_J e^{-E_J/kT}}{\sum_{-J}^{+J} e^{-E_J/kT}} \quad [\because \mu_J = g\mu_B M_J] \quad (4.25)$$

where $E_J = -M_J g\mu_B H$ (4.26)

$$\therefore M = Ng\mu_B \frac{\sum_{-J}^{+J} M_J e^{\frac{g\mu_B M_J H}{kT}}}{\sum_{-J}^{+J} e^{\frac{g\mu_B M_J H}{kT}}} \quad (4.27)$$

Now, two cases may arise:

Case I: When $g\mu_B M_J H \ll kT$

Under this condition the exponentials in Eq. (4.27) may be approximated as, $e^{\frac{g\mu_B M_J H}{kT}} = 1 + \frac{g\mu_B M_J H}{kT}$ [for moderate temperature and field] and we can find the magnetization per unit volume as,

$$\begin{aligned}
M &= Ng\mu_B \frac{\sum_{-J}^{+J} M_J \left(1 + \frac{g\mu_B M_J H}{kT}\right)}{\sum_{-J}^{+J} \left(1 + \frac{g\mu_B M_J H}{kT}\right)} = Ng\mu_B \frac{g\mu_B H}{kT} \frac{\sum_{-J}^{+J} M_J^2}{2J+1} \\
&= \frac{Ng^2\mu_B^2 H}{kT} \frac{J(J+1)(2J+1)}{3(2J+1)} \\
&\quad \left[\because \sum_{-J}^{+J} M_J^2 = 2 \sum_0^{+J} M_J^2 = 2 \frac{J(J+1)(2J+1)}{6} = \frac{J(J+1)(2J+1)}{3} \right]
\end{aligned}$$

or,

$$M = \frac{Ng^2\mu_B^2 J(J+1)H}{3kT} \quad (4.28)$$

Thus, we get the paramagnetic susceptibility as,

$$\chi = \frac{M}{H} = \frac{Ng^2\mu_B^2 J(J+1)}{3kT} = \frac{N\mu_J^2}{3kT} \quad (4.29)$$

where,

$$\mu_J^2 = g^2\mu_B^2 J(J+1) \quad (4.30)$$

or,

$$\chi = \frac{C}{T} \quad (4.31)$$

where,

$$C = \frac{N\mu_J^2}{3k} \quad (4.32)$$

is a constant known as the Curie constant. Eq. (4.31) is identical with the classical results and is the well known Curie law which states that the susceptibility of paramagnetic substances varies inversely as the absolute temperature.

Total magnetic moment μ_J associated with J is given by,

$$\mu_J^2 = g^2\mu_B^2 J(J+1) = p_{eff}^2 \mu_B^2 \quad (4.33)$$

where

$$p_{eff}^2 = g^2 J(J+1) \quad (4.34)$$

Thus we note that from susceptibility measurements in the range where the Curie law holds, it is possible to determine the effective number of Bohr magnetons as,

$$p_{eff} = g\sqrt{J(J+1)} \quad (4.35)$$

It is a theoretical result. Experimentally,

$$\chi = \frac{N\mu_J^2}{3kT} = \frac{Np_{eff}^2 \mu_B^2}{3kT} \quad (4.36)$$

\therefore

$$p_{eff} = \sqrt{\frac{3kT\chi}{N}} \frac{1}{\mu_B} \quad (4.37)$$

Case II: When $g\mu_B M_J H > kT$

Under this condition *i.e.*, at low temperatures and strong magnetic fields, we get the magnetization per unit volume as,

$$M = Ng\mu_B \frac{\sum_{-J}^{+J} M_J e^{\frac{g\mu_B M_J H}{kT}}}{\sum_{-J}^{+J} e^{\frac{g\mu_B M_J H}{kT}}}$$

Let, $\frac{g\mu_B H}{kT} = x$

$$\begin{aligned} \text{Thus, } M &= Ng\mu_B \frac{\sum_{-J}^{+J} M_J e^{M_J x}}{\sum_{-J}^{+J} e^{M_J x}} = Ng\mu_B \frac{d}{dx} \ln \sum_{-J}^{+J} e^{M_J x} \\ &= Ng\mu_B \frac{d}{dx} \ln \left[e^{Jx} + e^{(J-1)x} + \dots + e^{-Jx} \right] \\ &= Ng\mu_B \frac{d}{dx} \ln \left[e^{Jx} \left\{ 1 + e^{-x} + e^{-2x} + \dots + e^{-2Jx} \right\} \right] \\ &= Ng\mu_B \frac{d}{dx} \ln \left[e^{Jx} \frac{1 - (e^{-x})^{2J+1}}{1 - e^{-x}} \right] = Ng\mu_B \frac{d}{dx} \ln \left[\frac{e^{\left(J+\frac{1}{2}\right)x} - e^{-\left(J+\frac{1}{2}\right)x}}{e^{\frac{x}{2}} - e^{-\frac{x}{2}}} \right] \\ &= Ng\mu_B \frac{d}{dx} \ln \left[\frac{\sinh \left(J + \frac{1}{2} \right) x}{\sinh x} \right] = Ng\mu_B \left[\left(J + \frac{1}{2} \right) \coth \left(J + \frac{1}{2} \right) x - \frac{1}{2} \coth \frac{x}{2} \right] \\ &= Ng\mu_B J \left[\frac{2J+1}{2J} \coth \left(\frac{2J+1}{2} \right) x - \frac{1}{2J} \coth \frac{x}{2} \right] \\ &= Ng\mu_B J \left[\frac{2J+1}{2J} \coth \left(\frac{2J+1}{2J} \right) xJ - \frac{1}{2J} \coth \frac{xJ}{2J} \right] \\ &= Ng\mu_B J \left[\frac{2J+1}{2J} \coth \left(\frac{2J+1}{2J} \right) y - \frac{1}{2J} \coth \frac{y}{2J} \right] \end{aligned}$$

$$\text{or, } M = Ng\mu_B JB_J(y) \quad (4.38)$$

$$\text{where } y = xJ = \frac{g\mu_B HJ}{kT} \quad (4.39)$$

Here
$$B_J(y) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J}y\right) - \frac{1}{2J} \coth\frac{y}{2J} \quad (4.40)$$

is the *Brillouin function*. Fig. 4.6 shows a plot of this function with y .

As $y \rightarrow \infty$ (i.e., $T \rightarrow 0, H \rightarrow \infty, J \rightarrow \infty$)

$$B_J(y) \rightarrow 1$$

and we get,

$$M = Ng\mu_B J = N\mu_J = M_S \quad (4.41)$$

In this situation all dipoles are aligned parallel to the magnetic field. The corresponding magnetization M_S in the material is called the saturation magnetization.

Failure of Langevin's theory of paramagnetism: Langevin's theory of paramagnetism assumes that the dipoles are large distant apart so that they have negligible mutual interaction. So, this theory fails to explain the large mutual interaction of paramagnetic dipoles and hence the complicated temperature dependence of paramagnetic susceptibility. Later Weiss modified the Langevin's theory with his molecular field theory.

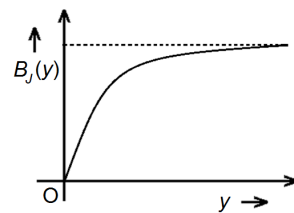


Fig. 4.6: $B_J(y)$ - y curve



EXAMPLE 4.12

Example 4.12 A paramagnetic material has 10^{28} atoms/m³. Its susceptibility at 330 K is 3.7×10^{-4} . Calculate the susceptibility at 300 K.

Solution

From Curie law we have, $\frac{\chi_1}{\chi_2} = \frac{T_2}{T_1}$

Here, $T_1 = 330\text{K}$, $T_2 = 300\text{K}$ and $\chi_1 = 3.7 \times 10^{-4}$.

$$\begin{aligned} \text{Thus, } \chi_2 &= \frac{T_1}{T_2} \chi_1 = \frac{330}{300} \times 3.7 \times 10^{-4} \\ &= 4.07 \times 10^{-4}. \end{aligned}$$

EXAMPLE 4.13

Example 4.13 Calculate the effective Bohr magneton for Gd^{+3} ($4f^7 5s^2 5p^6$).

Solution

Here, $L = 0$ and $J = L + S = S = 7/2$.

Thus, the effective Bohr magneton for Gd^{+3} is

$$\begin{aligned} \mu_{\text{eff}} &= g\sqrt{J(J+1)} \\ &= 2\sqrt{\frac{7}{2} \times \frac{9}{2}} = 7.94. \end{aligned}$$

Example 4.14 Find the total magnetic moment for an ion in $2P_{1/2}$ state.

Solution

For, $2P_{1/2}$ state $L = 1$, $S = \frac{1}{2}$ and $J = \frac{1}{2}$.

$$\text{Thus, } g = 1 + \frac{\frac{1}{2}\left(\frac{1}{2}+1\right) + \frac{1}{2}\left(\frac{1}{2}+1\right) - 1(1+1)}{2 \times \frac{1}{2}\left(\frac{1}{2}+1\right)} = 1 + \frac{\frac{3}{2} - 2}{\frac{3}{2}} = \frac{2}{3}.$$

So, the total magnetic moment

$$\begin{aligned} \mu_J &= p_{\text{eff}} \mu_B = g \sqrt{J(J+1)} \mu_B \\ &= \frac{2}{3} \sqrt{\frac{1}{2} \times \frac{3}{2}} \times 9.27 \times 10^{-24} \text{ Am}^2 = 16.05 \times 10^{-24} \text{ Am}^2. \end{aligned}$$

EXAMPLE 4.14

4.11 CURIE-WEISS LAW

Weiss in 1907 modified the Langevin theory by postulating that the effective magnetizing field consists of the externally applied magnetic field H , which orients the molecules plus an internal or molecular field H_{int} resulting from the interaction between the neighboring atomic dipoles. Weiss also assumed that the field H_{int} is proportional to the magnetization M i.e.,

$$H_T = H_{\text{ext}} + H_{\text{int}} = H + N_W M \quad (4.42)$$

where N_W is the Weiss constant.

Thus magnetization,

$$M = \frac{C}{T} H_T = \frac{C[H + N_W M]}{T} \quad (4.43)$$

or,

$$M \left[1 - \frac{CN_W}{T} \right] = \frac{C}{T} H$$

So, susceptibility

$$\chi = \frac{M}{H} = \frac{\frac{C}{T}}{\left[1 - \frac{CN_W}{T} \right]} = \frac{C}{T - CN_W}$$

or,

$$\chi = \frac{C}{T - \theta} \quad (4.44)$$

where $\theta = CN_W$ is the paramagnetic Curie temperature.

This is the temperature below which a paramagnetic substance behaves like a diamagnetic substance and the susceptibility of the paramagnetic substance becomes negative. Eq. (4.44) is known as the *Curie-Weiss law* of paramagnetism. It is applicable for paramagnetic material when its temperature is above the Curie temperature.



4.12 FERROMAGNETIC MATERIALS

In ferromagnetic materials, the magnetization versus magnetic field relationship exhibits hysteresis (Fig. 4.7). Ferromagnetic domains have illustrated in Fig. 4.8. It may be noted that the ferromagnetic materials can retain magnetism even after the removal of the external magnetizing field. These types of substances show very high intensity of magnetization. *Fe*, *Ni*, *Co*, *Gd* and *Dy* are ferromagnetic although there are relatively large number of ferromagnetic alloys and oxides. Susceptibility of these types of materials is a function of magnetic field and absolute temperature. Above the critical temperature θ_f known as the ferromagnetic Curie temperature, the spontaneous magnetization vanishes and the material becomes paramagnetic.

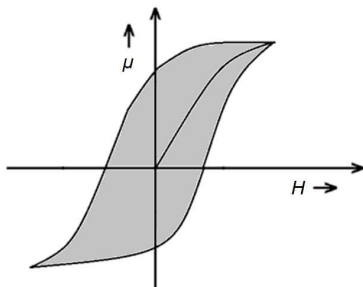


Fig. 4.7: μ - H (hysteresis) curve for ferromagnetic substances

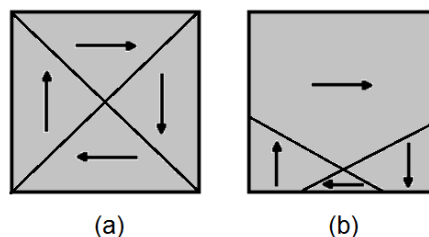


Fig. 4.8: Ferromagnetic domains

Far above the Curie temperature the susceptibility follows the *Curie-Weiss law* of paramagnetism,

$$\chi = \frac{C}{T - \theta}$$

where the symbol C is the so called Curie constant, the temperature θ is the Curie temperature for paramagnetic substance which is generally some degrees $> \theta_f$.



4.12.1 Domain Theory

The domain theory of ferromagnetism is centered about the following two hypotheses put forward in 1907 by Weiss:

- i) A ferromagnetic specimen of macroscopic dimension contains in general a number of small regions (domains) which are spontaneously magnetized; the magnitude of spontaneous magnetization of the specimen is determined by the vector sum of the magnetic moments of each domain. Ferromagnetic substances are intrinsically magnetized even in the absence of an external magnetic field due to the presence of these domains. Generally a piece of a magnetic material (e.g., iron) does not behave as a magnet as the magnetic domains are so arranged within it that their effect is neutralized resulting in a zero net magnetic moment [Fig. 4.8 (a)]. As shown in Fig. 4.8 (a), the domains also have equal size in absence of any magnetic field.
- ii) When small magnetic field is applied, the size of the domain which has their magnetic orientation along the magnetic field increases at the expense of those which are not oriented

favorably [Fig. 4.8 (b)]. Within each domain the spontaneous magnetization is due to the existence of a *molecular field* which tends to produce a parallel alignment of the atomic dipole resulting in a non-zero net magnetic moment. As the strength of the magnetic field is further increased, all the dipoles are oriented parallel to the magnetizing field in a single domain and the magnetism reaches a saturation stage at a particular magnetic field beyond which there is no increase in magnetism.

4.13 WEISS MOLECULAR FIELD THEORY

For the derivation of molecular field theory of ferromagnetism Weiss put forward the following two hypotheses:

- i) Magnetic moments of domains are randomly distributed and thus the net value may be zero.
- ii) There is some type of feeble internal magnetic field which helps to align the atomic magnets in one direction.

Spontaneous magnetization implies cooperation between the atomic dipoles within a single domain *i.e.*, there must be some kind of interaction between the atoms which produces the tendency for parallel alignment of the atomic magnetic dipoles. In order to obtain a phenomenological description of spontaneous magnetization, Weiss assumed that the molecular field H_T acting on a given dipole may be written in the form,

$$H_T = H_{ext} + H_{int} = H_{ext} + N_W M \quad (4.45)$$

where H_{ext} is the applied field, H_{int} is proportional to the magnetization M *i.e.*, $H_{int} = N_W M$ which provides the cooperative effect and N_W is the Weiss molecular field constant.

Now, consider a solid containing N atoms per unit volume, each with a total angular momentum quantum number J , which includes the total orbital and spin contribution (L and S). Then the net magnetization will be,

$$M = Ng\mu_B JB_J(y) \quad (4.46)$$

where
$$B_J(y) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J}y\right) - \frac{1}{2J} \coth\frac{y}{2J} \quad (4.47)$$

and
$$y = xJ = \frac{g\mu_B HJ}{kT} = \frac{\mu_J H}{kT} \quad [\mu_J = g\mu_B J]$$

or,
$$y = \frac{\mu_J [H_{ext} + N_W M]}{kT} \quad (4.48)$$

Now, two cases may arise:

Case I: When $T < \theta_f$: As long as we are interested in spontaneous magnetization $H_{ext} = 0$ and we may write the saturation magnetization as,

$$M_S = N\mu_J B_J\left(\frac{\mu_J N_W M_S}{kT}\right) = M_m B_J\left(\frac{\mu_J N_W M_S}{kT}\right) \quad (4.49)$$

where $M_m = N\mu_J$ is the maximum value of magnetization *i.e.*, the total magnetization.

$$\therefore \frac{M_S}{M_m} = B_J \left(\frac{\mu_J N_W M_S}{kT} \right) \quad (4.50)$$

Again we have

$$\frac{M_S}{M_m} = \frac{y k T}{\mu_J N_W M_m} \quad (4.51)$$

Since M_S must satisfy both the above equations, its value at a given temperature may be obtained from the point of intersection of the corresponding M_S vs y curve as shown in Fig.

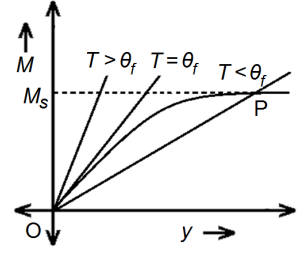


Fig. 4.9: M - y curve for (a) $T < \theta_f$, (b) $T = \theta_f$ and (c) $T > \theta_f$

4.9. Eq. (4.51) represents a straight line, the slope of the line being proportional to temperature T . From the curve it follows that for $T < \theta_f$, we get non-vanishing values of M although $H_{ext} = 0$.

Hence, for $T < \theta_f$ spontaneous magnetization is the result. For $T = \theta_f$ the slope of the straight line given by Eq. (4.50) is equal to that of the tangent of curve for Eq. (4.51) at the origin. Thus for $T \geq \theta_f$, the spontaneous magnetization vanishes and the substance behaves like a paramagnet. It is evident that there must be a relation between the Curie temperature θ_f and the molecular field constant N_W . In fact, one expects θ_f to increase with N_W because the tendency for parallel alignment increases as N_W becomes larger. Now to establish a relationship between θ_f and N_W let for $y \ll 1$ (near the origin of Fig. 4.9), the Brillouin function is approximately given by,

$$B_J(y) = \frac{J+1}{3J} y \quad (4.52)$$

So, from Eq. (4.51) and Eq. (4.52) we get,

$$B_J(y) = \frac{y k \theta_f}{\mu_J N_W M_m} = \frac{J+1}{3J} y \quad (4.53)$$

$$\begin{aligned} \text{or, } \theta_f &= \frac{\mu_J N_W (J+1) M_m}{3Jk} = \frac{g \mu_B J N_W (J+1) N \mu_J}{3Jk} \\ &= \frac{Ng \mu_B N_W (J+1) g \mu_B J}{3k} = \frac{Ng^2 \mu_B^2 J(J+1) N_W}{3k} \\ &= \frac{N \mu_J^2 N_W}{3k} \end{aligned} \quad (4.54)$$

$$\text{or, } \theta_f = C N_W \quad (4.55)$$

$$\text{where, } C = \frac{N \mu_J^2}{3k} \text{ is a constant.} \quad (4.56)$$

$$\text{Now, } \frac{M_S}{M_m} = B_J \left(\frac{\mu_J N_W M_S}{kT} \right) = B_J \left(\frac{M_S}{T} \frac{3J}{J+1} \frac{\theta_f}{M_m} \right) \quad (4.57)$$

Similarly from Eq. (4.51),

$$\frac{M_s}{M_m} = \frac{y k T}{\mu_J N_W M_m} = \frac{y k T}{\mu_J^2 N_W N} = \frac{y k T}{g^2 \mu_B^2 J^2 N_W N} = \frac{y T (J + 1)}{3 J \theta_f} \quad (4.58)$$

It is important to note that for a given value of J , we can get a universal curve when $\frac{M_s}{M_m}$ is plotted as a function of $\frac{T}{\theta_f}$ which is evident from Eq. (4.57) and Eq. (4.58). In Fig. 4.10 we have represented such curves for $J = \frac{1}{2}$, $J = 1$ and $J = \infty$, the latter case corresponds to classical freely rotating dipoles. $J = \frac{1}{2}$ curve is

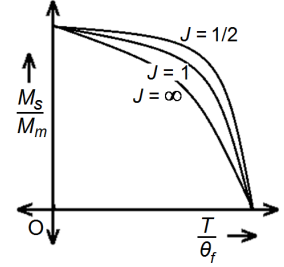


Fig. 4.10: M_s/M_m - T/θ_f curve for different J values

best fitted with the experimental results, indicating that the magnetization is essentially associated with the electron spin rather than with the orbital momentum of the electrons. That this indeed the case has been confirmed by gyromagnetic experiments. In such experiments, one either reverses the magnetization of a freely suspended specimen or observes the resulting rotation or one rotates the specimen to observe the resulting magnetization; the former called the Einstein-de-Hass method and the latter is the Barnett method. From such experiments, one obtain the g -value *i.e.*, the ratio between the magnetic moment and the angular momentum.

For the electron spin $g = 2$ and for the orbital motion $g = 1$.

Case II: When $T > \theta_f$: Here,

$$M = NgJ\mu_B B_J(y) = NgJ\mu_B \frac{(J+1)y}{3J} \quad \left[\because B_J(y) = \frac{J+1}{3J} y \quad \text{for } y \ll 1 \right]$$

$$\text{or,} \quad M = NgJ\mu_B \frac{(J+1)}{3J} \frac{gJ\mu_B H}{kT} = \frac{N\mu_J^2}{3kT} H \quad \left[\because y = \frac{gJ\mu_B H}{kT} \right]$$

$$\text{or,} \quad M = \frac{N\mu_J^2}{3kT} [H_{ext} + N_W M]$$

$$\text{or,} \quad M \left[1 - \frac{N\mu_J^2 N_W}{3kT} \right] = \frac{N\mu_J^2}{3kT} H_{ext} \quad (4.59)$$

Thus the magnetic susceptibility is given by,

$$\chi = \frac{M}{H_{ext}} = \frac{\frac{N\mu_J^2}{3kT}}{\left[1 - \frac{N\mu_J^2 N_W}{3kT} \right]} = \frac{\frac{N\mu_J^2}{3k}}{\left[T - \frac{N\mu_J^2}{3k} N_W \right]}$$

$$\text{or,} \quad \chi = \frac{C}{T - \theta_f} \quad (4.60)$$

where, $C = \frac{N\mu_J^2}{3k}$ is a constant

and $\theta_f = CN_W$. (4.61)

EXAMPLE 4.15

Example 4.15 Calculate the saturation magnetization for a rare earth element having 10^{27} atoms per unit volume, if the magnetic moment of the element is 1 Bohr magneton.

Solution

Here, $N = 10^{27}$ atoms/m³ and $M_m = 9.27 \times 10^{-24}$ Am².

So, the saturation magnetization of the material is

$$\begin{aligned} M_S &= N\mu_0 M_m = 10^{27} \times 4\pi \times 10^{-7} \times 9.27 \times 10^{-24} \\ &= 1.164 \times 10^{-2} \text{ Wb/m}^2. \end{aligned}$$

EXAMPLE 4.16

Example 4.16 Calculate the magnetic moment of a material with 10^{27} atoms per unit volume, if the saturation magnetization in it is 1 Wb/m².

Solution

Here, $N = 10^{27}$ atoms/m³ and $M_S = 1$ Wb/m².

So, the magnetic moment of the material is

$$M_m = \frac{M_S}{N\mu_0} = \frac{1}{10^{27} \times 4\pi \times 10^{-7}} = 7.96 \times 10^{-24} \text{ Am}^2.$$

4.14 DIFFERENCES OF THREE MAGNETIC MATERIALS

Diamagnetic materials	Paramagnetic materials	Ferromagnetic materials
i) Slightly repelled by magnets. ii) Magnetic susceptibility is independent of the absolute temperature. iii) Magnetic permeability $\mu < 1$. iv) Magnetic susceptibility $\chi < 1$. v) Does not show hysteresis. vi) Does not possess Curie temperature. vii) Eg. H, He, Au, Bi, Sb, Cu, H ₂ O etc.	i) Slightly attracted by magnets. ii) Magnetic susceptibility is inversely proportional to the absolute temperature. iii) Magnetic permeability $\mu > 1$. iv) Magnetic susceptibility $\chi > 1$. v) Does not show hysteresis. vi) Does not possess Curie temperature. vii) Eg. Na atoms, gaseous NO etc.	i) Strongly attracted by magnets. ii) Magnetic susceptibility shows complex dependence on temperature. iii) Magnetic permeability $\mu \gg 1$. iv) Magnetic susceptibility $\chi \gg 1$. v) Shows hysteresis. vi) Have a finite Curie temperature. vii) Eg. Fe, Ni, Co etc.

4.15 B-H CURVE

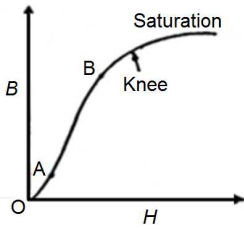


Fig. 4.11: B-H curve of a ferromagnetic substance

The B - H curve (or magnetization curve) of ferromagnetic substance indicates the manner in which the flux density (B) varies with magnetizing force (H).

Fig. 4.11 shows the general shape of B - H curve of a ferromagnetic material. The non-linearity of the curve indicates that the relative permeability $\mu_r (= B/\mu_0 H)$ of a ferromagnetic material is not constant; but depends very largely upon the flux density. While carrying out magnetic calculations, it should be ensured that the value of μ_r and H are taken at the working flux density.

For this purpose, the B - H curve of the material in question may be very helpful. The use of B - H curves permits the calculations of magnetic circuits with a fair degree of ease.

4.15.1 Magnetic Calculations from B-H Curves

The solution of magnetic circuits can be easily obtained by using B - H curves in the following way:

- i) Corresponding to the flux density B in the material, finding the magnetizing force H from the B - H curve of the material,
- ii) Computing the magnetic length l and
- iii) Computing the $m.m.f.$ required ($= H l$).

4.16 MAGNETIC HYSTERESIS

When a ferromagnetic material is experienced a cycle of magnetization, the flux density B in the material lags behind the applied magnetizing force H . This phenomenon is called *hysteresis*. The phenomenon of lagging of flux density (B) behind the magnetizing force (H) in a magnetic material subjected to cycles of magnetization is known as *magnetic hysteresis*.

4.16.1 Hysteresis Loop

Consider an un-magnetized iron bar AB wound with N turns as shown in Fig. 4.12 (a). The magnetizing force $H (= NI/l)$ produced by this solenoid can be changed by varying the current through the coil. When the iron piece is subjected to one cycle of magnetization, the resultant B - H curve traces a loop $abcdefa$ called *hysteresis loop*.

- i) When current in the solenoid is zero, $H = 0$, as H is increased the flux density (B) also increases until the point of maximum flux density ($+B_{max}$) is reached. The B - H curve of iron follows the path Oa .
- ii) If H is gradually reduced the flux density B does not decrease along aO but follows the path ab . At point b , the magnetizing force H is zero but flux density in the material has a finite value $+B_r (= Ob)$ called residual flux density. In other words, B lags behind H . The greater the lag, the greater is the residual magnetism.
- iii) To de-magnetize the iron piece (*i.e.* to remove the residual magnetism Ob), the magnetizing force H is reversed by reversing the current through the coil. When H is gradually increased in the reverse direction, the B - H curve follows the path bc so that when $H = Oc$, the residual magnetism is zero. The value of $H (= Oc)$ required to wipe out residual magnetism is known as coercive force (H_c).

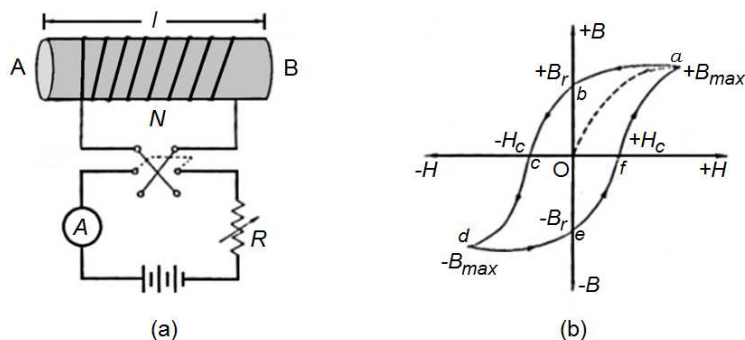


Fig. 4.12: (a) Iron bar subjected to magnetization cycle and (b) its B-H curve

- iv) If H is further increased in the negative direction, the material again saturates (point d) in the negative direction. Reducing H to zero and then increasing it in the positive direction completes the curve $defa$. Thus when an iron piece is subjected to one cycle of magnetization, the B - H curve traces a closed loop $abedefa$ called hysteresis loop. It is clear from the hysteresis loop that B lags behind H .

4.16.2 Hysteresis Loss

If we take a magnetic material and if it is subjected to a cycle of magnetization there will be an energy loss owing to the molecular friction within the material. It means the domains or the so called molecular magnets of the material resist in a way so that it is turned first in one direction and thereby to the other direction. For the purpose of overcoming this opposition energy is expended in the material. This loss comes out in the form of heat which is the well known *hysteresis loss*. This loss can be noted in different types of electrical machines whose iron parts take the role for contributing to the cycles of magnetization. Because of the effect of hysteresis loss there will be a rise of temperature of the said machine.

- Using alternating current, for practical purposes, transformers and most of the electric motors are operated. In all these devices, the flux is associated with the continuous iron changes thus producing the hysteresis loss in these types of machines.
- Hysteresis loss also occurs when an iron part rotates in a constant magnetic field *e.g.*, d.c. machines.

Calculation of Hysteresis Loss: The area of hysteresis loop represents the energy loss/m³/cycle. If the field is in free space, the stored energy is returned to the circuit when the field collapses. If the field is in a magnetic material, not all the energy supplied can be returned; part of it having been converted into heat due to hysteresis effect.

Let l = length of the iron bar, A = cross-sectional area of bar and N = no. of turns of wire of solenoid. Suppose at any instant, the current i is flowing through the solenoid.

Then,

$$H = \frac{Ni}{l}$$

(4.62)

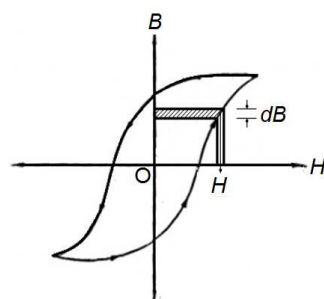


Fig. 4.13: Hysteresis loss

or,
$$i = \frac{HI}{N} \quad (4.63)$$

Suppose the current increases by di in a small time dt . This will increase flux density by dB and hence an increase in flux $d\phi (= AdB)$. This causes an *e.m.f.* e to be induced in the solenoid.

$$e = N \frac{d\phi}{dt} = NA \frac{dB}{dt}$$

By Lenz's law, this *e.m.f.* opposes the current i so that energy dW is spent in overcoming this opposing *e.m.f.* $dW = ei dt = \left(NA \frac{dB}{dt} \right) \times \left(\frac{HI}{N} \right) \times dt = Al \times H \times dB$
 $= V \times (H \times dB) \text{ J}$ where $Al = V = \text{volume of iron bar}$.

Now $H \times dB$ is the area of the shaded strip (Fig. 4.13). For one cycle of magnetization, the area $H \times dB$ will be equal to the area of hysteresis loop.

\therefore Hysteresis energy loss/cycle,

$$W_h = V \times (\text{area of loop}) \text{ joules} \quad (4.64)$$

If f is the frequency of reversal of magnetization, then hysteresis power loss,

$$P_h = W_h \times f = V \times (\text{area of loop}) \times f \quad (4.65)$$



Example 4.17 If the loss of energy/unit volume/cycle of hysteresis is 0.4 J/m^3 and the volume of the core of a transformer is 10^{-3} m^3 , find the loss of energy per hour when it is fed with AC of 50Hz.

Solution

Here, $A = 0.4 \text{ J/m}^3$, $f = 50 \text{ Hz}$ and $V = 10^{-3} \text{ m}^3$.

So, hysteresis power loss per hour = $VAf \times 3600 = 10^{-3} \times 0.4 \times 50 \times 3600 = 72 \text{ J/s}$.

EXAMPLE 4.17

Example 4.18 The hysteresis loop of a ferromagnetic specimen has an area 500 J/m^3 . If the density of the specimen is 7400 kg/m^3 and the weight of the specimen is 2 kg , find the rate of loss of energy at 100 Hz .

Solution

If f is the frequency of reversal of magnetization then hysteresis power loss,

$$P_h = W_h \times f = V \times (\text{area of loop}) \times f = VAf$$

Here, $A = 500 \text{ J/m}^3$, $f = 100 \text{ Hz}$ and $V = M/d = (2/7400) \text{ m}^3$.

So, hysteresis power loss = $\frac{2}{7400} \times 500 \times 100 = 13.51 \text{ J/s}$.

EXAMPLE 4.18

Importance of Hysteresis Loop: The shape and size of the hysteresis loop largely depends upon the nature of the material. The choice of a magnetic material for a particular application often depends upon the shape and size of the hysteresis loop.

- i) If the hysteresis loop area of a magnetic material is smaller the hysteresis loss becomes less. For silicon steel the hysteresis loop area is very small [Fig. 4.14 (a)] and that is why silicon steel is largely used for transformer cores and also for rotating machines subjected to fast reversals of magnetization.
- ii) For hard steel the hysteresis loop [Fig. 4.14 (b)] represents high retentivity and coercivity and that is why the hard steel is very suitable to prepare permanent magnets. But due to the large area of the loop, there is greater hysteresis loss. For this reason, hard steel is not suitable for the construction of electrical machines.
- iii) For wrought iron the hysteresis loop [Fig. 4.14 (c)] exhibits fairly good residual magnetism and coercivity and so it is considered appropriate for making cores of electromagnets.

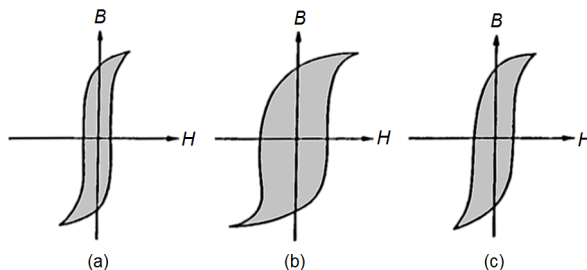


Fig. 4.14: Hysteresis curve for (a) silicon steel, (b) hard steel and (c) wrought iron

The shape and size of the hysteresis loop also depends upon

- a) the maximum value of flux density established and
- b) the initial magnetic state of the material.

4.17 ANTIFERROMAGNETISM

The theory of ferromagnetism is based on the assumption that the exchange integral is positive. When the exchange integral is negative (antiparallel orientation of neighboring spins), we get an antiferromagnetic substance. Such systems were first investigated theoretically by Neel and Bitter and extended by Van Vleck. In 1938, Bizette, Squire and Tsai first experimentally discovered it as a property of MnO . The antiferromagnetic substance MnF_2 shows a maximum susceptibility with temperature. This is shown in Fig. 4.15.

The most direct experimental evidence for the basic picture of antiferromagnetism has been confirmed from neutron diffraction experiments. When neutrons are incident on a crystal they are not only scattered by the atomic nuclei but also by interacting between the neutron spin and paramagnetic ions which may be present. Consequently, the ordered antiferromagnetic state gives rise to extra diffraction lines just as one observe extra X-ray diffraction lines for ordered alloys.

The intensity of these extra lines decreases as the temperature increases because the antiferromagnetic order diminishes. Above the antiferromagnetic temperature the extra lines disappear. For antiferromagnet below the Neel temperature, the atoms are spontaneously magnetized and above it the substance becomes a paramagnet.

4.18 NEEL'S MOLECULAR FIELD THEORY

Let us consider the two sub-lattice model in which all the nearest neighbors of an atom A is found to be an atom B and *vice-versa* (Fig. 4.16). However, we shall assume that, besides an antiferromagnetic AB interaction, there are also antiferromagnetic AA and BB interactions. For an N atom system, there should be $N/2$ atoms for A sub-lattice and $N/2$ atoms for B sub-lattice.

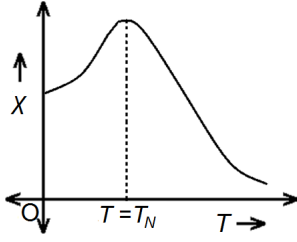


Fig. 4.15: χ - T curve for anti-ferromagnetic material

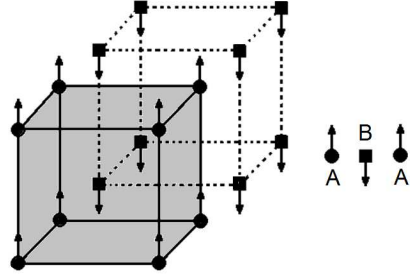


Fig. 4.16: Two sublattice model: Neel's theory

Now, the molecular internal field at A site is,

$$\vec{H}_{mA} = -N_{AB}\vec{M}_B - N_{AA}\vec{M}_A = -N_{AB}\vec{M}_B - N_{ii}\vec{M}_A \quad (4.66)$$

and the molecular internal field at B site is,

$$\vec{H}_{mB} = -N_{BA}\vec{M}_A - N_{BB}\vec{M}_B = -N_{AB}\vec{M}_A - N_{ii}\vec{M}_B \quad (4.67)$$

where $N_{AA} = N_{BB} = N_{ii}$ and $N_{AB} = N_{BA}$ are the molecular field constants.

Now the field at A site is thus,

$$\vec{H}_A = \vec{H} - N_{AB}\vec{M}_B - N_{ii}\vec{M}_A \quad (4.68)$$

and the field at B site is thus,

$$\vec{H}_B = \vec{H} - N_{AB}\vec{M}_A - N_{ii}\vec{M}_B \quad (4.69)$$

Now, two temperature regions may be discussed:

Case I: When $T > T_N$

For this case *i.e.*, when the temperature is above the Neel temperature, we are far apart from saturation and the magnetization of the A lattice may be written as,

$$\begin{aligned} \vec{M}_A &= \frac{N}{2} g \mu_B S_A B_S(x_A) = \frac{N}{2} g \mu_B S \frac{S+1}{3S} x_A \\ &= \frac{N}{2} g \mu_B \frac{S+1}{3} \frac{g \mu_B S_A \vec{H}_A}{kT} \quad \left[\because x_A = \frac{g \mu_B S_A \vec{H}_A}{kT} \right] \\ &= \frac{Ng^2 \mu_B^2 (S+1)}{6kT} \vec{H}_A \\ &= \frac{Ng^2 \mu_B^2 (S+1)}{6kT} \left[\vec{H} - N_{AB}\vec{M}_B - N_{ii}\vec{M}_A \right] \\ &= \frac{C}{2T} \left[\vec{H} - N_{AB}\vec{M}_B - N_{ii}\vec{M}_A \right] \quad \left[\because C = \frac{Ng^2 \mu_B^2 (S+1)}{3k} \right] \end{aligned} \quad (4.70)$$

Similarly, the magnetization of the B lattice may be written as,

$$\begin{aligned}
 \bar{M}_B &= \frac{N}{2} g \mu_B S_B B_S(x_B) = \frac{N}{2} g \mu_B S \frac{S+1}{3S} x_B \\
 &= \frac{N}{2} g \mu_B \frac{S+1}{3} \frac{g \mu_B S_B \bar{H}_B}{kT} \left[\because x_B = \frac{g \mu_B S_B \bar{H}_B}{kT} \right] \\
 &= \frac{N g^2 \mu_B^2 (S+1)}{6kT} \bar{H}_B = \frac{N g^2 \mu_B^2 (S+1)}{6kT} [\bar{H} - N_{AB} \bar{M}_A - N_{ii} \bar{M}_B] \\
 &= \frac{C}{2T} [\bar{H} - N_{AB} \bar{M}_A - N_{ii} \bar{M}_B] \left[\because C = \frac{N g^2 \mu_B^2 (S+1)}{3k} \right] \quad (4.71)
 \end{aligned}$$

Thus, the total magnetization of the two sub-lattices may be expressed as,

$$\begin{aligned}
 \bar{M} &= \bar{M}_A + \bar{M}_B = \frac{C}{2T} [2H - N_{AB}(\bar{M}_A + \bar{M}_B) - N_{ii}(\bar{M}_A + \bar{M}_B)] \\
 &= \frac{C}{2T} [2H - N_{AB} \bar{M} - N_{ii} \bar{M}] \\
 &= \frac{C}{2T} [2H - (N_{AB} + N_{ii}) \bar{M}] \quad (4.72)
 \end{aligned}$$

$$\text{or, } \bar{M} \left[1 + \frac{C}{2T} (N_{AB} + N_{ii}) \right] = \frac{C}{T} \bar{H} \quad (4.73)$$

If we assume \bar{M} and \bar{H} are parallel then the equations will become scalar and we get the susceptibility as,

$$\begin{aligned}
 \chi &= \frac{\bar{M}}{\bar{H}} = \frac{\frac{C}{T}}{\left[1 + \frac{C}{2T} (N_{AB} + N_{ii}) \right]} \\
 &= \frac{C}{T + \frac{C}{2} (N_{AB} + N_{ii})}
 \end{aligned}$$

$$\text{or, } \chi = \frac{C}{T + \theta} \quad (4.74)$$

$$\text{where } \theta = \frac{C}{2} (N_{AB} + N_{ii}) \quad (4.75)$$

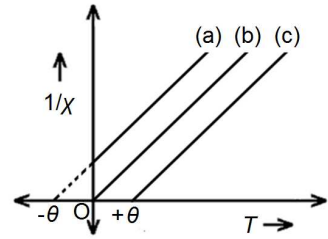


Fig. 4.17: $1/\chi$ - T curve for

(a) anti-ferromagnetic, (b) paramagnetic and (c) ferromagnetic substances

Thus when compared with the ferromagnetic susceptibility above the critical temperature, it is observed that the ferromagnetic case contains $(T + \theta)$ rather than $(T - \theta)$. Moreover, the Curie constant C is twice the Curie constant of the individual A or B lattice. In order to differentiate between the paramagnetic, ferromagnetic and antiferromagnetic behavior we can plot curve of Fig. 4.17 between $1/\chi$ versus T .

4.19 FERRIMAGNETIC MATERIALS

Ferrimagnetic materials also show anti-parallel alignment of moments (like anti-ferromagnetic material) at particular atomic site and a small difference between neighboring domains makes a possible magnetic field in the material (Fig. 4.18).

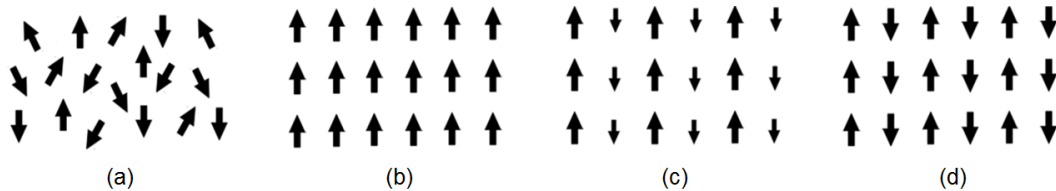


Fig. 4.18: Spin alignments for (a) paramagnetic, (b) ferromagnetic, (c) ferrimagnetic and (d) anti-ferromagnetic substances

Ferrites exhibits ferrimagnetism. It can also occur in molecular magnets. The oldest known ferrimagnetic substance is magnetite (Fe_3O_4). The Curie temperature of ferrimagnetic material (*e.g.* 580°C for magnetite) is less than that of a ferromagnetic material (*e.g.* 1131°C for Co).

These materials also follow a temperature dependence of magnetization and susceptibility near Curie point in a similar manner as in the ferromagnetic materials. They possess significantly large magnetization below the Curie temperature. Plot of M-H curves for ferromagnetic, ferromagnetic, paramagnetic, anti-ferromagnetic and diamagnetic substances is shown in Fig. 4.19.

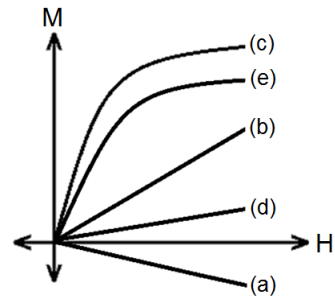


Fig. 4.19: M-H curve for (a) diamagnetic (b) paramagnetic, (c) ferromagnetic, (d) anti-ferromagnetic and (e) ferrimagnetic substances

4.20 FERRITES

Ferrites are nonmetallic, ceramic-like and described by the formula $\text{M}(\text{Fe}_x\text{O}_y)$, where M represents any metal that forms divalent bonds, such as nickel ferrite (NiFe_2O_4).

Some properties of ferrites include:

Significant saturation magnetization

- High electrical resistivity
- Low electrical losses
- Very good chemical stability

Ferrite is used in:

- Permanent magnets
- Transformers and toroidal inductors (as ferrite cores)
- Computer memory elements
- Solid-state devices

Hysteresis curve of ferrites is in the form of rectangle.

Difference between ferrites and ferromagnetic materials

Ferrites differ from ferromagnetic materials in the following manner:

- i) Magnetic induction in ferrites is much smaller than ferromagnets.
- ii) Ferrites have higher relative permeability and lower hysteresis loss compared to that of ferromagnets.
- iii) Ferrites have negligible electrical conductivity compared to that of ferromagnets.

Ferrites are classified as ‘soft magnetic materials’ and ‘hard magnetic materials’ in terms of their magnetic properties.

4.20.1 Soft Magnetic Materials

Movement of domain walls in these types of materials helps them to be magnetized and demagnetized easily. So, these types of materials can be used for making temporary magnets (*i.e.*, electromagnets). They do not possess any void and their structure is homogeneous so that the materials are not affected by impurities. Some other important properties of these types of materials are:

- i) They have low hysteresis loss due to small hysteresis area.
- ii) Susceptibility and permeability are high.
- iii) Coercivity and retentivity are less.
- iv) Stored magnetic energy is less.
- v) Due to high resistivity the loss for eddy current is less.
- vi) Due to low coercivity and retentivity, they cannot be used to make permanent magnets.

4.20.2 Hard Magnetic Materials

Instead of the removal of the external magnetic field, these types of materials can retain their magnetism and practically they are difficult to demagnetize. Hence this type of material can be used to produce permanent magnets. In permanent magnets, the magnetic materials are heated to a desired temperature and then quenched so that the movement of the domain walls can be prevented. The strength of hard magnetic materials can be increased by adding certain impurities to it. Some other important properties of these types of materials are:

- i) They have large hysteresis loss due to large hysteresis loop area.
- ii) Susceptibility and permeability are low.
- iii) Coercivity and retentivity values are large.
- iv) Stored magnetic energy is high.
- v) They exhibit high B-H value.
- vi) Loss due to eddy current is high.

UNIT SUMMARY

- **Magnetic induction or magnetic flux density (B)**
The amount of magnetic flux in an area taken perpendicular to the its direction
- **Magnetic field intensity (H)**

$$H = \frac{B}{\mu_0}$$
- **Magnetization (M)**

$$M = \frac{\mu}{V} = \frac{m}{\alpha}$$
- **Magnetic susceptibility**

$$\chi = \frac{M}{H}$$
- **Relation between B , H and M**

$$\vec{B} = \mu_0 (\vec{M} + \vec{H})$$
- **Magnetic materials and their classifications**
Diamagnetic: Permanent dipoles are absent in this type of material
Paramagnetic: This type of material possesses permanent dipoles
Ferromagnetic: This type of material possesses hysteresis
Ferrimagnetic: Above Curie temperature they become paramagnetic
Anti-ferrimagnetic: Below Curie temperature the susceptibility increases with temperature and above Curie temperature it decreases with temperature
- **Theory of permanent magnetic dipoles**
Spin angular momentum: $\mu = g \left(\frac{e}{2m} \right) \frac{\hbar}{2}$
Nuclear spin angular momentum: $\mu_n = \frac{e\hbar}{2M}$
- **Hund's rule**
All filled electronic shells are not contributing to the magnetic moment of the atom and it arises only due to the partially filled electronic shells
- **Langevin's theory of diamagnetism**

$$\vec{\mu}_{in} = -\frac{\mu_0 e^2 r^2}{6m} \vec{H} \quad \chi_{dia} = -\frac{N \mu_0 Z e^2}{6m} r^2$$
- **Quantum theory (Langevin theory) of Paramagnetism**

$$\chi = \frac{C}{T}$$
- **Weiss molecular field theory: Curie-Weiss law**

$$\chi = \frac{C}{T - \theta}$$

- **Ferromagnetism-Weiss molecular field theory of ferromagnetism**

$$\chi = \frac{C}{T - \theta_f}$$

- **Difference of diamagnetic, paramagnetic and ferromagnetic substances**

Diamagnetic: Magnetic permeability and susceptibility < 1

Paramagnetic: Magnetic permeability and susceptibility > 1

Ferromagnetic: Magnetic permeability and susceptibility $\gg 1$

- **B-H Curve**

The B-H curve (or magnetization curve) of ferromagnetic substance indicates the manner in which the flux density (B) varies with magnetizing force (H)

- **Magnetic hysteresis**

$$W_h = V \times (\text{area of loop}) \text{ Joules}$$

- **Antiferromagnetism**

For antiferromagnet below the Neel temperature, the atoms are spontaneously magnetized and above it the substance becomes a paramagnet

- **Neel's molecular field theory of anti-ferromagnetism**

$$\chi = \frac{C}{T + \theta}$$

- **Ferrimagnetic materials**

They possess significantly large magnetization below the Curie temperature

- **Ferrites**

Ferrites are nonmetallic, ceramic-like, usually ferromagnetic compounds of ferric oxide with other oxides, especially a compound characterized by extremely high electrical resistivity.

Ferrites are often classified as 'soft magnetic materials' and 'hard magnetic materials' in terms of their magnetic properties.

EXERCISES

Multiple Choice Questions

4.1 Magnetic susceptibility of ferromagnetic substance is

- (a) $\chi = \frac{1}{T}$ (b) $\chi = \frac{c}{T - \theta}$ (c) $\chi = \frac{c}{T + \theta}$ (d) none of these

4.2 The expression for Bohr magneton is

- (a) $\frac{eh}{2m}$ (b) $\frac{eh}{4m}$ (c) $\frac{eh}{4\pi m}$ (d) $\frac{eh}{2\pi m}$

4.3 Curie-Weiss law is obeyed by

- (a) paramagnetic materials (b) anti-ferromagnetic materials
(c) ferromagnetic materials above the Curie temperature
(d) ferromagnetic materials below the Curie temperature

- 4.4 Material which show negative magnetic susceptibility is
 (a) paramagnetic (b) diamagnetic (c) ferromagnetic (d) anti-ferromagnetic
- 4.5 The Curie-Weiss law for anti-ferromagnetism is
 (a) $\chi_m = \frac{C}{T - \theta_N}$ (b) $\chi_m = \frac{C}{T}$ (c) $\chi_m = \frac{C}{T + \theta_N}$ (d) $\chi_m = \frac{CT}{T + \theta_N}$
- 4.6 Paramagnetic susceptibility varies as
 (a) $1/T$ (b) T (c) T^2 (d) none of these
- 4.7 Unit of magnetic field intensity is
 (a) Am^2 (b) A/m (c) Wb/m^2 (d) Wb/m
- 4.8 \vec{B} , \vec{H} and \vec{M} are related as
 (a) $\vec{B} = \frac{\vec{H}}{\mu_0} - \vec{M}$ (b) $\vec{H} = \frac{\vec{M}}{\mu_0} - \vec{B}$ (c) $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ (d) $\vec{H} = \vec{B} - \mu_0 \vec{M}$
- 4.9 1 Bohr magneton is equal to
 (a) $7.29 \times 10^{-24} \text{ Am}^2$ (b) $7.92 \times 10^{-24} \text{ Am}^2$ (c) $9.72 \times 10^{-24} \text{ Am}^2$ (d) $9.27 \times 10^{-24} \text{ Am}^2$
- 4.10 Free space permeability is equal to
 (a) $4\pi \times 10^{-7} \text{ H/m}$ (b) $4\pi \times 10^{-7} \text{ H-m}$ (c) $4\pi \times 10^7 \text{ H-m}$ (d) $4\pi \times 10^7 \text{ H/m}$
- 4.11 Transition of ferromagnetic to paramagnetic state occurs at
 (a) Curie-Weiss temperature (b) Curie temperature
 (c) Neel temperature (d) none of these
- 4.12 For diamagnetic substances, the susceptibility is
 (a) small and positive (b) large and positive (c) small and negative (d) large and negative
- 4.13 Which of the following magnetic materials don't possess permanent magnetic dipoles?
 (a) diamagnetic (b) paramagnetic (c) ferromagnetic (d) ferrimagnetic
- 4.14 Paramagnetic materials behave like diamagnetic material
 (a) at Curie temperature (b) below Curie temperature
 (c) above Curie temperature (d) none of these
- 4.15 Transition of ferrimagnetic to ferromagnetic state occurs at temperature below
 (a) Curie-Weiss temperature (b) Curie temperature
 (c) Neel temperature (d) none of these
- 4.16 Antiparallel alignment of dipoles with unequal magnitude occurs in which of the following material?
 (a) diamagnetic (b) paramagnetic (c) ferromagnetic (d) ferrimagnetic
- 4.17 Electromagnets should have
 (a) low coercivity and low retentivity (b) low coercivity and high retentivity
 (c) high coercivity and low retentivity (d) high coercivity and high retentivity
- 4.18 Which of the following material shows hysteresis?
 (a) diamagnetic (b) paramagnetic (c) ferromagnetic (d) none of these

- 4.19 Among the following which steel is easier to magnetize?
(a) non-grain oriented (b) grain oriented (c) cast iron (d) stainless steel
- 4.20 In high frequency applications, ferrites is preferred to a ferromagnetic material because it has
(a) high permeability (b) high resistivity
(c) high saturation magnetization (d) square hysteresis loop
- 4.21 Hard ferrites are used for making
(a) transformer core (b) electrical machinery
(c) light weight permanent magnet (d) high frequency equipments
- 4.22 The temperature below which certain material are antiferromagnetic and above which they are paramagnetic is called
(a) Curie temperature (b) Neel temperature
(c) transition temperature (d) Weiss temperature
- 4.23 For antiferromagnetic substances susceptibility is maximum at
(a) Curie temperature (b) Neel temperature
(c) transition temperature (d) Weiss temperature
- 4.24 Curie-Weiss law holds good
(a) at Curie temperature (b) below Curie temperature
(c) above Curie temperature (d) above Neel temperature
- 4.25 For soft magnetic material area of B-H loop is
(a) equal to that of hard magnetic material (b) less than that of hard magnetic material
(c) greater than that of hard magnetic material (d) none of these
- 4.26 In ferromagnetic materials, susceptibility is
(a) very small and positive (b) very small and negative
(c) very large and positive (d) very large and negative
- 4.27 Materials do not contain permanent magnetic dipole moment are
(a) paramagnetic (b) ferromagnetic (c) ferrimagnetic (d) diamagnetic
- 4.28 Spontaneous magnetization is shown by the
(a) paramagnetic compounds (b) ferromagnetic compounds
(c) antiferromagnetic compounds (d) ferrimagnetic compounds
- 4.29 Ferrites are
(a) ferromagnetic materials (b) ferrimagnetic materials
(c) paramagnetic materials (d) diamagnetic materials
- 4.30 In paramagnetic materials χ_m is
(a) very small and positive (b) very small and negative
(c) very large and positive (d) very large and negative
- 4.31 Curie-Weiss law for ferromagnetism is
(a) $\chi = \frac{C}{T - \theta}$ (b) $\chi = \frac{C}{T}$ (c) $\chi = \frac{C}{T + \theta}$ (d) $\chi = \frac{CT}{T - \theta}$

- 4.32 The relative permeability of a specimen is 0.95. It is
 (a) paramagnetic (b) ferromagnetic (c) ferrimagnetic (d) diamagnetic

Answers of Multiple Choice Questions

4.1 (b), 4.2 (c), 4.3 (c), 4.4 (b), 4.5 (c), 4.6 (a), 4.7 (b), 4.8 (c), 4.9 (d), 4.10 (a), 4.11 (b), 4.12 (c), 4.13 (a), 4.14 (b), 4.15 (b), 4.16 (d), 4.17 (a), 4.18 (c), 4.19 (b), 4.20 (b), 4.21 (d), 4.22 (b), 4.23 (b), 4.24 (a), 4.25 (c), 4.26 (c), 4.27 (d), 4.28 (b), 4.29 (b), 4.30 (a), 4.31 (a), 4.32 (d)

Short and Long Answer Type Questions

Category I

- 4.1 Outline the failure of Weiss molecular field theory.
- 4.2 Using Larmor frequency obtain an expression for induced magnetic moment in terms of it.
- 4.3 From the statement of Curie-Weiss law find the Curie temperature.
- 4.4 Differentiate diamagnetic, paramagnetic and ferromagnetic substances.
- 4.5 For ferromagnetic substance obtain the relation among \vec{B} , \vec{M} and μ_r .
- 4.6 Give two examples and applications of soft and hard magnetic materials.
- 4.7 Mention the use of hysteresis loss for different practical purposes.
- 4.8 Draw the B-H curve for a ferromagnetic material and identify the retentive and coercive field on the curve. Find energy loss per cycle.
- 4.9 Draw the hysteresis curve for hard steel and wrought iron and give your comments.
- 4.10 Mention the role of magnetic flux density.
- 4.11 Elaborate the important roles of different types of magnetic materials with examples.
- 4.12 Distinguish the following terms: a) permeability, b) magnetic susceptibility, c) magnetic moment d) magnetization.
- 4.13 Explain the properties of ferrites and mention some applications.
- 4.14 Derive Curie's law of paramagnetism in the framework of Langevin's theory.
- 4.15 Explain the reason behind the negative susceptibility of diamagnetic material.
- 4.16 Explain ferromagnetic property on the basis of magnetic domain theory and find the numerical value of Bohr magneton.
- 4.17 Distinguish between soft magnetic material and hard magnetic material.
- 4.18 Give the technique to determine the susceptibility for a ferromagnetic substance.
- 4.19 Find out the loss of energy per unit volume due to hysteresis.
- 4.20 Following Langevin's classical theory, find an expression for diamagnetic susceptibility. Discuss the temperature dependence of diamagnetic susceptibility.
- 4.21 Explain what do you understand by hysteresis, remanence and coercivity? What is hysteresis loop? How will you determine the value of remanence and coercivity from a loop?

- 4.22 What are ferrites? How do ferrites differ from ferromagnetic substances? Why do we use ferrite material in radios and other communication equipment? Discuss their other applications.
- 4.23 Mention the characteristics of the hysteresis curve with a necessary diagram.
- 4.24 In hydrogen atom, obtain simple expression for magnetic moment associated with the orbital motion of an electron.
- 4.25 Distinguish between ferromagnetic and antiferromagnetic materials.
- 4.26 Mention the various uses of ferrites.
- 4.27 What are the achievements of the domain theory?
- 4.28 Outline a simple explanation of spontaneous magnetization of ferromagnetic below the Curie temperature and Curie Weiss law above Curie temperature.

Category II

- 4.29 From the spontaneous magnetization what we can get for practical purposes?
- 4.30 Elaborate the origin of diamagnetism in the light of Lenz's law.
- 4.31 Make your comments on Curie-Weiss law of ferromagnetism.
- 4.32 Focus the utility of Curie's law of paramagnetism.
- 4.33 Are all orientations of magnetic dipoles possible in quantum theory? Explain.
- 4.34 Why is the diamagnetism almost independent of temperature?
- 4.35 On the basis of Weiss molecular field theory elaborate the concept of susceptibility of a ferromagnetic material.
- 4.36 What is Bohr magneton? Show that Bohr magneton represents the magnetic moment of an elementary dipole.

Numerical Problems

- 4.1 Calculate the effective Bohr magneton for Gd^{+3} . The electronic configuration for Gd^{+3} is $4f^7 5s^2 5p^6$.
- 4.2 If the magnetic field intensity and susceptibility of a material is 10^5 A/m and 10^{-4} respectively, calculate the magnetization field and flux density.
- 4.3 A sample of iron having length 5 cm cross sectional area $10^{-4} m^2$ develops a magnetic moment of $2000 A/m^2$. Find the intensity of magnetization and magnetic induction if the magnetic field intensity is 10^5 A/m.
- 4.4 If the magnetic susceptibility of a medium is 1000, find the absolute and relative permeability.
- 4.5 A paramagnetic material has 10^{28} atoms/ m^3 . Its susceptibility at 330 K is 3.7×10^{-4} . Calculate the susceptibility at 300 K. [Ans: 4.07×10^{-4}]
- 4.6 Calculate the saturation magnetization for a rare earth element having 10^{28} atoms per unit volume, if the magnetic moment of the element is $4\mu_B$.
- 4.7 If a magnetic field at the inside of a solenoid is 10^{-4} T when it is empty and 0.2 T when it is filled with iron, calculate the relative permeability of iron.

- 4.8 The magnetization within a bar of some metal alloy is 1.2×10^6 A/m when the magnetic field is 200 A/m. Calculate the magnetic susceptibility and magnetic induction within the alloy.
- 4.9 A magnetizing field of 2000 A/m produces a magnetic flux 10^8 Wb in bar of iron of cross sectional area 0.5 cm^2 . Find the susceptibility and permeability of the bar.
- 4.10 Find the Curie constant for iron, if the saturation magnetization is 1.5×10^6 A/m and each atom of iron has a magnetic moment of 4 Bohr magneton.

PRACTICAL

To calculate the energy loss of a magnetic material using B-H Curve

Aim

To draw the B-H curve of a magnetic material (transformer core and ferrite core) and to calculate energy loss

Apparatus

Resistors, Capacitors, Transformer cores, Multimeter, Cathode Ray Oscilloscope

Theory

The energy loss of the core of the transformer is given by

$$E = \frac{N_1}{N_2} \times \frac{R_2}{R_1} \times \frac{C_2}{AL} \times S_V \times S_H \times \text{area of the loop} \quad (i)$$

where N_1 and N_2 are respectively represents the number of turns in the primary and the secondary coils of the transformer, R_1 and R_2 are the resistances in the primary and the secondary circuit, C_2 is the capacitor, L is length and A is the area of cross section of the specimen, S_H and S_V are the horizontal and vertical sensitivities of Cathode Ray Oscilloscope (CRO).

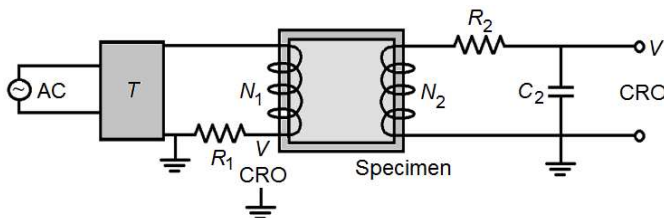


Fig. (i)

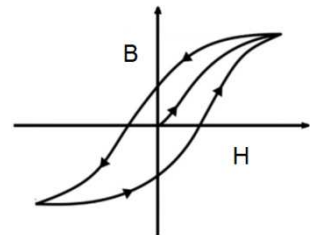


Fig. (ii)

Procedure

1. Construct the circuit according to the circuit diagram. The step down transformer (T) connected to the AC source supplies input of the primary of the core.

The voltage across R_1 and the voltage across C_2 are respectively feed to the horizontal and the vertical inputs of the CRO.

2. Now adjust properly both horizontal as well as vertical gain controls of the CRO for getting a loop of suitable size.
3. Plot the B-H curve on a graph paper.
4. Finally, determine the horizontal sensitivity S_H and vertical sensitivity S_V in V/cm without disturbing the gain controls.

Observations

Table 1: Determination of different parameters for transformer core and ferrite core

Parameter	Transformer Core	Ferrite Core
N_1		
N_2		
R_1		
R_2		
C_2		
A		
L		
S_V		
S_H		

Precautions

1. The specimen should be near to the probe at the center of the magnetizing coil.
2. If the area of the loop is taken in cm^2 , the sensitivity should be expressed in V/cm otherwise the area of the coil should be expressed in m^2 .

Results

The energy loss for the transformer core = J/cycle/V.

The energy loss for the ferrite core = J/cycle/V.

Application

The idea can be utilized for the design of transformer with low energy losses.

KNOW MORE

Magnetostatics is the study of magnetic fields when the currents are not changing with time. It is analogous to electrostatics where the charges are stationary.

Magnetostatic focusing can be obtained either by using a permanent magnet or by sending current through a coil of wire whose axis coincides with the beam axis.

Activity

A direct current produces a static magnetic field in the space surrounding the current carrying region. For applications where the magnitude of the direct current can be taken to be a constant or to vary slowly with time, coupling between magnetic and electric fields can be ignored. Magnetostatic analysis gives a solution for applications where certain assumptions are valid.

Electromagnetic elements must be used for modeling the response of all the regions. In order to get accurate solutions, the outer boundary of the space being modeled must be at least a few characteristic length scales away from the region of interest on all sides. The user-defined nodes only define the geometry of the elements while the degrees of freedom of the element are not related to these nodes, which has implications for applying the boundary conditions

Interesting facts

In the analysis of magnetostatic, the stiffness matrix can be very ill-conditioned; *i.e.*, it can have many singularities. A special iterative solution technique may be applied to prevent the ill-conditioned matrix from negatively impacting the computed magnetic fields. There can be situations sometimes when the default numerical scheme fails to converge.

A stabilization scheme can help to mitigate the effects of the ill-conditioning. Higher values of the stabilization factor lead to more stabilization and the lower values of the stabilization factor lead to less stabilization.

Analogy

Magnetostatic analysis gives the magnetic flux density and the magnetic field at a given value of the impressed direct current.

Timelines

600 BC - 1599: Humans discovered the magnetic lodestone and the attracting properties of amber. Advanced societies, in particular the Chinese and the Europeans exploited the properties of magnets in compasses, a tool that makes possible exploration of the seas, “new worlds” and the nature of Earth’s magnetic poles.

1600 - 1699: The Scientific Revolution took hold, facilitating the groundbreaking work of luminaries such as William Gilbert, who took the first truly scientific approach to the study of magnetism and electricity and wrote extensively of his findings.

1700 - 1749: Aided by tools such as static electricity machines and Leyden jars, scientists continued their experiments into the fundamentals of magnetism and electricity.

1775 - 1799: Scientists took important steps toward a clear understanding of electricity and some fruitful missteps including an elaborate theory on animal magnetism that sets the stage for a groundbreaking invention.

1820 - 1829: Hans Christian Oersted’s accidental discovery that an electrical current moves a compass needle rocked the scientific world; a spate of experiments followed, immediately leading to the first electromagnet and electric motor.

1830 - 1839: The first telegraphs were constructed and Michael Faraday produces much of his brilliant and enduring research into electricity and magnetism, inventing the first primitive transformer and generator.

Applications (Real Life / Industrial)

Magnetostatics as a special case of Maxwell's equations is one important application. Starting from Maxwell's equations and assuming that charges are either fixed or move as a steady current, the equations can be separated into two equations for the electric field and two for the magnetic field but these fields are independent of time and each other. Another important application is the re-introducing Faraday's law. A usual technique is to solve a series of magnetostatic problems at incremental time steps and then use these solutions to approximate. However, this method is not a true solution of Maxwell's equations but can provide a good approximation for slowly changing fields.

Case Study (Environmental / Sustainability / Social / Ethical Issues)

It is important that predefined temperature and field variables can be specified in a magnetostatic analysis. However, user-defined fields that allow the value of field variables at a material point to be redefined through user subroutine. These values affect only temperature and field-variable-dependent material properties, if any.

Inquisitiveness and Curiosity Topics

Electromagnetic elements must be used for the purpose of modeling all regions in any magnetostatic analysis. Unlike conventional finite elements, for using node-based interpolation, these elements are taken to use edge-based interpolation with the tangential components of the magnetic vector potential along element edges and thus serving as the primary degrees of freedom. Electromagnetic elements are available in standard two dimensions (planar only) and three dimensions. The planar elements can be formulated in terms of an in-plane magnetic vector potential, where the magnetic flux density and magnetic field vectors have only an out-of-plane component.

REFERENCES AND SUGGESTED READINGS

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3. W. Saslow, *Electricity, Magnetism and Light*, Harcourt/Academic-Elsevier, Stamford, 2002.
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8. https://nptel.ac.in/content/storage2/courses/112108150/pdf/Web_Pages/WEBP_M16.pdf
9. <https://unlcms.unl.edu/cas/physics/tsymbal/teaching/EM-913/section6-Magnetostatics.pdf>
10. <http://siva.bgk.uni-obuda.hu/~szakacs/segedanyagok/0910/MEEN/coey-magnetism.pdf>

5

Faraday's Law

UNIT SPECIFICS

We have emphasized the following aspects in this unit:

- Magnetic flux, flux density;
- Eddy currents;
- Laws of electromagnetic induction;
- Its integral and differential form;
- Lenz's law and conservation of energy;
- Induced *e.m.f.* and the method of its production;
- Self-inductance, mutual inductance, their units and dimensions;
- Dependence and co-efficient of coupling;
- Energy stored in a coil placed in a magnetic field.

The practical applications of the topics are discussed for generating further curiosity and creativity as well as improving problem solving capacity.

Besides giving a large number of multiple choice questions as well as questions of short and long answer types marked in two categories following lower and higher order of Bloom's taxonomy, assignments through a number of numerical problems, a list of references and suggested readings are given in the unit so that one can go through them for practice. It is important to note that for getting more information on various topics of interest some QR codes have been provided in different sections which can be scanned for relevant supportive knowledge.

After the related practical, based on the content, there is a "Know More" section. This section has been judiciously designed so that the supplementary information provided in this part becomes beneficial for the users of the book. This section mainly highlights the initial activity, examples of some interesting facts, analogy, history of the development of the subject focusing the salient observations and finding, timelines starting from the development of the concerned topics up to the recent time, applications of the subject matter for our day-to-day real life or/and industrial applications on variety of aspects, case study related to environmental, sustainability, social and ethical issues whichever applicable, and finally inquisitiveness and curiosity topics of the unit.

RATIONALE

This unit on Faraday's law will help students to get a clear idea about some basic terminologies like magnetic flux, flux density and eddy currents. Laws of electromagnetic induction and Lenz's law related to electromagnetic induction, self and mutual inductance and their applications to transformer have special importance for all practical purposes, like calculation of energy stored in a coil placed in a magnetic field.

Faraday's Law of Induction explains clearly how an electric current is responsible to produce a magnetic field. It also considers how inside a conductor a changing magnetic field generates an electric current. Faraday discovered magnetic induction that makes possible to construct and design the electric motors, generators and transformers which may be accepted as the foundation of modern technology. By using the concept of induction, we have an electric power grid and many other related items we plug into it. Subsequently the law was used into Maxwell's equations for interpreting the relationship between electricity and magnetism.

PRE-REQUISITES

Mathematics: Vector Calculus (Class XII)

Physics: Electromagnetism (Class XII)

UNIT OUTCOMES

List of outcomes of this unit is as follows:

U5-O1: Define magnetic flux, flux density, eddy currents

U5-O2: Explain laws of electromagnetic induction, its integral and differential form, Lenz's law and conservation of energy

U5-O3: Explain about induced *e.m.f.* and the method of its production

U5-O4: Explain about self-inductance, mutual inductance, their units and dimensions, dependence and co-efficient of coupling

U5-O5: Calculate energy stored in a coil placed in a magnetic field

Unit-5 Outcomes	EXPECTED MAPPING WITH COURSE OUTCOMES (1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation)					
	CO-1	CO-2	CO-3	CO-4	CO-5	CO-6
U5-O1	3	-	2	1	-	-
U5-O2	2	-	3	1	-	-
U5-O3	2	-	1	1	-	-
U5-O4	2	-	1	-	-	-
U5-O5	2	-	-	-	-	-

5.1 INTRODUCTION

When some current flows through a conductor, a magnetic field is set up around it. If the current carrying conductor is in the form of a coil, it will behave similar to a bar magnet. The polarity of the coil can be reversed by altering the direction of current flow in it. The region in space through which magnetic flux can travel around the coil is referred as the magnetic circuit. If the magnetic flux linking a conductor is changed, an electromagnetic force (*e.m.f.*) is induced in it. If now the conductor forms a complete loop or makes a circuit, a current will flow in it. This phenomenon is termed as *electromagnetic induction*. The primary requirement for an electromagnetic induction is the variation of flux linking the conductor or coil. As a result the current in the conductor (or coil) will remain so long this change occurs. The *e.m.f.* thus obtained is known as the induced *e.m.f.* So far the magnetic flux linked with the coil changes the induced *e.m.f.* or current lasts in the coil.

5.2 MAGNETIC FLUX

Magnetic flux linked with a surface is the measurement of the number of magnetic field lines crossing the surface normally when kept in a magnetic field and is measured by the product of the component of the magnetic field normal to the surface and the surface area.

The flux due to the magnetic field (\vec{B}) through the surface area ($d\vec{S}$) is defined as,

$$\begin{aligned} d\phi &= \vec{B} \cdot d\vec{S} \\ &= B dS \cos \theta \end{aligned} \quad (5.1)$$

The magnetic flux through any surface will always be highest when the field lines are normal to the surface. If the field lines are parallel to the surface then the magnetic flux linked with the surface is zero. It may be noted that the magnetic flux is a scalar quantity.

5.3 MAGNETIC FLUX DENSITY

Magnetic flux density may be defined as the flux through unit area placed in a magnetic field. It is alternately known as the magnetic induction vector and is mathematically defined as,

$$B = \frac{d\phi}{dS} \quad (5.2)$$

and hence the magnetic field (B) is also termed as magnetic flux density and it is a vector quantity.

5.3.1 Units and Dimensions

The SI unit of magnetic flux density or magnetic field is telsa (T) or $\text{N} \cdot \text{m}^{-1} \text{A}^{-1}$ and the SI unit of magnetic flux is tesla- m^2 or weber (Wb).

Dimensions of magnetic flux density is $[MT^{-2}I^{-1}]$ while the dimensions of the magnetic flux is

$$[\phi] = [BdS] = \left[\frac{F}{Il} dS \right] = \left[\frac{MLT^{-2}}{IL} \times L^2 \right]$$

$$\therefore [\phi] = [MT^2I^{-1}L^2]$$

Definition of weber: It is the magnetic flux linked with an area of 1 m^2 kept perpendicular to a magnetic field of strength 1 T.

5.4 FARADAY'S LAWS

Faraday performed a series of experiments to demonstrate the phenomenon of electromagnetic induction. He put his experimental finding into two laws, called Faraday's laws of electromagnetic induction.

First law: If a conductor is kept in a varying magnetic field, there will be an induction of electromotive force.

Second law: In any circuit, the magnitude of the induced *e.m.f.* is equal to the time rate of change of magnetic flux linked with the circuit. Mathematically,

$$e = \frac{d\phi}{dt} \quad (5.3)$$

Modified form of Faraday's second law (Lenz's law): Direction of the induced *e.m.f.* or current linked with a circuit will always oppose the change in magnetic flux produced in it. Therefore,

$$e = -\frac{d\phi}{dt} \quad (5.4)$$

Explanation of Faraday's law: Suppose a coil has N turns and the flux linked with the coil changes from ϕ_1 to ϕ_2 in t second. Therefore the net change in flux is $(N\phi_2 - N\phi_1)$ and rate of change of flux is $\frac{N\phi_2 - N\phi_1}{t}$.

According to Faraday's law induced *e.m.f.* is

$$e = \frac{N\phi_2 - N\phi_1}{t} = \frac{N(\phi_2 - \phi_1)}{t}$$

or,
$$e = \frac{d}{dt}(N\phi) = N \frac{d\phi}{dt} \quad (5.5)$$

It is a usual practice to give a minus sign to the right-hand side of the above expression. The minus sign comes from Lenz's law and indicates that the voltage is induced in a direction to oppose the change in flux produced in it.

$\therefore e = -N \frac{d\phi}{dt} \text{ volts} \quad (5.6)$



EXAMPLE 5.1

Example 5.1 15 mWb flux is linked with a coil of 150 turns and the flux is reversed in 5 ms. Find the average induced *e.m.f.* in the coil.

Solution

Change in flux $d\phi = 15 - (-15) = 30 \text{ mWb} = 30 \times 10^{-3} \text{ Wb}$.

Time taken for the change $dt = 5 \text{ ms} = 5 \times 10^{-3} \text{ s}$.

$$\begin{aligned} \therefore e &= N \frac{d\phi}{dt} \\ &= 150 \times \frac{30 \times 10^{-3}}{5 \times 10^{-3}} = 900 \text{ V.} \end{aligned}$$

Example 5.2 A coil with 50 turns is wound on a magnetic circuit. If the 1 A current flowing in this coil is reversed in 1 s, calculate the average induced *e.m.f.* in it. Take the reluctance of the circuit as 2000 AT/Wb.

Solution

$$\text{Flux in the coil} = \frac{m.m.f.}{\text{reluctance}} = \frac{50 \times 1}{2000} = 0.025 \text{ Wb.}$$

when 1 A current in the coil is reversed, flux through the coil is also reversed.

$$\text{Now,} \quad e = N \frac{d\phi}{dt}$$

$$\text{Here,} \quad N = 500; d\phi = 0.025 - (-0.025) = 0.05 \text{ Wb and } dt = 1 \text{ s.}$$

$$\therefore e = 500 \times \frac{0.05}{1} = 25 \text{ V.}$$

EXAMPLE 5.2

Example 5.3 The field winding of 4-pole DC generator consists of 4 coils connected in series, each coil being wound with 1200 turns. In an excited field the magnetic flux is 0.04 Wb/pole. When a speed is applied the field switch is opened in such a manner so that the flux is dropped to the residual value of 0.004 Wb/pole in a time of 0.1 s. Find the mean value of *e.m.f.* induced across field winding terminals.

Solution

Total no. of turns, $N = 1200 \times 4 = 4800$, total initial flux $= 4 \times 0.04 = 0.16 \text{ Wb}$

and total residual flux $= 4 \times 0.004 = 0.016 \text{ Wb.}$

Change in flux, $d\phi = 0.16 - 0.016 = 0.144 \text{ Wb.}$

Time taken, $dt = 0.1 \text{ s.}$

$$\therefore \text{Induced } e.m.f. \quad e = N \frac{d\phi}{dt} = 4800 \times \frac{0.144}{0.1} = 6912 \text{ V.}$$

EXAMPLE 5.3

5.4.1 Integral and Differential form of Faraday's Law

According to Faraday's law of electromagnetic induction, the expression for induced *e.m.f.* due to a change in magnetic flux in a coil is given by

$$e = - \frac{d\phi}{dt}$$

where ϕ is the magnetic flux through the surface S bounded by the curved path C and is given by,

$$\phi = \iint_S \vec{B} \cdot \hat{n} dS = \iint_S \vec{B} \cdot d\vec{S} \quad (5.7)$$

where $d\vec{S}$ is normal on the surface S .

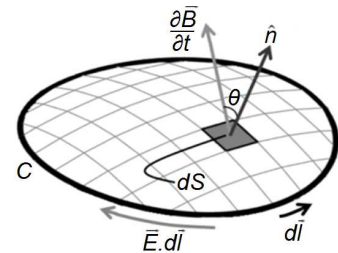


Fig. 5.1: Illustration of Faraday's law of e.m. induction

Thus,
$$e = -\frac{d}{dt} \left(\iint_S \vec{B} \cdot d\vec{S} \right) \quad (5.8)$$

Now if \vec{E} be the electric field along S then the induced *e.m.f.* may be written as,

$$e = \oint_C \vec{E} \cdot d\vec{l} \quad (5.9)$$

So, by comparing Eq. (5.8) and Eq. (5.9) we get,

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \left(\iint_S \vec{B} \cdot d\vec{S} \right) \quad (5.10)$$

Eq. (5.10) is known as the integral form of Faraday's law of electromagnetic induction.

Again using Stokes' theorem of vector calculus we have,

$$\oint_C \vec{E} \cdot d\vec{l} = \iint_S \vec{\nabla} \times \vec{E} \cdot d\vec{S} \quad (5.11)$$

So, by comparing Eq. (5.10) and Eq. (5.11) we get,

$$\iint_S \vec{\nabla} \times \vec{E} \cdot d\vec{S} = -\frac{d}{dt} \left(\iint_S \vec{B} \cdot d\vec{S} \right)$$

or,
$$\iint_S \vec{\nabla} \times \vec{E} \cdot d\vec{S} + \iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = 0$$

or,
$$\iint_S \left(\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} \right) \cdot d\vec{S} = 0$$

As, the above equation is true for any arbitrary $d\vec{S}$, so to make it valid we must have

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

or,
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (5.12)$$

Eq. (5.12) is known as the differential form of Faraday's law of electromagnetic induction.

5.4.2 Induced *e.m.f.* and Current

The direction of induced *e.m.f.* and hence current in a conductor or coil can be determined applying one of the following:

- Lenz's law
- Fleming's right-hand rule



5.5 LENZ'S LAW

Emil Lenz, a German scientist gave the following simple rule known as Lenz's law to find the direction of induced current:

The flow of an induced current will be in a direction so that it will oppose the particular cause which was responsible to produce it. The cause that was responsible to produce the current represents the change of flux linking the coil. Therefore, the direction of induced current will be such that its own magnetic field opposes the change in flux that produced the induced current.

Let us apply Lenz's law to Fig. 5.2. In this case the magnetic N-pole is approaching towards the coil. The direction of the induced current, from Lenz's law, will determine in a manner so that flux set up by it will oppose the change in original flux. It becomes possible if the left hand side of the coil be close to N-pole of the magnet.

Once we know the magnetic polarity of the side of the coil, we can get the direction of the induced current for the coil, applying the right hand rule.



5.5.1 Lenz's Law and Conservation of Energy

Lenz's law follows the law of conservation of energy. Let a bar magnet with the N-pole facing the coil (Fig. 5.2) is just pushed towards the coil such that a small current is induced in it. Let the direction of the induced current is such that instead of N-pole, S-pole is generated at the near end of the magnet.

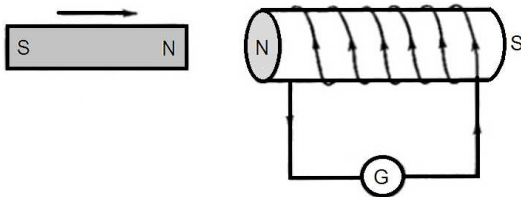


Fig. 5.2: Verification of Lenz's law

As a result the magnet will move towards the coil due to the attraction with increasing kinetic energy and thereby the current will be increased gradually and hence the magnetic field strength will also increase without doing any external work. This is clearly the violation of conservation of energy principle.

Hence the direction of the induced current will be such that only N-pole can be generated at the near end so that it can oppose the movement of magnet due to which the *e.m.f.* is induced.

5.6 FLEMING'S RIGHT HAND RULE

For a conductor moving right angle to a steady magnetic field, the direction of the induced *e.m.f.* or induced current can be determined by using Fleming's right hand rule. According to this rule when you stretch out the forefinger, middle finger and thumb of your right hand perpendicular to one another; if the forefinger directs towards the magnetic field and thumb directs towards the motion of the conductor, then the middle finger will be directed towards the induced current.

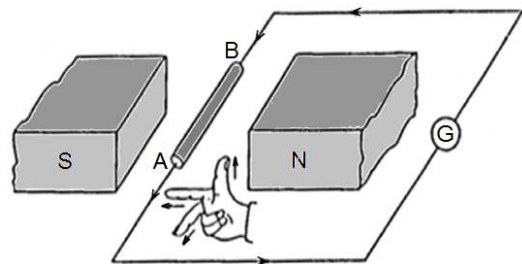


Fig. 5.3: Verification of Fleming's right hand rule

Fig. 5.3 describes the upward motion of a conductor AB moving perpendicular to a uniform magnetic field as developed due the magnetic pole pieces N and S.

According to the Fleming's right hand rule, here the direction of induced current will be from B to A. For downward motion of the conductor, keeping the magnetic field direction fixed, as per Fleming's right hand rule, the direction of induced current will be from A to B.

5.7 INDUCED E.M.F.

If the flux linking a coil or conductor is altered, there will be an induced electromotive force which can be found by using the following methods:

- Either by moving the conductor in a steady magnetic field such that there is a change in the linked flux. The *e.m.f.* thus induced is known as the dynamically induced *e.m.f.* It is called so as the *e.m.f.* induced in the conductor, is actually moving (e.g., AC or DC generator).
- The conductor is kept stationary while the magnetic field is varying. The *electromotive force* induced in this manner is known as statically induced electromotive force. For this we can give the example of a transformer. It is named so as the electromotive force is induced through this process in any stationary conductor.

It should be mentioned that in both the cases the magnitude of induced electromotive force can be expressed as, $N \frac{d\phi}{dt}$

i) Dynamically Induced *e.m.f.*: Consider a single conductor of length l metres moving at right angles to a uniform magnetic field of B Wb/m² with a velocity of v m/s [Fig. 5.4 (a)]. Suppose the conductor moves through a small distance dx in time dt seconds.

Then area swept by the conductor is $= l \times dx$.

\therefore Flux cut, $d\phi = \text{Field} \times \text{Area swept} = Bldx$ Wb

According to Faraday's laws of electromagnetic induction, *e.m.f.* e induced in the conductor is given by;

$$e = N \frac{d\phi}{dt} = Bl \frac{dx}{dt} \quad (\because N = 1) \quad (5.13)$$

or,
$$e = Blv \quad \left(\because \frac{dx}{dt} = v \right) \quad (5.14)$$

If the conductor moves at an angle θ to the magnetic field [Fig. 5.4 (b)], then the velocity at which the conductor moves across the field is $v \sin \theta$.

$$\therefore e = Blv \sin \theta \quad (5.15)$$

The direction of induced *e.m.f.* can be determined by Fleming's right hand rule. If the conductor is moved parallel to the magnetic field, there would be no change in flux and hence no *e.m.f.* would be induced.

As this induced *e.m.f.* is owing to the motion of the conductor, it is called *motional e.m.f.* It disappears when the motion of the conductor stops.

If the ends of the conductor is connected to the external circuit of resistance R and if r be the internal resistance of the conductor, then according to Ohm's law the induced current i is given by,

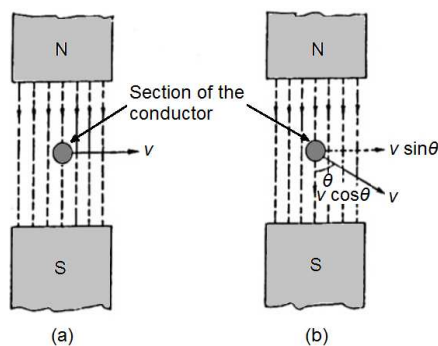


Fig. 5.4: Dynamically induced *e.m.f.*

$$i = \frac{e}{R + r} = \frac{Blv}{R + r} \quad (5.16)$$

Direction of the current through the conductor may be obtained from Fleming's right hand rule.

Explanation: When the conductor moves through the magnetic field, all the free electrons of the conductor move in the same direction. Hence the Lorentz's force acting on each electron is

$$\vec{F} = -e(\vec{v} \times \vec{B})$$

where \vec{v} represents the velocity of the electron. Thus an *e.m.f.* is induced between the ends of the conductor.



Example 5.4 The velocity of a conductor of length 25 cm moving perpendicular to a uniform flux density of 5 T is 2 cm/s. Calculate the *e.m.f.* induced in it.

Solution

Given, length, $l = 25 \text{ cm} = 0.25 \text{ m}$ and velocity, $v = 2 \text{ cm/s} = 0.02 \text{ m/s}$.

So, induced *e.m.f.*, $E = Blv$

$$= 5 \times 0.25 \times 0.02 = 0.025 \text{ V} = 25 \text{ mV.}$$

EXAMPLE 5.4

Example 5.5 A conductor is of length 120 mm. It must be moved at right angles to a magnetic field of flux density 0.6 T and to induce in it an *e.m.f.* of 1.8 V is applied. Determine the speed.

Solution

Speed of the conductor,

$$v = \frac{E}{Bl} = \frac{1.8}{0.6 \times 0.12} = 25 \text{ m/s.}$$

EXAMPLE 5.5

Example 5.6 A conductor of length 25 cm is moving with a uniform speed of 8 m/s through a 1.2 T uniform magnetic field. Calculate the current flowing through the conductor when (a) it is open-circuited and (b) it is connected to a 15Ω load resistance.

Solution

We have induced *e.m.f.*, $E = Blv$

$$= 1.2 \times 0.25 \times 8 = 2.4 \text{ V.}$$

(a) When the conductor is open circuited, no current will flow through it.

(b) When the conductor is connected to a 15Ω load resistance current flowing through it will be,

$$I = \frac{E}{R} = \frac{2.4}{15} = 0.16 \text{ A.}$$

EXAMPLE 5.6

EXAMPLE 5.7

Example 5.7 A 500 mm long straight conductor is moving with constant 5 m/s velocity perpendicular to both its length and a uniform magnetic field. Calculate the flux density of the magnetic field, if the *e.m.f.* induced in the conductor is 2.5 V. When the conductor forms a closed circuit of total resistance 5 Ω , obtain the force experienced by the conductor.

Solution

Induced *e.m.f.*, $E = 2.5$ V, velocity, $v = 5$ m/s and length, $l = 500$ mm = 0.5 m.

So, flux density $B = \frac{E}{lv} = \frac{2.5}{0.5 \times 5} = 1$ T.

Current, $I = \frac{E}{R} = \frac{2.5}{5} = 0.5$ A.

Force on conductor, $F = BIl = 1 \times 0.5 \times 0.5 = 0.25$ N.

EXAMPLE 5.8

Example 5.8 A car is moving at a speed of 40 km/h. If the back axle of the car is 2.2 m long, calculate the *e.m.f.* generated in the axle due to motion. Assume that the vertical component of the earth's magnetic field is 24 μ T.

Solution

Generated *e.m.f.*, $E = Blv = 24 \times 10^{-6} \times 2.2 \times \frac{60 \times 10^3}{60 \times 60} = 0.88$ mV.

EXAMPLE 5.9

Example 5.9 A moving conductor has a velocity of 20 m/s for an angle (a) 90° (b) 45° and (c) 30° when placed in a magnetic field. The field is created between two square-faced poles with side length 2.5 cm. When the flux on the pole face is 60 mWb, obtain the magnitude of the induced electromotive force for all the cases.

Solution

Induced *e.m.f.*, $E = Blv \sin \theta$

(a) When $\theta = 90^\circ$, $E = Blv \sin 90^\circ = \frac{\phi}{A} \times l \times v \times 1 = \frac{60 \times 10^{-3}}{(2.5)^2 \times 10^{-4}} \times 2.5 \times 10^{-2} \times 20 = 48$ V.

(b) When $\theta = 45^\circ$, $E = Blv \sin 45^\circ = 48 \sin 45^\circ = 33.9$ V.

(c) When $\theta = 30^\circ$, $E = Blv \sin 30^\circ = 48 \sin 30^\circ = 24$ V.

EXAMPLE 5.10

Example 5.10 A conductor of length 0.5 m kept at right angle to a uniform magnetic field of flux density 1 Wb/m² and having a velocity of 40 m/s. Find the induced electromotive force in the conductor. Also determine the induced *e.m.f.* when the conductor moves at an angle 60° to the field.

Solution

(i) $E = Blv = 1 \times 0.5 \times 40 = 20$ V.

(ii) $E = Blv \sin \theta = 1 \times 0.5 \times 40 \times \sin 60^\circ = 17.32$ V.

Example 5.11 A conductor has a length of 400 mm. It is moved at 70° to a 0.85 T magnetic field. If it has a velocity of 115 km/h, find (a) the induced voltage, and (b) force acting on the conductor if connected to an $8\ \Omega$ resistor.

Solution

(a) Induced voltage,

$$E = Blv \sin \theta = 0.85 \times 0.4 \times \left(\frac{115 \times 1000}{60 \times 60} \right) \times \sin 70^\circ$$

$$= 10.206\text{ V.}$$

(b) Force on conductor,

$$F = BIl \sin \theta = B \times \left(\frac{E}{R} \right) \times l \times \sin \theta$$

$$= 0.85 \times \left(\frac{10.206}{8} \right) \times 0.4 \times \sin 70^\circ = 0.408\text{ N.}$$

EXAMPLE 5.11

Example 5.12 An aircraft has a wing span of 56 m. It is flying horizontally at a speed of 810 km/hr and the vertical component of earth's magnetic field is $4 \times 10^{-4}\text{ Wb/m}^2$. Calculate the potential difference between the wing tips of the aircraft.

Solution

Induced *e.m.f.* $E = Blv$

Here, $B = 4 \times 10^{-4}\text{ Wb/m}^2$, $l = 56\text{ cm}$ and $v = 810 \times 10^3 / 3600 = 225\text{ m/s}$.

\therefore Induced *e.m.f.* $E = 4 \times 10^{-4} \times 56 \times 225 = 5.04\text{ V}$.

or, Potential difference = 5.04V.

EXAMPLE 5.12

ii) Statically Induced *e.m.f.*: If the conductor is maintained at the stationary condition and the associated field is altered, the *electromotive force* induced in the conductor is named as statically induced *electromotive force* which is sub-categorized as:

- self-induced *electromotive force* and
- mutually induced *electromotive force*.

a) Self-Induced *e.m.f.*: The *e.m.f.* induced in a coil due to the change of its own flux linked with it is called self-induced *e.m.f.* When a coil is carrying current (Fig. 5.5), a magnetic field is established through the coil. If the current in the coil changes, then flux linking the coil also changes. Hence by Faraday's law, an *e.m.f.* will be induced in the coil. This is known as self-induced *e.m.f.*

The magnitude of this self-induced *e.m.f.* is

$$e_s = N \frac{d\phi}{dt} \quad (5.17)$$

The direction of induced *e.m.f.* (by Lenz's law) is always such so as to oppose the cause responsible for inducing the *e.m.f.*; i.e., change of current (and hence field) in the coil.

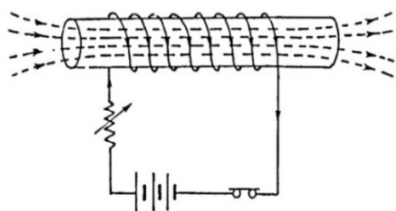


Fig. 5.5: Self induction

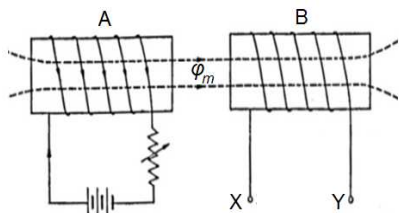


Fig. 5.6: Mutual induction

When current in a coil changes, an *e.m.f.* is induced in it, which opposes the change in current in the coil. This important property of the coil is called its self-inductance. It may be noted that self-induced *e.m.f.* and hence inductance does not prevent the current from changing; it serves only to delay the change. Thus, after the switch is closed (Fig. 5.5), the current will rise from zero to final steady value taking some time. This delay is due to self-induced *e.m.f.* or inductance of the coil. When current in the coil is steady or constant, the magnetic field also becomes constant and self-induced *e.m.f.* drops to zero.

b) Mutually Induced *e.m.f.*: The induced *e.m.f.* in a coil for the change in current in the neighbouring coil is termed as mutually induced *e.m.f.* Let us consider two coils A and B kept adjacent to each other as revealed in Fig. 5.6. A portion of the magnetic flux as produced by the coil A goes through or links with the other coil B. This flux is the same for both the coils A and B and is known as the mutual flux (ϕ_m).

If we now vary the current in coil A, the mutual flux will also alters and so the *e.m.f.* will be induced in both the coils. The *e.m.f.* induced in coil A is known as self-induced *e.m.f.* as already pointed out. On the other hand, the *e.m.f.* which is induced in the coil B is known as mutually induced *e.m.f.* From Faraday's laws the magnitude of mutually induced *e.m.f.* can be expressed as,

$$e_M = N_B \frac{d\phi_m}{dt} \quad (5.18)$$

where N_B represents the number of turns of the coil B and $\frac{d\phi_m}{dt}$ gives the rate of change of mutual flux which is the flux common to both the coils.

From Lenz's law, the direction of the mutually induced electromotive force becomes such that it opposes the cause producing it. That very cause which produces the mutually induced electromotive force in coil B is the changing mutual flux created by the coil A. Thus when the circuit is closed, the direction of induced current in coil B will be the flux set up by it. This will oppose the varying mutual flux as created by the coil A. Regarding this the following points are important to note:

- (i) The mutually induced electromotive force in coil B remains so long as the current in coil A is varying. If the current in the coil A is steady, the mutual flux also exhibits steady value and mutually induced electromotive force drops down to the zero value.
- (ii) The property of two neighboring coils to induce voltage in one coil due to the change of current in the other is termed as mutual inductance.

Methods of producing *e.m.f.*: There are three methods of producing *e.m.f.* which are as follows:

- (i) By altering the magnitude of the magnetic field.
- (ii) By altering the elementary surface area *i.e.*, by stretching or shrinking the coil.
- (iii) By altering the relative orientation of the surface and the magnetic field *i.e.*, the angle between the direction of magnetic field and the normal to the elementary surface.

5.8 SELF INDUCTANCE

Whenever a current is flowing in a closed loop it produces a magnetic field and thus it has its own flux through the area bounded by its own loop. If now the current changes with time, the flux also changes through the loop and hence an *e.m.f.* is induced according to Faraday's law of electromagnetic induction. This process is known as self induction.

Definition: It is the property of a coil or loop due to which it opposes the change in the current flowing through it by inducing an *e.m.f.* in it. For this it may be regarded as the inertia of electricity.

5.8.1 Coefficient of Self Induction

If ϕ be the magnetic flux linked with a coil at any instant t when the current through the coil is I then,

$$\phi \propto I$$

$$\text{or,} \quad \phi = LI \quad (5.19)$$

where the constant of proportionality L is known as the co-efficient of self induction or the self inductance of the coil and is a characteristic constant of the coil.

If $I = 1$ then $L = \phi$.

Thus, *self induction* of a coil may be considered as the magnetic flux linked with the coil when unit current flows through it.

Inductance is due to the self-induced *e.m.f.* in the coil itself by the changing current. For an increase of current in the coil, there will be a set up of the self induced electromotive force. Interestingly, it will set up in such a manner so that it can oppose the current increase. Thus the self induced electromotive force will be opposite to the voltage applied. Similarly, for a decrease in current in the coil, self-induced *e.m.f.* will be in such a direction so as to oppose the decrease in current *i.e.* self-induced *e.m.f.* will be in the same direction as the applied voltage. It may be noted that self-inductance does not prevent the current from changing; it serves only to delay the change.

The greater the self-induced *e.m.f.* $\left(= N \frac{d\phi}{dt} \right)$, the greater the self-inductance of the coil and hence

larger is the opposition to the changing current. Hence inductance of a coil depends upon the following factors:

- (i) Shape and number of turns.
- (ii) μ_r of the material surrounding the coil.
- (iii) The speed with which magnetic field changes.

Expression for L: Consider a coil of N turns carrying a current of I amperes. If current in the coil changes, the flux linkages of the coil will also change. This will set up a self-induced *e.m.f.* (e) in the coil and is given by;

$$e = N \frac{d\phi}{dt} = \frac{d}{dt} (N\phi) \quad (5.20)$$

Since flux linkages ($= N\phi$) is due to the current in the coil, it follows that it will be proportional to I .

$$\therefore \quad e \propto \frac{dI}{dt}$$

$$\text{or,} \quad e = L \frac{dI}{dt} \quad (5.21)$$

where L is a constant called self-inductance of the coil. The SI unit of self inductance is henry (H). If in Eq. (5.21), $e = 1$ volt and $\frac{dI}{dt} = 1$ A/s, then $L = 1$ H.

Hence a coil (or circuit) has an inductance of 1 henry if an *e.m.f.* of 1 volt is induced in it when current through it changes at the rate of 1 ampere per second.

Dimension of self inductance: We have, $L = \frac{e}{\frac{dI}{dt}}$ [from Eq. (5.21)]

So, the dimensions of self inductance can be obtained as,

$$[L] = \left[\frac{\text{emf} \times \text{time}}{\text{current}} \right] = \left[\frac{\text{work} \times \text{time}}{\text{charge} \times \text{current}} \right] = \left[\frac{\text{work} \times \text{time}}{\text{charge}^2 \times \text{time}} \right]$$

$$\therefore [L] = [ML^2T^{-2}I^{-2}]$$



Other Expressions for L: Apart from Eq. (5.21), the value of L can be determined by one of the following two ways:

(i) First method: If the flux linkages of the coil and current are known, then inductance can be determined as under:

$$e = L \frac{dI}{dt} = \frac{d}{dt} (LI)$$

$$\text{also} \quad e = N \frac{d\phi}{dt} = \frac{d}{dt} (N\phi)$$

From the two expressions, we have, $LI = N\phi$

$$\therefore L = \frac{N\phi}{I} \quad (5.22)$$

Thus, inductance is the flux linkages of the coil per ampere.

If $N\phi = 1$ Wb-turn and $I = 1$ A, then $L = 1$ H.

Hence a coil has an inductance of 1 henry if a current of 1 A in the coil sets up flux linkages of 1 Wb-turn.

EXAMPLE 5.13

Example 5.13 If 5 A current flowing in a coil of 250 turns produces a magnetic flux of $100 \mu\text{Wb}$, calculate the self inductance of the coil. If the current is reversed in 2.5 ms find also the induced *e.m.f.* in the coil.

Solution

$$\text{Inductance,} \quad L = \frac{N\phi}{I} = \frac{250 \times 100 \times 10^{-6}}{5} = 0.005 \text{ H} = 5 \text{ mH.}$$

$$\text{Induced } e.m.f., \quad E = L \frac{dI}{dt} = 0.005 \times \frac{5 - (-5)}{2.5 \times 10^{-3}} = 20 \text{ V.}$$

Example 5.14 The self inductance of a coil of 400 turns is measured as 8 mH. What are the numbers of turns required for producing a 6 mH coil if the same core is used to do that?

Solution

Since inductance, $L = \frac{N\phi}{I}$ (for constant ϕ and I)

$$\therefore L \propto N$$

$$\text{or, } 8 \times 10^{-3} \propto 400$$

$$\text{or, } 8 \times 10^{-3} = k \cdot 400$$

$$\text{or, } k = \frac{8 \times 10^{-3}}{400} = 20 \times 10^{-6}.$$

$$\text{Hence, when } L = 6 \times 10^{-3} \text{ H, then } L = kN$$

$$\text{or, } 6 \times 10^{-3} = 20 \times 10^{-6} N$$

$$\text{from which, number of turns, } N = \frac{6 \times 10^{-3}}{20 \times 10^{-6}} = 300 \text{ turns.}$$

EXAMPLE 5.14

Example 5.15 A coil wound on an iron core of permeability 400 has 150 turns and a cross-sectional area of 5 cm². Calculate the inductance of the coil. Given, a steady current of 3 mA produces a magnetic field of 10 lines/cm² and consider air as the medium.

Solution $\mu_i = \frac{\text{flux density in iron}}{\text{flux density in air}} = \frac{B_i}{10}$

$$\therefore B_i = 10 \times \mu_i = 10 \times 400 = 4000 \text{ lines/cm}^2.$$

Flux produced by 3 mA current in the iron core is

$$\phi = B_i \times a = 4000 \times 5 = 20,000 \text{ lines} = 2 \times 10^{-4} \text{ Wb.}$$

$$\therefore L = \frac{N\phi}{I} = \frac{150 \times 2 \times 10^{-4}}{3 \times 10^{-3}} = 10 \text{ H.}$$

EXAMPLE 5.15

(ii) Second method: The inductance of a magnetic circuit can be found in terms of its physical dimensions. Consider an iron-cored solenoid of dimensions as shown in Fig. 5.7. Inductance of the solenoid is given by,

$$L = N \frac{d\phi}{dI}$$

$$\text{Now, } \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{l / a\mu_0\mu_r} \quad (5.23)$$

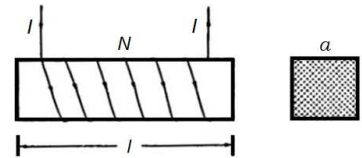


Fig. 5.7: Section of a solenoid

Differentiating ϕ with respect to I , we get,

$$\frac{d\phi}{dI} = \frac{N a \mu_0 \mu_r}{l}$$

$$\therefore L = N \frac{N a \mu_0 \mu_r}{l} = \frac{N^2 a \mu_0 \mu_r}{l} \quad (5.24)$$

$$\text{or, } L = \frac{N^2}{l / a \mu_0 \mu_r} = \frac{N^2}{\text{reluctance (S)}} \quad (5.25)$$

It may be noted that inductance is directly proportional to square of the number of turns and inversely proportional to reluctance of the magnetic path.

EXAMPLE 5.16

Example 5.16 There is a wooden toroid whose mean diameter is 400 mm having an area of cross section 400 mm^2 . It is uniformly wound with a coil which has 100 turns and carrying a current of 2 A. Determine (i) the self inductance of the given coil and also find the (ii) induced electromotive force in the coil when the current is uniformly reduced to zero in a time of 10 ms.

Solution

Here $l = 0.4 \pi \text{ m}$ and $a = 400 \times 10^{-6} \text{ m}^2$.

$$(i) \quad L = \frac{N^2 a \mu_0 \mu_r}{l} = \frac{(1000)^2 \times 400 \times 10^{-6} \times 4\pi \times 10^{-7} \times 1}{0.4 \pi} = 0.4 \times 10^{-3} \text{ H.}$$

$$(ii) \quad e = L \frac{dI}{dt} = 0.4 \times 10^{-3} \frac{2 - 0}{10 \times 10^{-3}} = 0.08 \text{ V.}$$

EXAMPLE 5.17

Example 5.17 A 300-turn coil has a resistance of 6Ω and an inductance of 0.5 H. Determine the new resistance and new inductance if one-third of the turns are removed. Assume all the turns have the same circumference.

Solution

Since the resistance of a coil is directly proportional to its length, we have,

$$R_1 / R_2 = N_1 / N_2$$

$$\therefore 6 / R_2 = 300 / 200$$

$$\therefore R_2 = \frac{200}{300} \times 6 = 4 \Omega.$$

$$\text{Now, } \frac{L_1}{L_2} = \frac{N_1^2 / S}{N_2^2 / S}$$

$$\text{Assuming the reluctance of magnetic path to be constant, } \frac{0.5}{L_2} = \frac{(300)^2}{(200)^2}$$

$$\therefore L_2 = 0.22 \text{ H.}$$

Dependence of self inductance of a coil: Self inductance of a coil depends on:

- (i) length (l) of the coil,
- (iii) number of turns (n) in the coil,
- (ii) area of cross section (A) of the coil and
- (iv) nature of material of the core on which the coil is wound.

For example, the self inductance of a long solenoid (when air field) of radius r is given by,

$$L = \mu_0 n^2 l A = \mu_0 \pi r^2 n^2 l \quad (5.26)$$

where $A = \pi r^2$ is the cross sectional area of the solenoid, n is the number of turns per unit length.

If the core is filled with a material of permeability μ then,

$$L = \mu n^2 l A = \mu_0 \mu_r \pi r^2 n^2 l \quad (5.27)$$

where μ_0 represents the permeability of free space and μ_r the relative permeability of the medium.

5.9 MUTUAL INDUCTANCE

Mutual induction may be considered as the phenomenon of production of induced *e.m.f.* in one coil due to the varying current in the neighboring coil. If the current through one coil changes with time, an *e.m.f.* is induced in the other neighboring coil due to the change of the magnetic flux linked with it. The *e.m.f.* so induced in the second coil opposes the change in current through the first coil and this phenomenon is known as *mutual induction*. We can consider the mutual induction as developed owing to the property of the two coils.

Definition: By virtue of this property each of the coils opposes any change in the flow of current through the other developing an induced electromotive force.

5.9.1 Coefficient of Mutual Induction

Let, I be the current flowing through the primary coil at an instant t for which if the magnetic flux linked with the secondary coil at that instant is φ . Then,

$$\varphi \propto I$$

$$\text{or,} \quad \varphi = MI \quad (5.28)$$

where the constant of proportionality M is known as the co-efficient of mutual induction or the *mutual inductance* of the two coils (primary and secondary) and is a characteristic constant of the coils.

If $I = 1$ then $M = \varphi$.

Thus, *mutual inductance* of a pair of coils is defined as the magnetic flux linked with one coil, for a unit flow of current through the other coil.

The unit of mutual inductance is Wb/A or volt-s/amp and is commonly known as henry (H).

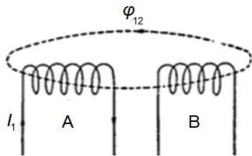


Fig. 5.8: Two coils A and B placed adjacent to each other

Consider two coils A and B placed adjacent to each other as shown in Fig. 5.8. If current I_1 flows in the coil A, a flux is set up and a part φ_{12} (mutual flux) of this flux links with the coil B. If current in coil A changes, the mutual flux also changes and hence *e.m.f.* is induced in coil B. This *e.m.f.* induced in coil B is termed as mutually induced *e.m.f.*

The converse of this action is also true *i.e.*, a change of current in coil B will produce a mutually induced *e.m.f.* in coil A. Just as self-induced *e.m.f.* is responsible for self-inductance (L), similarly mutually induced *e.m.f.* is responsible for mutual inductance (M). Let, two coils A and B have inductances L_A and L_B respectively. In addition, there is also an inductance, called mutual inductance M between the coils due to mutually induced *e.m.f.* between them. The effect of mutual inductance is to either increase (L_A+M and L_B+M) or decrease (L_A-M and L_B-M) the inductance of the two coils; decided by the arrangement of the coils.

Expression for M: Mutually induced *e.m.f.* in coil B \propto rate of change of current in coil A

$$\text{or, } e_M \propto \frac{dI_1}{dt}$$

$$\text{or, } e_M = M \frac{dI_1}{dt} \quad (5.29)$$

where M is a constant called mutual inductance between the two coils.

Definition of henry: If in Eq. (5.29), $e_M = 1$ volt and $\frac{dI_1}{dt} = 1$ A/s, then $M = 1$ H.

Hence mutual inductance between two coils is 1 henry if current change at the rate of 1 A/s in one coil induces an *e.m.f.* of 1 V in the other coil.

Dimension of mutual inductance: From Eq. (5.29) we have, $M = \frac{e_M}{\frac{dI_1}{dt}}$

\therefore Dimensions of mutual inductance will be,

$$[M] = \left[\frac{\text{emf} \times \text{time}}{\text{current}} \right] = \left[\frac{\text{work} \times \text{time}}{\text{charge} \times \text{current}} \right] = \left[\frac{\text{work} \times \text{time}}{\text{charge}^2 \times \text{time}} \right]$$

$$\therefore [M] = [ML^2T^{-2}I^{-2}]$$

Other Expressions for M: Apart from Eq. (5.29), for M , mutual inductance can alternately be determined by one of the following two methods:

(i) First method: In Fig. 5.9 for two magnetically coupled coils A and B

$$e_M = M \frac{dI_1}{dt} = \frac{d}{dt} (MI_1)$$

$$\text{also } e_M = N_2 \frac{d\phi_{12}}{dt} = \frac{d}{dt} (N_2 \phi_{12})$$

From these two expressions we have, $MI_1 = N_2\phi_{12}$

$$\text{or, } M = \frac{N_2 \phi_{12}}{I_1} \quad (5.30)$$

Definition of henry: If $N_2\phi_{12} = 1$ Wb and $I_1 = 1$ A, then $M = 1$

H. Hence mutual inductance between two coils A and B is 1 henry, if 1 Wb-turn flux is linked in coil B due to a current of 1 A flowing in coil A.

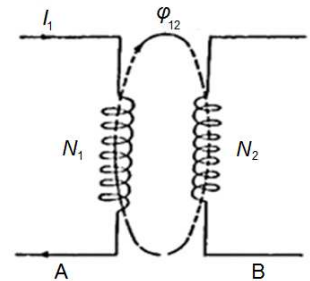


Fig. 5.9: Two magnetically coupled coils A and B

(ii) Second method: Mutual inductance between the two coils can also be determined in terms of physical dimensions of the magnetic circuit. Fig. 5.9 shows two magnetically coupled coils A and B having N_1 and N_2 turns respectively.

Suppose l and a are the length and area of cross-section of the magnetic circuit respectively. Let μ_r be the relative permeability of the material of which the magnetic circuit is composed.

$$\text{Mutual flux, } \phi_{12} = \frac{m.m.f.}{\text{reluctance}} = \frac{N_1 I_1}{l / a \mu_0 \mu_r} \quad (5.31)$$

$$\text{or, } \frac{\phi_{12}}{I_1} = \frac{N_1 a \mu_0 \mu_r}{l}$$

$$\therefore M = \frac{N_1 N_2 a \mu_0 \mu_r}{l} \quad [\text{as, } M = \frac{N_2 \phi_{12}}{I_1}] \quad (5.32)$$

$$\text{or, } M = \frac{N_1 N_2}{l / a \mu_0 \mu_r} = \frac{N_1 N_2}{\text{reluctance (S)}} \quad (5.33)$$

From Eq. (5.33) it is clear that the mutual inductance varies inversely with the reluctance of the magnetic circuit.

Example 5.18 Cross-sectional area of 300 mm long iron ring is 500 mm^2 . It is wound with 100 turns. Find the (a) required amount of current to set up a $500 \mu\text{Wb}$ flux in the coil, (b) system inductance, and (c) induced *e.m.f.* when the field collapses within 1 ms. Take, relative permeability to be 1600.

Solution

(a) Reluctance of ring,

$$S = \frac{l}{\mu_0 \mu_r A} = \frac{300 \times 10^{-3}}{4\pi \times 10^{-7} \times 1600 \times 500 \times 10^{-6}} = 298.4 \text{ kA/Wb.}$$

Again, $NI = \phi S$ from which current,

$$I = \frac{\phi S}{N} = \frac{500 \times 10^{-6} \times 298415.52}{100} = 1.492 \text{ A.}$$

$$\text{(b) Inductance, } L = \frac{N^2}{S} = \frac{(100)^2}{298.4 \times 1000} = 33.51 \text{ mH.}$$

$$\text{(c) Induced e.m.f., } E = -N \frac{d\phi}{dt} = -100 \times \frac{500 \times 10^{-6}}{1 \times 10^{-3}} = -50 \text{ V.}$$

EXAMPLE 5.18

Example 5.19 If the current flowing through the primary coil of a magnetic circuit having 400 turns is permitted to enhance linearly from 10 mA to 35 mA in 100 ms, an electromotive force of 75 mV is observed to induce the secondary coil, which is left open circuited, having 240 turns. Find (i) the value of mutual inductance between the two coils, (ii) the reluctance and (iii) the self inductance of the secondary coil.

EXAMPLE 5.19

EXAMPLE 5.19

Solution

$$(a) \text{ Mutual inductance, } M = \frac{E_M}{\frac{dI_1}{dt}} = \frac{75 \times 10^{-3}}{\frac{(35-10) \times 10^{-3}}{100 \times 10^{-3}}} = 0.30 \text{ H.}$$

$$(b) \text{ Reluctance, } S = \frac{N_1 N_2}{M} = \frac{400 \times 240}{0.30} = 320000 \text{ A/Wb} = 320 \text{ kA/Wb.}$$

$$(c) \text{ Secondary self inductance, } L_2 = \frac{N_2^2}{S} = \frac{(240)^2}{320000} = 0.18 \text{ H.}$$

EXAMPLE 5.20

Example 5.20 A solenoid of 70 cm in length and of 2100 turns has a radius of 4.5 cm. A second coil of 750 turns is wound upon the middle part of the solenoid. Find (i) self-inductance of solenoid (ii) and mutual inductance between the two coils.

Solution

Since μ_r is not given, let us take it as 1.

Here, area $a = \pi r^2 = \pi (4.5 \times 10^{-2})^2 \text{ m}^2$.

$$(i) \text{ Inductance } L = \frac{N^2 \alpha \mu_0 \mu_r}{l} \\ = \frac{(2100)^2 \times \pi \times (4.5 \times 10^{-2})^2 \times 4\pi \times 10^{-7} \times 1}{0.7} = 51 \times 10^{-3} \text{ H.}$$

(ii) As the second coil is wound on the middle part of the solenoid, the co-efficient of coupling is unity *i.e.* the whole of the flux produced by solenoid links the second coil.

$$M = \frac{N_1 N_2 \alpha \mu_0 \mu_r}{l} \\ = \frac{2100 \times 750 \times \pi \times (4.5 \times 10^{-2})^2 \times 4\pi \times 10^{-7} \times 1}{0.7} = 18.2 \times 10^{-3} \text{ H.}$$

EXAMPLE 5.21

Example 5.21 Two coils are marked as A and B having turns 100 and 1000 respectively. They are wound side by side on a closed iron circuit of cross-sectional area 8 cm^2 and mean length 80 cm. The relative permeability of iron is given as 900. Determine the mutual inductance between the coils. What will be the induced *e.m.f.* in coil B if current in coil A is increased uniformly from 0 to 10 A in 0.02 s?

Solution

Mutual inductance between the coils

$$M = \frac{N_1 N_2 a \mu_0 \mu_r}{l} = \frac{100 \times 1000 \times 8 \times 10^{-4} \times 4\pi \times 10^{-7} \times 900}{0.8} = 0.113 \text{ H.}$$

$$\text{and induced } e.m.f. \ e_B = M \frac{dI_1}{dt} = 0.113 \times \frac{10 - 0}{0.02} = 56.5 \text{ V.}$$

5.10 COEFFICIENT OF COUPLING

It is defined as the fraction of magnetic flux produced by the current in one coil that links the other coil. Consider two coils A and B having turns N_1 and N_2 respectively. Their individual coefficients of self inductance are L_1 and L_2 respectively and mutual inductance between them is M .

$$\text{Then} \quad M = \frac{N_1 \phi_1}{I_2}$$

$$\text{or,} \quad M = \frac{N_1 K_1 \phi_2}{I_2} \quad (5.34)$$

where ϕ_2 is the flux due to the current I_2 flowing in the coil 2. However, only a fraction $K_1 \phi_2$ links the in coil 1.

The mutual inductance between the coil 1 and coil 2 may also be written as

$$M = \frac{N_2 \phi_2}{I_1} = \frac{N_2 K_2 \phi_1}{I_1} \quad (5.35)$$

where ϕ_1 is the flux produced by the current I_1 flowing in the coil 1. However, only a fraction $K_2 \phi_1$ links the in coil 2. Multiplying Eq. (5.34) and Eq. (5.35) we get

$$\begin{aligned} M^2 &= \frac{N_1 N_2 \phi_1 \phi_2}{I_1 I_2} K_1 K_2 = K_1 K_2 \left(\frac{N_1 \phi_1}{I_1} \right) \left(\frac{N_2 \phi_2}{I_2} \right) \\ &= K_1 K_2 L_1 L_2 \quad \left[\because L_1 = \frac{N_1 \phi_1}{I_1} \text{ and } L_2 = \frac{N_2 \phi_2}{I_2} \right] \end{aligned}$$

$$\therefore M = \sqrt{K_1 K_2} \times \sqrt{L_1 L_2} \quad (5.36)$$

So the coefficient of coupling,

$$K = \sqrt{K_1 K_2} = \frac{M}{\sqrt{L_1 L_2}} \quad (5.37)$$

Hence the coefficient of coupling is dependent upon the mutual inductance as well as the self-inductance of the individual coils.

Dependence of mutual inductance of two coils: Mutual inductance as produced by two coils depends on:

- (i) distance (d) between the coils,
- (ii) orientation of the coils,
- (iii) size and shape of the coils,
- (iv) number of turns of the two coils and
- (v) nature of material on which the coils are wound.

For example, the mutual inductance between two long coaxial solenoids is,

$$M = \mu_0 n_1 n_2 A d \quad (\text{for air medium}) \quad (5.38) (a)$$

$$= \mu n_1 n_2 A d \quad (\text{for other medium}) \quad (5.38) (b)$$

where n_1 and n_2 are the number of turns per unit length in the primary and secondary coils respectively, L is the length of the solenoid, A is the area of the inner coil, μ_0 is the permeability of free space and μ is the permeability of the said medium.

5.11 ENERGY STORED IN A COIL IN A MAGNETIC FIELD

In order to produce a magnetic field around a coil, energy is required; but no energy is needed to maintain it. This energy is stored in the magnetic field and is not used up. For a decrease in the current there is also a decrease in the flux linked, causing the stored energy to be returned to the circuit. Now, let us consider an inductor connected to a dc source as shown in Fig. 5.10 (a).

The inductor is equivalent to inductance L in series with a small resistance R as shown in Fig. 5.10 (b). The energy supplied to the circuit is spent in two ways:

- (i) A part of the supplied energy is spent to meet $I^2 R$ losses and cannot be recovered.
- (ii) The remaining part is spent to create flux around the coil (or inductor) and is stored in the magnetic field. When the field collapses, the stored energy is returned to the circuit.

Mathematical Expression: Suppose at any instant the current in the coil is i and is increasing at the rate of $\frac{di}{dt}$. Then *e.m.f.* e across L is given by;

$$e = L \frac{di}{dt}$$

$$\therefore \text{Instantaneous power, } p = ei = Li \frac{di}{dt} \quad (5.39)$$

During a short interval of time dt , the energy dW put into the magnetic field is equal to power multiplied by time *i.e.*,

$$dW = p \cdot dt = \left(Li \frac{di}{dt} \right) dt = Li \, di$$

The total energy put into the magnetic field from the time when current is zero until it has attained the final steady value I is,

$$W = \int_0^I Li \, di = \frac{1}{2} LI^2 \quad (5.40)$$

$$\therefore \text{Energy stored in magnetic field} = \frac{1}{2} LI^2 \text{ joules}$$

Note that energy stored will be in joules if inductance (L) and current (I) are in henry and ampere respectively. If current in an inductor varies, the stored energy rises and falls in step with the current. Thus, whenever current increases, the coil absorbs energy and whenever current falls, energy is returned to the circuit.

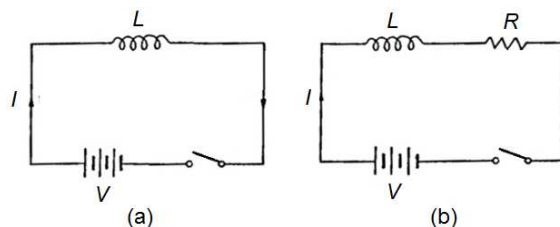


Fig. 5.10: (a) An inductor connected to a dc source, (b) energy stored in a coil in a magnetic field

Example 5.22 Find out the energy stored in a coil of inductance 0.5 H, if the current flowing through it is 50 mA.

Solution

$$\text{Energy stored, } W = \frac{1}{2} LI^2 = \frac{1}{2} \times 0.5 \times (50 \times 10^{-3})^2 = 0.625 \text{ mJ.}$$

EXAMPLE 5.22

Example 5.23 Find out the inductance of a coil, if a 2 A current flowing through the coil produces 60 J energy.

Solution

$$\text{We have, } W = \frac{1}{2} LI^2$$

So, inductance of the coil will be,

$$L = \frac{2W}{I^2} = \frac{2 \times 60}{(2)^2} = 30 \text{ H.}$$

EXAMPLE 5.23

Example 5.24 When 5 A is passed through a coil of 1200 turns 30 mWb flux is linked. Find the (a) inductance, (b) energy stored and (c) average induced *e.m.f.* in the coil if the current is reduced to zero in 250 ms.

Solution

$$\text{(a) Inductance of coil, } L = \frac{N\phi}{I} = \frac{1200 \times 30 \times 10^{-3}}{5} = 7.2 \text{ H.}$$

$$\text{(b) Energy stored, } W = \frac{1}{2} LI^2 = \frac{1}{2} \times 7.2 \times (5)^2 = 90 \text{ J.}$$

$$\text{(c) Induced e.m.f., } E = L \frac{dI}{dt} = 7.2 \times \frac{5-0}{0.25} = 144 \text{ V.}$$

EXAMPLE 5.24

Example 5.25 The diameter of an air-cored solenoid is 5 cm and its length is 50 cm. It is wound with 1000 turns. If a current of 1 A flows in the solenoid, find the (a) inductance and (b) energy stored.

Solution

$$\text{Here } a = \frac{\pi d^2}{4} = \frac{\pi}{4} \times 25 \times 10^{-4} = 19.63 \times 10^{-4} \text{ m}^2.$$

$$\text{(a) Inductance } L = \frac{N^2 a \mu_0 \mu_r}{l} = \frac{(1000)^2 \times 19.63 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1}{0.5} = 4.94 \text{ mH.}$$

$$\text{(b) Energy stored } \frac{1}{2} LI^2 = \frac{1}{2} \times 4.94 \times 10^{-3} \times (1)^2 = 2.47 \text{ mJ.}$$

EXAMPLE 5.25

5.12 EDDY CURRENTS

Whenever there is a change in magnetic field inside a conductor as a consequence of Faraday's law of electromagnetic induction, loops of electric current is induced within the conductor. These are known as the eddy currents. These flows in the perpendicular direction of the magnetic field within the conductor in closed loops. Because of the time-varying magnetic field the eddy currents are induced to the closely situated stationary conductors. This eddy current may also be induced owing to the relative motion between a close conductor and the concerned field produced by the magnet. In a given loop the magnitude of these current depends directly on:

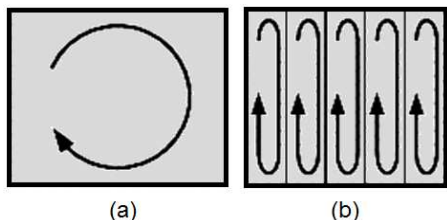


Fig. 5.11: (a) A conducting block with high eddy currents and (b) a laminated block with low eddy currents

- i) the magnetic field strength,
 - ii) the loop area, and
 - iii) the rate of change in magnetic flux,
- and is inversely proportional to -
- i) the resistivity of the material.

Eddy current produces a magnetic field following Lenz's law which opposes the produced field itself. The eddy current thus takes the role to counter back the original source of the magnetic field.

This phenomenon can be well employed in electrical brakes. These are generally used to stop the rotating power tools suddenly, after they are turned off.

The current flowing through the resistance of the conductor also dissipates energy as heat in the material. Eddy current is the sources of energy loss in alternating current (AC) machineries like the inductors, transformers, electric motors, electric generators etc. due to dissipation of heat while current flows through the resistance of those machineries. In order to minimize the effect of eddy currents some special constructions, like the laminated magnetic cores, are recommended.

In induction type of heating equipments with a view to heat the objects, eddy currents can be successfully utilized. They can also be used to detect flaws or cracks in metal parts by the use of proper testing arrangements.

5.13 ELECTROMAGNETIC BRAKING

Magnetic braking works due to induced currents and Lenz's law. If a metal plate is attached to the end of a pendulum and the pendulum is allowed to swing, its speed will largely decrease when it goes between the poles of a magnet. When the plate goes through the magnetic field, an electric field is induced in the plate and circulating eddy currents are thereby generated.

These currents act to oppose the change in flux through the plate to make Lenz's law valid. The currents in turn make the plate heated, thereby reducing the kinetic energy of the plate. This is the basic principle behind *electromagnetic braking*.

The above phenomenon is used for variety of purposes such as, for separating the nonmagnetic metals from the solid waste or for eliminating the vibrations produced in any spacecrafts. When electric current is passed through the electromagnet and the electromagnet is moved along the rail, eddy currents are generated in the rail. These eddy currents generate an opposing magnetic field, providing braking force.

UNIT SUMMARY

- **Magnetic flux**

$$d\phi = \vec{B} \cdot d\vec{S} = B dS \cos \theta$$

- **Magnetic flux density**

$$B = \frac{d\phi}{dS}$$

- **Faraday's law of Electromagnetic Induction**

$$e = \frac{d}{dt}(N\phi) = N \frac{d\phi}{dt}$$

- **Lenz's law**

$$e = -N \frac{d\phi}{dt} \text{ volts}$$

- **Fleming's Right-Hand rule**

Stretching out the forefinger, middle finger and thumb of the right hand at right angles to one another; if the forefinger points in the direction of magnetic field, thumb in the direction of motion of the conductor, then the middle finger will point in the direction of induced current.

- **Induced E.M.F.**

Dynamically induced *e.m.f.* $e = Blv \sin \theta$

- **Self-Inductance**

$$L = \frac{e}{\frac{dI}{dt}}$$

- **Mutual Inductance**

$$M = \frac{e_M}{\frac{dI_1}{dt}}$$

- **Coefficient of Coupling**

$$K = \sqrt{K_1 K_2} = \frac{M}{\sqrt{L_1 L_2}}$$

- **Energy stored in a coil placed in a magnetic field**

$$\frac{1}{2} LI^2 \text{ joules}$$

- **Eddy currents**

Eddy currents are a source of energy loss in alternating current (AC) inductors, transformers, electric motors and generators

- **Electromagnetic braking**

This phenomenon can be used to eliminate vibrations in spacecrafts and to separate nonmagnetic metals from solid waste.

EXERCISES

Multiple Choice Questions

- 5.1 Electromotive force induced by motion of conductor across magnetic field is called
(a) *e.m.f.* (b) motional *e.m.f.* (c) static *e.m.f.* (d) rotational *e.m.f.*
- 5.2 Self induced *e.m.f.* is sometimes also known as
(a) induced *e.m.f.* (b) deduce *e.m.f.* (c) back *e.m.f.* (d) both (a) and (b)
- 5.3 If the magnetic field is stationary and the conductor is in motion then the *emf* induced is called
(a) dynamically induced *e.m.f.* (b) statically induced *e.m.f.*
(c) self induced *e.m.f.* (d) mutually induced *e.m.f.*
- 5.4 Which among the following sentences is correct?
(a) Lenz's law is used to find the direction of induced current
(b) Fleming's right hand rule is used to find the direction of induced current
(c) Fleming's left hand rule is used to find the direction of force
(d) all of these
- 5.5 If current in a conductor increases then according to Lenz's law self-induced voltage will
(a) aid the increasing current (b) tend to decrease the amount of current
(c) produce current opposite to the increasing current
(d) aid the applied voltage
- 5.6 The magnitude of the induced *e.m.f.* in a conductor depends on the
(a) flux density of the magnetic field (b) amount of flux cut
(c) amount of flux linkages (d) rate of change of flux linkages
- 5.7 In case of an inductance, current is proportional to
(a) voltage across the inductance (b) magnetic field
(c) both (a) and (b) (d) neither (a) nor (b)
- 5.8 The self inductances of two coils are 8 mH and 18 mH. If the coefficients of coupling is 0.5, the mutual inductance of the coils is
(a) 4 mH (b) 5 mH (c) 6 mH (d) 12 mH
- 5.9 In case all the flux from the current in coil 1 links with coil 2, the co-efficient of coupling will be
(a) 2.0 (b) 1.0 (c) 0.5 (d) zero
- 5.10 The magnetic flux linked with a coil at any instant t is given by $\phi_t = 5t^3 - 100t + 200$. The *emf* induced in the coil at $t = 2$ seconds is
(a) 200 V (b) 40 V (c) 20 V (d) - 20 V

- 5.11 The co-efficient of coupling between two air core coils depends on
 (a) self-inductance of two coils only (b) mutual inductance between two coils only
 (c) mutual inductance and self inductance of two coils
 (d) none of the above
- 5.12 Magnitude of induced *e.m.f.* depends on
 (a) speed of motion of coil and magnet (b) number of turns of coil
 (c) current passing through coil (d) both (a) and (b)
- 5.13 Lenz's law is a consequence of the law of conservation of
 (a) induced current (b) charge (c) energy (d) induced *e.m.f.*

Answers of Multiple Choice Questions

5.1 (b), 5.2 (c), 5.3 (a), 5.4 (d), 5.5 (c), 5.6 (d), 5.7 (b), 5.8 (d), 5.9 (b), 5.10 (b), 5.11 (c), 5.12 (d), 5.13 (c)

Short and Long Answer Type Questions

Category I

- 5.1 Explain Faraday's laws of electromagnetic induction. Obtain its integral form and hence find out the differential form.
- 5.2 Find out an expression for energy stored in a magnetic field.
- 5.3 On what factors the magnitude of eddy current depends?
- 5.4 Give some applications of magnetic effect of electric current.
- 5.5 Mention some applications of electromagnetic braking.
- 5.6 Show that self inductance is directly proportional to square of the number of turns and inversely proportional to reluctance of the magnetic path.
- 5.7 Mutual inductance depends on the reluctance of the magnetic circuit - Justify.
- 5.8 How coefficient of coupling depends upon the mutual inductance between two coils also upon the self-inductance of these coils?
- 5.9 Explain the basic principle used in electromagnetic braking.
- 5.10 Find out the SI unit of magnetic flux and magnetic flux density from their definition.
- 5.11 From the statement of Lenz's law establish principle of conservation of energy.
- 5.12 Briefly outline Fleming's right-hand rule.
- 5.13 Express your idea about dynamically and statically induced *e.m.f.*
- 5.14 Give an outline about the coefficient of coupling between two coils, in your own way.
- 5.15 Briefly outline motional *e.m.f.* and obtain an expression for it. Can you obtain the direction of induced *e.m.f.* and current?

- 5.16 Starting from the definition of self and mutual inductance find out an expression for them.
- 5.17 Justify the reason for a variation of magnetic flux from some maximum to zero value.
- 5.18 Give your opinion for the necessity of modifying the Faraday's law as was done by Lenz.
- 5.19 "The integral and differential forms of Faraday's law of electromagnetic induction are used for some specific purposes." Give your interpretation.
- 5.20 In the light of Lenz's law how can you consider the principle of conservation of energy?
- 5.21 The Fleming's right hand rule suitable to find the direction of induced *e.m.f.* and hence current when the conductor moves at right angles to a stationary magnetic field. Are you of same opinion? Elaborate.
- 5.22 "Direction of the current through the conductor may be obtained from Fleming's right hand rule" – Justify.
- 5.23 Mutually induced *e.m.f.* can be achieved in practice using certain simple technique. Give your opinion with proper explanation.
- 5.24 Give a simple method for demonstrating self inductance.
- 5.25 Mention the parameters on which inductance of a coil depend.
- 5.26 Elaborate the dependence of the mutual inductance of two coils.
- 5.27 An inductor equivalent to an inductance L in series with a small resistance R is connected. The energy supplied to the circuit is spent in two ways. Outline those two ways.
- 5.28 Give the way for flow of eddy currents within conductors.
- 5.29 Electromagnetic braking can be used for eliminating vibrations in spacecrafts. Briefly mention the method.

Category II

- 5.30 Can you define henry from the definition of self-induced *e.m.f.* and mutually induced *e.m.f.*?
- 5.31 Mention the primary requirement for an electromagnetic induction.
- 5.32 Give the outline for measuring the magnetic flux for practical purposes.
- 5.33 A metallic bar slides frictionless on two parallel conducting rails at distance l apart. A resistor R is connected across the rails and a uniform magnetic field B , pointing into this plane fills the entire region. If the bar moves to the right a constant speed v , then what is the current in the resistor?
- 5.34 Magnetic flux is a scalar quantity while the magnetic flux density is a vector. Explain the reason. Give the units and dimensions of these two quantities.

Numerical Problems

- 5.1 When the current in the primary coil of 400 turns of a magnetic circuit increases linearly from 10 mA to 35 mA in 100 ms, an *e.m.f.* of 75 mV is induced into the secondary coil of 240 turns, which is left open circuited. Determine (a) the mutual inductance of the two coils, (b) the reluctance of the former, and (c) the self inductance of the secondary coil.

[Ans: 0.3 H, 320 kA/Wb, 0.18 H]

- 5.2 A coil of 200 turns of wire is wound on a magnetic circuit of reluctance 2000 AT/Wb. If a current of 1 A flowing in the coil is reversed in 10 ms, find the average *e.m.f.* induced in the coil. [Ans: 4 V]
- 5.3 A toroid of mean diameter 400 mm and cross-sectional area 400 mm² is uniformly wound with a coil of 1000 turns which carries a current of 2 A. Determine (i) self-inductance and (ii) the *e.m.f.* induced in the coil when the current is uniformly reduced to zero in 10 ms. [Ans: 0.4×10^{-3} H, 0.08 V]
- 5.4 A conductor of length 0.5 m situated in and at right angles to a uniform magnetic field of flux density 1 Wb/m² moves with a velocity of 40 m/s. Calculate the *e.m.f.* induced in the conductor. What will be the *e.m.f.* induced if the conductor moves at an angle 60° to the field? [Ans: 20 V, 17.32 V]
- 5.5 A 300-turn coil has a resistance of 6 Ω and an inductance of 0.5 H. Determine the new resistance and new inductance if one-third of the turns are removed. Assume all the turns have the same circumference. [Ans: 4 Ω, 0.22 H]
- 5.6 The energy stored in the magnetic field of an inductor is 80 J when the current flowing in the inductor is 2 A. Calculate the inductance of the coil. [Ans: 40 H]
- 5.7 An iron ring has a cross-sectional area of 500 mm² and a mean length of 300 mm. It is wound with 100 turns and its relative permeability is 1600. Calculate (a) the current required to set up a flux of 500 μWb in the coil, and (b) the inductance of the system, and (c) the induced *e.m.f.* if the field collapses in 1 ms. [Ans: 1.492 A, 33.51 mH, - 50 V]
- 5.8 A single turn coil having an area of 50 m² for each turn is held perpendicular in a uniform field of 0.008 T. Calculate the induced *e.m.f.* in the coil, if the field is removed in 0.4 s. [Ans: 10 V]

PRACTICALS

1. Study of Faraday's law and Lenz's law of electromagnetic induction

Aim

- i) Study of Faraday's law of electromagnetic induction and Lenz's law of electromagnetic induction qualitatively
- ii) To determine the direction of the current induced in a given coil due to a change in the magnetic flux (by inserting and removing an electromagnet or a bar magnet in the coil) inside it

Apparatus

Galvanometer, Coils (primary and secondary), Battery (1.5 V), Bar magnet, Aluminum and Iron rod, Connecting wires

Theory

According to Faraday's law of induction, if a permanent magnet, B_{original} , is inserted into (or pulled out of) a coil, a current I_{ind} is induced in the windings of this coil. The current is to be maintained if only the magnet is inserted into or taken out from the experimental coil.

The induced current present there will produce the coil as an electromagnet, B_{ind} . It has N and S poles and like any other magnet, it will exert forces on other magnets and magnetic materials. Following the Fleming's right hand rule, if one grasp the coil using the right hand in a manner so that fingers point the direction of the induced current, the thumb will direct towards the N-pole while the other end will represent the S-pole.

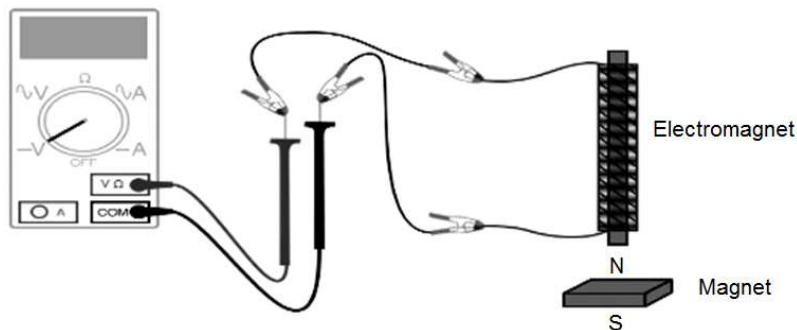


Fig. (i)

Procedure

- Conventionally when the electric current enters the ammeter (or the galvanometer) through the positive terminal, its needle deflects to the right direction. This convention will help us to find the direction of the induced current within the solenoid.
 - Look at the secondary, *i.e.*, the large solenoid for identifying the two small holes at the bottom wherefrom wires are coming out. These assist to find induced current direction of the solenoid. This measurement is too important as it permits to find the consistency with Lenz's law.
 - Now, the galvanometer is to be connected to the secondary and insert the N- pole in the solenoid to observe the deflection in galvanometer. After this part of your finding, figure out the direction of the induced current that will produce in the windings of the solenoid. Using Fleming's right hand rule, figure out the end of the coil to identify the N-pole and the S-pole. Draw a diagram to present the movement of the magnet, direction of induced current flowing in the solenoid as well as the N-pole and the S-poles of the solenoid.
 - See the speed of the bar magnet owing to the magnitude of the induced current.
 - Now move the magnet away from the coil and observe the records similar to those in last two steps.
 - Now hold the magnet with its S-pole close to the solenoid and repeat the last three steps.
 - Determine whether the induced field of the solenoid attracts or repels the permanent magnet in each case.
 - Replace the permanent magnet with an electromagnet (solenoid connected to a battery) and repeat the previous steps. Connect the primary coil to the battery as presented in Fig. (i).
- Repeat steps (3) to (6) by making use of the primary solenoid as magnet.
- After connecting the primary to the battery, put it in the secondary. Now disconnect it from the battery present in the circuit and observe the deflection. Draw two diagrams similar to the ones above.
 - Insert iron rod inside the primary and note its effect on the magnitude of induced current, when disconnected and reconnected to the battery.

Analysis

1. Draw diagrams to show the direction of the induced current inside the solenoid. Indicate from your study the end of the solenoid representing N and S. Next to the diagram, indicate whether the force between the bar magnet and the electromagnet is attraction or repulsion. Is this force consistent with the Lenz's law? You have to consider 10 such figures. Out of those, four utilizing permanent magnet, four utilizing the primary coil as the magnet while two other to be implemented by disconnecting and reconnecting the primary when it lies within the secondary.

2. Study of the resonance condition of a series LCR circuit

Aim

- To obtain the condition of resonance for the case of a series LCR circuit
- To determine the quality factor and the bandwidth for different resistance values

Apparatus

Resistors, Inductor, Capacitor, Connecting wires, Multi-meter; Function generator (0-30 kHz, sinusoidal)

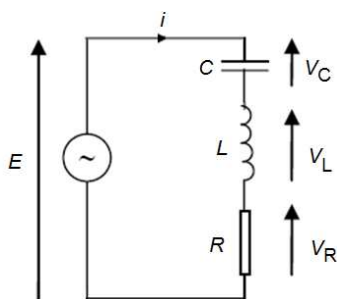


Fig. (i)

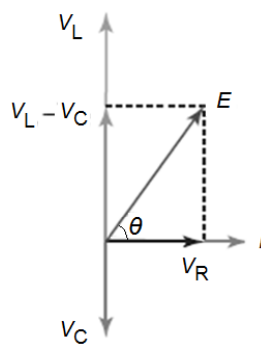


Fig. (ii)

Theory

LCR circuit refers to electrical circuit having a resistance (R), an inductance (L) and a capacitance (C), connected either in series or parallel. The circuit develops a harmonic oscillator for current, resulting the resonances at a particular frequency. This is an important property of the circuit to achieve the condition for resonance at a particular frequency which is referred as the resonance frequency f_{res} . It is a frequency at which the impedance of the circuit is minimum. Alternatively, it is defined as the frequency at which the impedance is purely resistive in nature. This occurs because at resonance the inductive reactance ($X_L = \omega L$) and capacitive reactance ($X_C = 1/\omega C$) are equal but of opposite sign and cancel each other. The corresponding angular frequency is given by:

$$\omega_{\text{res}} = 2\pi f_{\text{res}}$$

At resonance $X_L = X_C$.

$$\text{or, } \omega_{\text{res}} L = \frac{1}{\omega_{\text{res}} C}$$

or,
$$\omega_{res} = \frac{1}{\sqrt{LC}} \quad (i)$$

Here X_L and X_C are respectively at an angle $+90^\circ$ and -90° w.r.t. R . Thus the angular difference between them (X_L and X_C) is 180° which tends to cancel them out.

Introduction of the resistance increases the decay of these oscillations and this feature is commonly known as the damping. The resistance also reduces the peak resonant frequency. Likewise, the LCR circuit will also be damped if there is no driving AC power source in the circuit.

To analyze the study of resonance in a series LCR circuit bandwidth and Q -factor are the two important factors. The bandwidth measures the difference between the frequencies where the power through the circuit falls to half of the value at resonance. If ω_1 and ω_2 are the lower and upper half power frequencies, the bandwidth can be defined as

$$\Delta\omega = \omega_2 - \omega_1 \quad (ii)$$

Q -factor, on the other hand, is marked as the maximum energy stored in any circuit divided by the dissipated average energy per radian when resonance occurs. So,

$$Q = \frac{\omega_{res}}{\Delta\omega} \quad (iii)$$

Low Q circuits are therefore damped and lossy and high Q circuits are underdamped. Q is also related to bandwidth; low Q circuits are referred as wide band and high Q circuits are referred as narrow band. For an LCR circuit it can be shown that Q is given by

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} \quad (iv)$$

Procedure

1. Connect the circuit properly according to the circuit diagram and after making sure that the circuit is properly connected switch the power supply ON.
2. Set the multi-meter in AC mode and choose the appropriate range of the voltage (in V) on the dial.
3. Keeping the digital function generator in kHz/Frequency mode and setting the dc-offset voltage to ± 0 V, the peak-to-peak voltage for the ac input can be fixed at 6 V all throughout the measurements. This will ensure the *safety limits* for the voltages across the circuit elements.
4. Keeping the values of L and C constant, select any value of R . Measure the voltages V_R across R corresponding to various AC input frequencies. At the resonant frequency (f_{res}) there will be highest voltage value across the resistance R .
5. Draw a graph between the voltage V_R and frequency f and from the highest location of the voltage graph, to get the frequency of resonance f_{res} of LCR series circuit can be deduced.
6. Measure V_R , V_L and V_C at resonant frequency.
7. Repeat measurements according to the last three steps for each value of the resistor given. Compare the obtained V_R vs. f curves for chosen values of the resistances.

Results

Table 1: Variation of voltage with frequency for fixed value of resistors

Inductance =mH.

Capacitance = μ F.

Resistance (Ω)	Frequency f (Hz)	Voltage V_R (V)
R = fixed value		

Table 2: Voltages across R , L and C and corresponding resonant frequency

V_R (V)	V_L (V)	V_C (V)	f_{res} (Hz)

The quality factor (Q) can be calculated as: $Q = \frac{X_L}{R} = \frac{2\pi f_{res} L}{R}$ (v)

The bandwidth is given by: $BW = \frac{f_{res}}{Q}$ (vi)

Table 3: Comparison of bandwidth and quality factor for different values of resistors

Resistance (Ω)	Bandwidth (BW)	Quality factor (Q)

Precautions

1. Check all the connections properly (especially for the multi-meter and digital function generator).
2. Choose proper values of resistance, inductance and capacitance to perform the experiment.
3. Ensure that the voltage and current values are in the expected limits. Also be certain that the wattages of resistors are within the limit and not crossing the value. In a similar way, it is required to ascertain that the highest allowed voltage rating for the capacitor is within the limit and not exceeding.
4. Near f_{res} take readings for smaller steps in frequency in order to find the exact value of the maximum voltage V_{Rmax} and the frequency f_{res} at which resonance occurs.

KNOW MORE

Michael Faraday (born September 22, 1791 at Newington, Surrey, England; died August 25, 1867 at Hampton Court, Surrey) was an English physicist and chemist. His many experiments contributed largely to the understanding of electromagnetism.

Activity

Faraday showed that by keeping a conductor in a changing magnetic field, it would produce voltage across the conductor. He thus established a way to cause an electric current. This discovery was later applied to large number of devices we use today.

Interesting facts

Michael Faraday developed two fundamental components of magnetic storage. His inventions largely transformed the home, farm and factory. Faraday's discoveries revolutionized work for small-time farmers. Electricity minimized manual labor like pumping water, automated systems for tasks like milking cows kept farmers from crippling their hands. From cell phones to air conditioning, all the modern conveniences were not possible without Faraday's discoveries.

Analogy

Faraday invented the electric motor, transformer and generator. Without the knowledge of electromagnetic induction, we wouldn't have wireless energy transfer or pickups for the electrical guitar, either. In fact, Faraday turned electricity from amusement to practical and wide-ranging uses.

History

Faraday discovered electromagnetic induction in 1831.

Timelines

Contemporary documents

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- “Michael Faraday” (<http://www.chemheritage.org/discover/online-resources/chemistry-in-history/themes/electrochemistry/faraday.aspx>)
- “Joseph Henry Biography” (http://www.ieeehqn.org/wiki/index.php/Joseph_Henry)

Applications (Real Life / Industrial)

Michael Faraday (1791-1867) is well known for his discovery of the interaction between electricity and magnetism forming the principles of electro-magnetic induction and electro-magnetic rotation.

Both play vital roles in the magnetic recording and electric motor technologies of modern data storage systems.

Case Study (Environmental / Sustainability / Social / Ethical Issues)

In a series of lectures to the Royal Society in London, Faraday in 1831 described the results of his experiments. He demonstrated the production of a “current of electricity by ordinary magnets.”

Faraday used a liquid battery to send electric current through a small coil. When it was moved in or out of a larger coil, its magnetic field induced a transient voltage in the small coil which was identified by a galvanometer.

James Clerk Maxwell (1831-1879) mathematically expressed the time varying aspect of electromagnetic induction as a differential equation that was familiar as Faraday’s law.

Inquisitiveness and Curiosity Topics

Faraday, one of the greatest scientists of the 19th century, started his career as a chemist. He wrote a manual of practical chemistry which reflects his expertisation of the technical aspects of his art. He discovered many new organic compounds including benzene. He was the first to liquefy a “permanent” gas. However, his major contribution was in the field of electricity and magnetism and in fact he was the first person who was able to produce an electric current from a magnetic field.

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11. <https://nptel.ac.in/content/storage2/courses/115101005/downloads/lectures-doc/Lecture-29.pdf>
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6

Maxwell's Equations

UNIT SPECIFICS

This unit is focussed on the following main aspects:

- Continuity equation for current densities;
- Concept of displacement current;
- Maxwell's field equations;
- EM energy flow;
- Poynting theorem;
- Poynting vector;
- Momentum in electromagnetic fields;
- Resultant pressure in electromagnetic fields.

The applications of the topics are discussed for generating further curiosity and creativity as well as improving problem solving capacity.

Besides giving a large number of multiple choice questions as well as questions of short and long answer types marked in two categories following lower and higher order of Bloom's taxonomy, assignments through a number of numerical problems, a list of references and suggested readings are given in the unit so that one can go through them for practice. It is important to note that for getting more information on various topics of interest some QR codes have been provided in different sections which can be scanned for relevant supportive knowledge.

After the end of the unit, based on the content, there is a “Know More” section. This section has been judiciously designed so that the supplementary information provided in this part becomes beneficial for the users of the book. This section mainly highlights the initial activity, examples of some interesting facts, analogy, history of the development of the subject focusing the salient observations and finding, timelines starting from the development of the concerned topics up to the recent time, applications of the subject matter for our day-to-day real life or/and industrial applications on variety of aspects, case study related to environmental, sustainability, social and ethical issues whichever applicable, and finally inquisitiveness and curiosity topics of the unit.

RATIONALE

This unit on Maxwell's Equations will help students to get a theoretical idea about the continuity equation for current densities and the concept of displacement current which can help to explain Maxwell's field equations. Understanding of EM energy flow and Poynting vector as well as the calculation of momentum in electromagnetic fields and resultant pressure are other topics to develop idea about the propagation of electromagnetic waves.

All the four equations of Maxwell explain the electric and magnetic fields originating from the distributions of electric charges and currents as well as type of variation of those fields with time. These equations are considered as the mathematical distillation of so many experimental observations of electric and magnetic effects of charges and currents as reported over the decades. It is interesting that Maxwell's own contribution when analyzed all these equations is just the last term of the fourth equation. It justified for the first time that the changing electric and magnetic fields are interrelated to each other and these fields could propagate through space indefinitely. Maxwell's new terminology displacement current interpreted the propagation of EM wave through space in a self-sustaining manner and was able to predict their velocity same as the velocity of light.

PRE-REQUISITES

Mathematics: Vector Calculus (Class XII)

Physics: Electrostatics, Electromagnetism (Class XII)

UNIT OUTCOMES

List of outcomes of this unit is as follows:

U6-O1: Define Continuity equation for current densities

U6-O2: Explain concept of displacement current

U6-O3: Explain Maxwell's field equations

U6-O4: Explain about E.M. energy flow and Poynting Vector

U6-O5: Calculate momentum in electromagnetic fields and resultant pressure

Unit-6 Outcomes	EXPECTED MAPPING WITH COURSE OUTCOMES (1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation)					
	CO-1	CO-2	CO-3	CO-4	CO-5	CO-6
U6-O1	2	1	-	-	-	1
U6-O2	2	2	-	1	-	1
U6-O3	-	-	1	3	-	2
U6-O4	-	-	-	-	-	-
U6-O5	1	-	-	-	-	-

6.1 INTRODUCTION

From late 19th century scientists were largely convinced to the fact that electricity and magnetism are somehow related and unified. Wherever an electric charge is in motion electric current results and as a consequence magnetism is developed. The source for electric field is electric charge, whereas that for magnetic field is electric current *i.e.*, the moving charges. When there is any variation in the magnetic flux linked with a coil, the induction of an electromagnetic force so produced, is termed as the *electromagnetic induction*. The basic laws of electromagnetic induction were proposed by Michael Faraday to relate time-varying magnetic field with electric field. In this unit we shall deal with the time-varying electro-magnetic fields and consequently the behavior of electromagnetic waves by studying Maxwell's equations for electro-magnetic fields in different media.

6.2 DISPLACEMENT CURRENT

The term displacement current may be defined as the rate of change of the electric displacement field. It is a quantity which has important role in Maxwell's equations. The fundamental relationships among the static electric fields quantities can be expressed as:

$$\vec{\nabla} \times \vec{E} = 0; \vec{\nabla} \cdot \vec{D} = \rho \quad (6.1)$$

where for a linear and isotropic medium $\vec{D} = \epsilon \vec{E}$.

Similarly for the magnetostatic fields fundamental relationships among the field quantities can be summarized as:

$$\vec{\nabla} \cdot \vec{B} = 0; \vec{\nabla} \times \vec{H} = \vec{J} \quad (6.2)$$

where $\vec{B} = \mu \vec{H}$

Eq. (6.1) and Eq. (6.2) give the relationship among the electric and magnetic field quantities in the static field. For time varying case, the relationship among them may be expressed as,

$$\vec{\nabla} \times \vec{H} = \vec{J} \quad (6.3) \text{ (a)}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (6.3) \text{ (b)}$$

$$\vec{\nabla} \cdot \vec{D} = \rho \quad (6.3) \text{ (c)}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (6.3) \text{ (d)}$$

In addition, from the principle of conservation of charges we get the equation of continuity as

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \quad (6.4)$$

The Eq. (6.3) (a)-(d) must be consistent with Eq. (6.4).

Thus we observe that $\vec{\nabla} \cdot \vec{\nabla} \times \vec{H} = \vec{\nabla} \cdot \vec{J} = 0$.

This is because $\vec{\nabla} \cdot \vec{\nabla} \times \vec{A}$ becomes zero for any vector \vec{A} . Thus $\vec{\nabla} \times \vec{H} = \vec{J}$ applies only for the static case *i.e.*, for the scenario when $\frac{\partial \rho}{\partial t} = 0$.

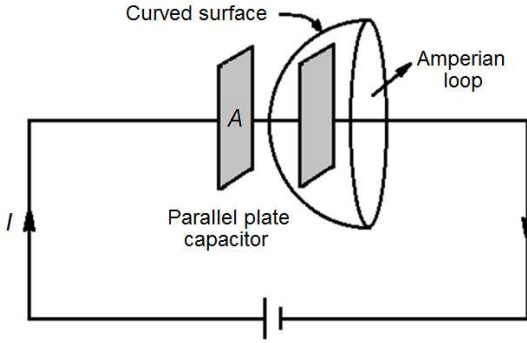


Fig. 6.1: Concept of displacement current

Let us assume that we are charging a capacitor (Fig. 6.1). For the loop we apply the Ampere's law as exhibited in the figure. Let I is the total current flowing through the said loop. But if we draw another curved surface as in Fig. 6.1, no current passes through this surface and hence enclosed current $I = 0$. As in this case for non-steady currents the concept of current enclosed by a loop is vague as it is dependent on the type of the surface used. In fact Ampere's law should also hold true for time varying case as well, then comes the idea of displacement current. Hence for time varying case,

$$\nabla \cdot \nabla \times \vec{H} = 0 = \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = \nabla \cdot \vec{J} + \frac{\partial}{\partial t} (\nabla \cdot \vec{D}) = \nabla \cdot \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$$

$$\text{So,} \quad \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (6.5)$$

The Eq. (6.5) is valid for static as well as for time varying case. This equation shows that if there is a time varying electric field it will produce a magnetic field even under the condition when \vec{J} is not present.

In the expression $\frac{\partial \vec{D}}{\partial t}$ has a dimension of current density (which is equal to A/m^2) and is referred to as the displacement current density. Displacement current plays the key role in making the total current continuous across the discontinuity in the conduction current. Displacement current is negligible as compared to the conduction current in case of a conductor at any lower frequency than optical one.

Displacement current essentially refers to the very momentary movement of charge when an electric field is applied to the material. It is given by

$$\vec{I}_d = \frac{dQ}{dt} = \frac{d}{dt} (\sigma A) = \frac{d}{dt} (\epsilon_0 A \vec{E})$$

$$\text{or,} \quad \vec{I}_d = \epsilon_0 A \frac{d\vec{E}}{dt} \quad (6.6)$$

Thus the displacement current density will be the displacement current per unit area and can be expressed as,

$$\vec{J}_d = \frac{\vec{I}_d}{A} = \epsilon_0 \frac{d\vec{E}}{dt} = \frac{\partial}{\partial t} (\epsilon_0 \vec{E}) = \frac{\partial \vec{D}}{\partial t}$$

$$\text{or,} \quad \vec{J}_d = \frac{d\vec{D}}{dt} \quad [\text{where } \vec{D} = \epsilon_0 \vec{E}] \quad (6.7)$$

Thus the displacement current density is the rate of change of electric displacement and the current set-up in a dielectric medium due to the time-varying electric field through it.

Example 6.1 Find the displacement current within a parallel plate capacitor in series with a resistor which carries current I . Area of the capacitor plates is to be considered as A and the dielectric is vacuum.

Solution

It is given by
$$I_d = \frac{dQ}{dt} = \frac{d}{dt}(CV) = \frac{d}{dt}\left(\frac{\epsilon_0 A}{d}V\right)$$

where $Q = CV$ and $C = \frac{\epsilon_0 A}{d}$

or,
$$I_d = \frac{\epsilon_0 A}{d} \frac{dV}{dt} = \epsilon_0 A \frac{dE}{dt}.$$

EXAMPLE 6.1

Example 6.2 If the electric field across a capacitor of plate area 0.05 m^2 changes at a rate of 10^{10} V/m , find the amount of displacement current.

Solution

Here, plate area $A = 0.05 \text{ m}^2$ and $\frac{dE}{dt} = 10^{10} \text{ Vm}^{-1}\text{s}^{-1}$.

So, the displacement current is given by

$$I_d = \epsilon_0 A \frac{dE}{dt} = 8.854 \times 10^{-12} \times 0.05 \times 10^{10} = 4.43 \text{ mA}.$$

EXAMPLE 6.2

Example 6.3 For a free space, the magnetic field is expressed as, $\vec{B} = B_0 \cos(\omega t - kz) \hat{y}$.

- What would be the displacement current if no free charge is there?
- Ignoring the integration constant find an expression for E .
- Verify that Faraday's law expressed in the differential form of the electromagnetic induction is satisfied by E and B .

Solution

We have, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

i) Thus, displacement current density $\vec{J} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B}$

and displacement current $\vec{I}_d = \vec{J}A$.

Here, $\vec{B} = B_0 \cos(\omega t - kz) \hat{y}$

EXAMPLE 6.3

So,
$$\vec{J} = \frac{1}{\mu_0} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & B_0 \cos(\omega t - kz) & 0 \end{vmatrix} = \frac{B_0 k}{\mu_0} \sin(\omega t - kz) \hat{x}$$

and
$$\vec{I}_d = \vec{J}A = \frac{AB_0 k}{\mu_0} \sin(\omega t - kz) \hat{x}$$

ii) Again we have,
$$\vec{I}_d = \epsilon_0 A \frac{d\vec{E}}{dt}$$

Thus,
$$\epsilon_0 A \frac{d\vec{E}}{dt} = \frac{AB_0 k}{\mu_0} \sin(\omega t - kz) \hat{x}$$

or,
$$\frac{d\vec{E}}{dt} = \frac{B_0 k}{\mu_0 \epsilon_0} \sin(\omega t - kz) \hat{x}$$

or,
$$\vec{E} = \int \frac{B_0 k}{\mu_0 \epsilon_0} \sin(\omega t - kz) \hat{x} = \frac{B_0 k}{\mu_0 \epsilon_0 \omega} \cos(\omega t - kz) \hat{x}$$

[neglecting integration constant]

iii) Now,
$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{B_0 k}{\mu_0 \epsilon_0 \omega} \cos(\omega t - kz) & 0 & 0 \end{vmatrix} = \frac{B_0 k^2}{\mu_0 \epsilon_0 \omega} \sin(\omega t - kz) \hat{y}$$

or,
$$\vec{\nabla} \times \vec{E} = \frac{B_0 k^2 c^2}{\omega} \sin(\omega t - kz) \hat{y} = B_0 \omega \sin(\omega t - kz) \hat{y}$$

$$\left[\because c^2 = \frac{1}{\mu_0 \epsilon_0}, \omega = ck \right]$$

Again,
$$\frac{\partial \vec{B}}{\partial t} = \frac{\partial \{B_0 \cos(\omega t - kz) \hat{y}\}}{\partial t} = -B_0 \omega \sin(\omega t - kz) \hat{y}$$

Thus, $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ and this is in agreement with the differential form of Faraday's law of electromagnetic induction.

6.2.1 Conduction Current versus Displacement Current

By conduction current we mean the electric current which flows through a conductor owing to an applied potential difference while displacement current represents the current that is included for interpreting the magnetic field inside the capacitor for mounting up of charges on its plates.

Some important differences between the conduction current and displacement current are given below:

- i) Conduction current density is given by $\vec{J}_C = \sigma \vec{E}$ and the displacement current density is given by $\vec{J}_d = \frac{d\vec{D}}{dt}$.
- ii) Conduction current is the actual current flowing through a circuit, but displacement current is produced due to the time-varying electric field and is an apparent one.
- iii) Ohm's law is applicable for the conduction current while it is not so for the displacement current.

6.3 MAXWELL'S EQUATIONS

James Clerk Maxwell (1831–1879) was a Scottish scientist whose most notable achievement was to formulate the classical theory of electromagnetism bringing electricity, magnetism and optics together. All phenomena of electromagnetism may be explained with the help of four different equations commonly known as Maxwell's equations. These are discussed below:

6.3.1 Gauss's Law in Electrostatics and Dielectrics

This is the first Maxwell's equation. It states that the electric flux linked with a closed loop is equal to the charge enclosed by it divided by the permittivity of free space *i.e.*,

$$\int_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} \quad [\text{for electrostatics } \epsilon_0 \vec{E}]$$

$$\text{or,} \quad \int_S \epsilon_0 \vec{E} \cdot d\vec{S} = \int_S \vec{D} \cdot d\vec{S} = q \quad [\because \vec{D} = \epsilon_0 \vec{E}]$$

$$\text{or,} \quad \int_S \vec{D} \cdot d\vec{S} = \int_V \rho dV \quad [\because \rho = \frac{dq}{dV}] \quad (6.8)$$

This is the integral form of the equation for dielectrics.

Again using the divergence theorem Eq. (6.8) may be written as,

$$\int_V \vec{\nabla} \cdot \vec{D} dV = \int_V \rho dV$$

Equating the integrand on both sides we get,

$$\vec{\nabla} \cdot \vec{D} = \rho \quad (6.9) (a)$$

$$\text{or,} \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \left[\text{as } \vec{D} = \epsilon_0 \vec{E} \right] \quad (6.9) (b)$$

Eq. (6.9) (a) and Eq. 6.9 (b) are respectively the differential form of the equations for dielectrics and for electrostatics.

6.3.2 Gauss's Law in Magnetostatics

This is the second Maxwell's equation. It states that the magnetic flux linked with a closed loop is equal to zero *i.e.*,

$$\int_s \vec{B} \cdot d\vec{S} = 0 \quad (6.10)$$

This is the integral form of the equation.

Again using the divergence theorem the above equation may be written as,

$$\int_V \vec{\nabla} \cdot \vec{B} dV = 0$$

$$\text{or,} \quad \vec{\nabla} \cdot \vec{B} = 0 \quad [\text{as the above equation is true for any arbitrary } dV] \quad (6.11)$$

This is the differential form of the equation.

6.3.3 Faraday's Law of Electromagnetic Induction

This is the third Maxwell's equation. It states that the induced *e.m.f.* (e) in a magnetic circuit is same as the rate of change of magnetic flux (ϕ) through it and it induces in such a way that the rate of change of flux is always opposed. Mathematically the law may be stated as,

$$e = -\frac{d\phi}{dt}$$

$$\text{or,} \quad \oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \left(\iint_S \vec{B} \cdot d\vec{S} \right) \quad (6.12)$$

This is known as the integral form of Faraday's law of electromagnetic (EM) induction.

Again using Stokes' theorem of vector calculus we have,

$$\oint_C \vec{E} \cdot d\vec{l} = \iint_S \vec{\nabla} \times \vec{E} \cdot d\vec{S}$$

So, by comparing we get,

$$\iint_S \vec{\nabla} \times \vec{E} \cdot d\vec{S} = -\frac{d}{dt} \left(\iint_S \vec{B} \cdot d\vec{S} \right)$$

$$\text{or,} \quad \iint_S \vec{\nabla} \times \vec{E} \cdot d\vec{S} + \iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = 0$$

$$\text{or,} \quad \iint_S \left(\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} \right) \cdot d\vec{S} = 0$$

As, the above equation is true for any arbitrary $d\vec{S}$, we have $\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$

$$\text{or,} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (6.13)$$

This is known as the differential form of Faraday's law of EM induction.

6.3.4 Modified Ampere's Circuital Law

Ampere's circuital law was modified by Maxwell and is known as Ampere-Maxwell law or modified Ampere's circuital law. According to this modified law, for a time-varying current in free space the line integral of magnetic induction around a closed path in a magnetic field is equal to μ_0 times the net steady current (I) through a surface enclosed by the path.

From Ampere's circuital law we have,

$$\oint_C \vec{B} \cdot d\vec{l} = \oint_C \mu_0 \vec{H} \cdot d\vec{l} = \mu_0 I$$

$$\text{or,} \quad \oint_C \vec{H} \cdot d\vec{l} = I = \int_C \vec{J}_c \cdot d\vec{S} \quad \left[\because J_c = \frac{dI}{dS} \right]$$

On applying Stokes' theorem we have from the above equation,

$$\int_S \vec{\nabla} \times \vec{H} \cdot d\vec{S} = \int_C \vec{J}_c \cdot d\vec{S}$$

For any arbitrary dS , equating the integrands we have, $\vec{\nabla} \times \vec{H} = \vec{J}_c$ which is valid for steady fields.

Now, taking the divergence on both sides of this equation we have,

$$\vec{\nabla} \cdot \vec{\nabla} \times \vec{H} = \vec{\nabla} \cdot \vec{J}_c = 0$$

Again from the continuity equation for steady fields we have,

$$\vec{\nabla} \cdot \vec{J}_c + \frac{\partial \rho}{\partial t} = 0$$

$$\text{or,} \quad \vec{\nabla} \cdot \vec{J}_c = -\frac{\partial \rho}{\partial t} = 0$$

$$\text{or,} \quad \frac{\partial \rho}{\partial t} = 0$$

Therefore for time-varying fields, the total current density should be equal to the conduction and displacement current densities and hence we need to introduce the fourth Maxwell's equation, which is basically the modified form of Ampere's circuital law.

It states that the line integral of magnetic field intensity is same as the algebraic sum of surface integral of conduction and displacement current density *i.e.*,

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J}_c \cdot d\vec{S} + \int_S \vec{J}_d \cdot d\vec{S}$$

$$\text{or,} \quad \oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J}_c \cdot d\vec{S} + \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} \quad (6.14)$$

This is the integral form of the equation. Now, using Stokes' theorem in vector calculus we have,

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{\nabla} \times \vec{H} \cdot \hat{n} dS = \int_S \vec{\nabla} \times \vec{H} \cdot d\vec{S} \quad (6.15)$$

So, comparing Eq. (6.14) and Eq. (6.15) we get

$$\int_S \vec{\nabla} \times \vec{H} \cdot d\vec{S} = \int_S \vec{J}_c \cdot d\vec{S} + \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} = \int_S \left(\vec{J}_c + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S} \quad (6.16)$$

The above equation is true for any arbitrary dS and hence the integrands on both sides of the equations will be equal and thus we have,

$$\vec{\nabla} \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t} \quad (6.17)$$

This is the differential form of the equation.

Example 6.4: Prove that $\vec{E} = \cos(y-t)\hat{k}$ and $\vec{B} = \cos(y-t)\hat{i}$ constitute possible electromagnetic wave.

Solution

We have $\vec{E} = \cos(y-t)\hat{k}$ and $\vec{B} = \cos(y-t)\hat{i}$

Now,
$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & \cos(y-t) \end{vmatrix} = -\hat{i} \sin(y-t)$$

and
$$\frac{\partial \vec{B}}{\partial t} = -\hat{i} \sin(y-t)$$

So,
$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

and hence $\vec{E} = \cos(y-t)\hat{k}$ and $\vec{B} = \cos(y-t)\hat{i}$ constitute possible electromagnetic wave.

EXAMPLE 6.4

6.4 SIGNIFICANCE OF MAXWELL'S EQUATIONS

It is known that electric field lines diverge from positive charges and converge on negative charges. The electric field induced by charges can be described clearly by the electric flux density since this quantity is most closely related to the charge. There is specific significance of all the Maxwell's equations. These are described below.

6.4.1 1st equation $\vec{\nabla} \cdot \vec{D} = \rho$ or, $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

- It states that through any closed surface the total electric flux is $1/\epsilon_0$ times the total amount of charge covered by it. This represents Gauss's law in electrostatics in differential form.
- It indicates that the charge density ρ is a scalar quantity.

- iii) If ρ is positive, divergence of electric field is positive and if ρ is negative, divergence of electric field is negative.

So, electric field lines starts from positive charges (sources) and end at negative charges (sink). Thus it gives a relation between electric field and charge distribution.

- iv) As the equation does not depend on time, it represents a steady state equation.

6.4.2 2nd equation $\vec{\nabla} \cdot \vec{B} = 0$

- i) It states that the net magnetic flux through any closed surface is zero. This represents the Gauss's law in magnetostatics in differential form.
- ii) It indicates that the total number of magnetic lines of force entering into any region within a covered area is equal to the number of lines of force leaving it. Hence an isolated magnetic pole *i.e.*, magnetic monopole cannot exists. It appears only in pairs.
- iii) Hence, there is no source or sink for magnetic lines of force.
- iv) As the equation does not depend on time, it is also a steady state equation.

6.4.3 3rd equation $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

- i) It indicates that an electric field is produced due to the time rate of change of magnetic flux.

So, the equation may also be written as, $e = -\frac{\partial \phi}{\partial t}$

This represents the Faraday's law of EM induction. This is basically the differential form of the equation.

- ii) It is a time dependent equation.

6.4.4 4th equation $\vec{\nabla} \times \vec{H} = \vec{J}_C + \frac{\partial \vec{D}}{\partial t}$

- i) It indicates that a magnetic field is produced due to the conduction current density \vec{J}_C and the time rate of change of electric displacement vector \vec{D} , *i.e.*, displacement current density \vec{J}_D , jointly as well as separately.
- ii) This is the modified differential form of Ampere's circuital law for steady current as well as varying current.
- iii) It is also a time dependent equation.

6.5 EM ENERGY DENSITY

The energy density of an EM wave is found to be proportional to the square of the amplitude of the electric or magnetic field. In any EM wave, the amplitude represents the highest field strength of both electric and magnetic fields. The wave energy can be obtained by using the wave amplitude as well as the energy carried by the concerned wave.

Electric and magnetic energy density per unit volume in a region of space is termed as the electric and magnetic energy density and are respectively given as,

$$U_e = \frac{1}{2} \vec{D} \cdot \vec{E} = \frac{1}{2} \epsilon \vec{E} \cdot \vec{E} = \frac{1}{2} \epsilon E^2 \quad (6.18)$$

and

$$U_m = \frac{1}{2} \vec{B} \cdot \vec{H} = \frac{1}{2} \mu \vec{H} \cdot \vec{H} = \frac{1}{2} \mu H^2 \quad (6.19)$$

Thus, the EM energy density *i.e.*, the electromagnetic energy per unit volume in a medium is

$$U_{em} = U_e + U_m = \frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \quad (6.20)$$

In free space the EM energy density is

$$U = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \quad (6.21)$$

6.6 POYNTING VECTOR

It is popularized by putting the name of the discoverer John Henry Poynting of the theory. Poynting vector provides the transfer of energy per unit time per unit area of any EM field. The SI unit of the Poynting vector is watt per square metre (W/m²).

Thus the Poynting vector may be defined as the rate of flow of EM energy per unit area normal to the direction of energy flow and is mathematically defined as,

$$\vec{P} = \vec{E} \times \vec{H} = \frac{\vec{E} \times \vec{B}}{\mu_0} \quad (6.22)$$

It is a time varying quantity and is not applicable for static fields. It gives the EM power flowing per unit area. Its SI unit is W/m² and it is perpendicular to both *E*-field and *H*-field.



EXAMPLE 6.5

Example 6.5 Find the value of Poynting vector at a distance of 1 m from a 100 W lamp.

Solution

The total radiated power from the lamp may be written as, $\phi = 4\pi R^2 P$

Here, $\phi = 100$ W and $R = 1$ m.

So, the Poynting vector is $P = \frac{\phi}{4\pi R^2} = \frac{100}{4 \times 3.14 \times (1)^2} = 7.96$ W/m².

EXAMPLE 6.6

Example 6.6 Find the value of Poynting vector at the Sun's surface if the radiated power from the Sun is 2.8×10^{26} W and the radius of the Sun is 7×10^8 m.

Solution

The total radiated power from the Sun may be written as, $\phi = 4\pi R^2 P$.

Here, $\phi = 2.8 \times 10^{26}$ W and $R = 7 \times 10^8$ m.

So, the Poynting vector is $P = \frac{\phi}{4\pi R^2} = \frac{2.8 \times 10^{26}}{4 \times 3.14 \times (7 \times 10^8)^2} = 4.55 \times 10^7$ W/m².

Example 6.7 A very long cylinder of radius a carries uniformly distributed current I , over the cross-section of the cylinder. If the current is allowed to pass along the cylinder axis, determine the electric field in the conductor. It is given that the magnetic field is situated just outside the conductor and the Poynting vector can be noted at the surface of the cylinder. You may assume that the conductivity of the material is σ .

Solution

Using Ampere's circuital law we have the magnetic

field H as
$$\oint_C \vec{H} \cdot d\vec{l} = I$$

or,
$$H \cdot 2\pi r = I$$

or,
$$H = \frac{I}{2\pi r}$$

at $r = a$ field is
$$H = \frac{I}{2\pi a} \hat{\phi}$$

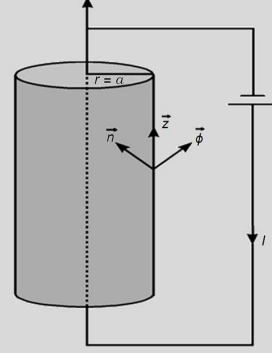
and direction of the field be tangential to the surface. We know electric field and current density is related with $\vec{J} = \sigma \vec{E}$ (let it is directed along Z-axis as shown in the figure).

As Poynting vector is $\vec{S} = \vec{E} \times \vec{H}$,

So on substituting \vec{E} and \vec{H} we have

$$\vec{P} = \frac{JI}{2\pi a \sigma} \hat{z} \times \hat{\phi} = \frac{JI}{2\pi a \sigma} \hat{n}$$

where \hat{n} is perpendicular to electric and magnetic field and acting inward to the conductor as shown in the figure.



6.7 POYNTING THEOREM

Statement: It states that the net flow of power in a given volume is equal to the time rate of decrease of stored EM energy in that volume minus the losses due to conduction.

Deduction: From Maxwell's equations we have,

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \text{ and } \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

So,
$$\vec{H} \cdot \vec{\nabla} \times \vec{E} = -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} \text{ and } \vec{E} \cdot \vec{\nabla} \times \vec{H} = \vec{E} \cdot \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$$

Thus,
$$\vec{H} \cdot \vec{\nabla} \times \vec{E} - \vec{E} \cdot \vec{\nabla} \times \vec{H} = -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{E} \cdot \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) = -\left(\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} \right) - \vec{E} \cdot \vec{J}$$

$$\begin{aligned} \text{or,} \quad \vec{\nabla} \cdot \vec{E} \times \vec{H} &= - \left(\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} \right) - \vec{E} \cdot \vec{J} \quad \left[\because \vec{H} \cdot \vec{\nabla} \times \vec{E} - \vec{E} \cdot \vec{\nabla} \times \vec{H} = \vec{\nabla} \cdot \vec{E} \times \vec{H} \right] \\ &= - \left(\mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} + \epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \right) - \vec{E} \cdot \vec{J} \end{aligned}$$

$$\text{or,} \quad \vec{\nabla} \cdot \vec{E} \times \vec{H} = - \frac{\partial}{\partial t} \left(\frac{1}{2} \mu H^2 + \frac{1}{2} \epsilon E^2 \right) - \vec{E} \cdot \vec{J} \quad (6.23) \text{ (a)}$$

$$\text{or,} \quad \vec{\nabla} \cdot \vec{P} = - \frac{\partial U_{em}}{\partial t} - \vec{E} \cdot \vec{J} \quad [\text{using Eq. (6.22)}] \quad (6.23) \text{ (b)}$$

For a particular volume of the medium integrating the above equation we get,

$$\int_V \vec{\nabla} \cdot \vec{P} dV = - \int_V \frac{\partial U_{em}}{\partial t} dV - \int_V \vec{E} \cdot \vec{J} dV$$

$$\text{or,} \quad \iint_S \vec{P} \cdot d\vec{S} = - \int_V \frac{\partial U_{em}}{\partial t} dV - \int_V \vec{E} \cdot \vec{J} dV \quad [\text{using Gauss's theorem}] \quad (6.24)$$

Eq. (6.24) is the mathematical form of Poynting theorem in which the first term represents the total power leaving the volume V bounded by the surface S , the second term represents the rate of decrease of EM energy in the volume V and the last term represents the ohmic power loss in the volume V (Fig. 6.2).

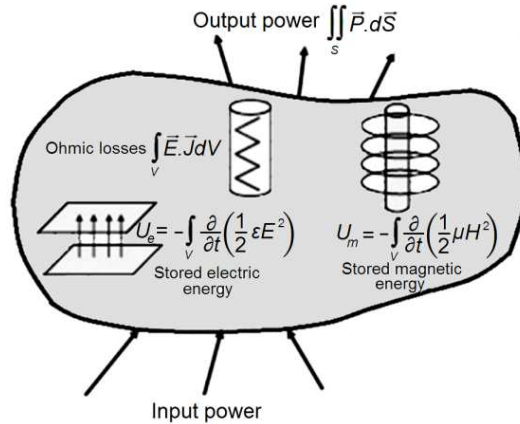


Fig. 6.2: Illustration of Poynting's theorem

6.8 ELECTROMAGNETIC POTENTIAL

The electric potential represents the work energy required for moving a unit electric charge from some arbitrary point to some specific point within an electric field. An EM potential combines both the electric scalar potential and the magnetic vector potential.

From Maxwell's second equation (Gauss's law in magnetostatics) we have, $\vec{\nabla} \cdot \vec{B} = 0$.

Again, from vector calculus, we have, the divergence of curl of a vector is zero *i.e.*, $\vec{\nabla} \cdot \vec{\nabla} \times \vec{A} = 0$.

Thus, $\vec{B} = \vec{\nabla} \times \vec{A}$ (6.25)

which gives the relation between magnetic field \vec{B} and magnetic vector potential \vec{A} .

Again from Maxwell's third equation (Faraday's law of EM induction) we have,

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{A}) = -\vec{\nabla} \times \frac{\partial \vec{A}}{\partial t}$$

or,
$$\vec{\nabla} \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$

Now, we have the curl gradient of a scalar function is zero *i.e.*, $\vec{\nabla} \times \vec{\nabla} \phi = 0$.

Thus, combining the above two equations we can write

$$\vec{\nabla} \times \left(\vec{\nabla} \phi + \vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$

or,
$$\vec{\nabla} \phi + \vec{E} + \frac{\partial \vec{A}}{\partial t} = 0$$

or,
$$\vec{E} = -\vec{\nabla} \phi - \frac{\partial \vec{A}}{\partial t}$$
 (6.26)

which gives the relation among electric field \vec{E} , magnetic scalar potential ϕ and vector potential \vec{A} .

Example 6.8 Starting from Maxwell's equation show that the electric field can be written (in terms of scalar and vector potential) as, $\vec{E} = -\vec{\nabla} \phi - \frac{\partial \vec{A}}{\partial t}$.

Solution

We have, $\vec{B} = \vec{\nabla} \times \vec{A}$.

Now, from Maxwell's third equation (Faraday's law of EM induction) we have,

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{A}) = -\vec{\nabla} \times \frac{\partial \vec{A}}{\partial t}$$

or,
$$\vec{\nabla} \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$

Again, from vector calculus the curl gradient of a scalar function is zero *i.e.*, $\vec{\nabla} \times \vec{\nabla} \phi = 0$.

Thus, combining we get
$$\vec{\nabla} \times \left(\vec{\nabla} \phi + \vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$

or,
$$\vec{\nabla} \phi + \vec{E} + \frac{\partial \vec{A}}{\partial t} = 0$$

or,
$$\vec{E} = -\vec{\nabla} \phi - \frac{\partial \vec{A}}{\partial t}.$$

Example 6.9 If ϕ is a scalar potential associated with the electric field \vec{E} and \vec{A} is the vector potential associated with the magnetic field \vec{B} , show that they must satisfy the equation $\nabla^2\phi + \frac{\partial}{\partial t}(\vec{\nabla} \cdot \vec{A}) = -\frac{\rho}{\epsilon_0}$.

Solution

We have, $\vec{B} = \vec{\nabla} \times \vec{A}$.

Now, from Maxwell's third equation (Faraday's law of EM induction) we have,

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{A}) = -\vec{\nabla} \times \frac{\partial \vec{A}}{\partial t}$$

or,
$$\vec{\nabla} \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$

Again, from vector calculus the curl gradient of a scalar function is zero *i.e.*, $\vec{\nabla} \times \vec{\nabla}\phi = 0$.

Thus, combining we get
$$\vec{\nabla} \times \left(\vec{\nabla}\phi + \vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$

or,
$$\vec{\nabla}\phi + \vec{E} + \frac{\partial \vec{A}}{\partial t} = 0$$

or,
$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$

Again from Maxwell's first equation we have,

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

or,
$$\vec{\nabla} \cdot \left(-\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} \right) = \frac{\rho}{\epsilon_0}$$

or,
$$\nabla^2\phi + \frac{\partial}{\partial t}(\vec{\nabla} \cdot \vec{A}) = -\frac{\rho}{\epsilon_0}.$$

EXAMPLE 6.9

6.9 MOMENTUM AND PRESSURE OF EM WAVES

The momentum of an EM wave is defined as the energy carried by the EM wave divided by the speed of light. If an EM wave is absorbed or reflected by the object the wave will transfer momentum to the object. If the wave incident on the object is longer, greater amount of momentum is transferred.

The energy density of the electric and magnetic fields are

$$U_e = \frac{1}{2} \vec{D} \cdot \vec{E} = \frac{1}{2} \epsilon \vec{E} \cdot \vec{E} = \frac{1}{2} \epsilon E^2 \quad \left[\because \vec{D} = \epsilon \vec{E} \text{ and } \vec{E} \cdot \vec{E} = E^2 \right]$$

and
$$U_m = \frac{1}{2} \vec{B} \cdot \vec{H} = \frac{1}{2} \mu \vec{H} \cdot \vec{H} = \frac{1}{2} \mu H^2 \quad \left[\because \vec{B} = \mu \vec{H} \text{ and } \vec{H} \cdot \vec{H} = H^2 \right]$$

Thus, the EM energy density is

$$U_{em} = U_e + U_m = \frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2$$

To avoid the intricacy of this time dependence a term radiation pressure P is introduced. This does not depend on time.

Momentum carried by EM waves may be expressed as,

$$p = \frac{U}{c} \quad (6.27)$$

and the corresponding radiation pressure exerted is given by,

$$P = \frac{F}{A} = \frac{1}{A} \frac{dp}{dt} = \frac{1}{Ac} \frac{dU}{dt} = \frac{S}{c} \quad \left[\because F = \frac{dp}{dt} \text{ and } S = \frac{1}{A} \frac{dU}{dt} \right] \quad (6.28)$$

Eq. (6.27) and Eq. (6.28) are only true for hitting an absorbing surface. For hitting a perfectly reflecting surface the values should be doubled and thus in that case, momentum transfer is $p = \frac{2U}{c}$

and radiation pressure is $P = \frac{2S}{c}$.

UNIT SUMMARY

- Concept of displacement current: Maxwell's field equations**

$$\text{Displacement current } \vec{I}_d = \epsilon_0 A \frac{d\vec{E}}{dt}$$

$$\text{Displacement current density } \vec{J}_d = \frac{d\vec{D}}{dt}$$

- Maxwell's equations of electromagnetic induction**

- Gauss's law in electrostatics and dielectrics

$$\text{Integral form } \int_S \vec{D} \cdot d\vec{S} = \int_V \rho dV \quad (\text{dielectric})$$

$$\int_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} \quad (\text{electrostatics})$$

$$\text{Differential form } \vec{\nabla} \cdot \vec{D} = \rho \quad (\text{dielectrics})$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (\text{electrostatics})$$

- Gauss's law in magnetostatics

$$\text{Integral form } \int_S \vec{B} \cdot d\vec{S} = 0$$

$$\text{Differential form } \vec{\nabla} \cdot \vec{B} = 0$$

- Faraday's law of electromagnetic induction

$$\text{Integral form } \oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \left(\int_S \vec{B} \cdot d\vec{S} \right)$$

$$\text{Differential form } \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- Modified Ampere's circuital law

$$\text{Integral form } \oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J}_c \cdot d\vec{S} + \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$$

$$\text{Differential form } \vec{\nabla} \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$$

- Physical significance of Maxwell's equations**

- Significance of Maxwell's first equation
Indicates that the charge density is a scalar quantity.
It represents a steady state equation.
- Significance of Maxwell's second equation
Indicates magnetic monopole cannot exist

It is also a steady state equation.

3. Significance of Maxwell's third equation

Electric field is produced due to change in magnetic flux

It is a time dependent equation

4. Significance of Maxwell's fourth equation

It indicates that a magnetic field can be produced due \vec{J}_C and \vec{J}_D

It is also a time dependent equation.

- **Electromagnetic energy density**

$$U = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2$$

- **Poynting Vector**

$$\vec{P} = \vec{E} \times \vec{H} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

- **Poynting theorem**

$$\iint_S \vec{P} \cdot d\vec{S} = - \int_V \frac{\partial U_{em}}{\partial t} dV - \int_V \vec{E} \cdot \vec{J} dV$$

- **Electromagnetic potential**

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$

- **Momentum carried by electromagnetic waves and resultant pressure**

$$\text{Momentum } p = \frac{U}{c}$$

$$\text{Radiation pressure } P = \frac{F}{A} = \frac{S}{c}$$

EXERCISES

Multiple Choice Questions

- 6.1 Displacement current arises due to

- (a) positive charges only (b) negative charges only
(c) time-varying electric field (d) time-varying magnetic field

- 6.2 Maxwell's electromagnetic wave equations in terms of electric field vector \vec{E} in free space is

- (a) $\nabla^2 \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$ (b) $\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$ (c) $\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$ (d) $\nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$

- 6.3 Ampere's circuital law is applicable when the current density is

- (a) constant over space (b) time independent (c) solenoidal (d) irrotational

6.4 The differential form of Faraday's law of electromagnetic induction is

(a) $\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$ (b) $\vec{\nabla} \times \vec{E} = 2\vec{B}$ (c) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (d) $\vec{\nabla} \times \vec{B} = -\frac{\partial \vec{E}}{\partial t}$

6.5 The energy associated with a magnetic field \vec{H} is

(a) $\frac{1}{2} H^2$ (b) $\mu_0 H^2$ (c) $\frac{1}{2} \mu_0 H^2$ (d) $\frac{1}{2\mu_0} H^2$

6.6 An electric field in a certain region has the components $E_x = ax - bz$, $E_y = -ay + bz$ and $E_z = b(y - x)$. Then which of the following statement is correct?

- (a) \vec{E} is an electrostatic field (b) there is free charge in space
(c) \vec{E} is irrotational (d) \vec{E} is solenoidal

6.7 The equation $\vec{\nabla} \times \vec{H} = \vec{J}_C + \frac{\partial \vec{D}}{\partial t}$ represents

- (a) Gauss's law (b) Maxwell's modification of Ampere's law
(c) Faraday's law (d) Coulomb's law

6.8 The energy associated with an electric field vector \vec{E} is

(a) $\frac{1}{2} E^2$ (b) $\frac{1}{2} \epsilon_0 E^2$ (c) $\mu_0 \epsilon_0 E^2$ (d) $\sqrt{\frac{\mu_0}{\epsilon_0}} E^2$

6.9 The SI unit of Poynting vector is

- (a) Wm^2 (b) $\text{Js}^{-1}\text{m}^{-2}$ (c) Wm (d) Ws^{-1}

6.10 The integral form of Maxwell's 3rd equation is:

- (a) Gauss' law (b) Biot Savart's law
(c) Faraday's law (d) Ampere's circuital law

6.11 The EM waves in a charge free conducting medium is a

- (a) standing waves (b) progressive waves (c) transverse waves (d) polarized waves

6.12 If in a 1 mF capacitor, an instantaneous displacement current of 1 A is to be established in the spaces between its plates then it is possible by

- (a) 10^6 A (b) 10^6 A/s (c) 10^6 volts (d) 10^6 volts/s

6.13 Which statement does not say that electrostatic field conservative?

- (a) the curl of E is identically zero
(b) the potential difference between two points is zero in an electrostatic field
(c) electrostatic field is gradient of a scalar potential
(d) work done in a closed path inside the field is zero

6.14 Which of the following is not a Maxwell's equation?

(a) $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ (b) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (c) $\vec{D} = \epsilon \vec{E}$ (d) $\vec{\nabla} \cdot \vec{D} = \rho$

- 6.15 The rate of energy flow is given by
 (a) Maxwell equation (b) Poynting vector (c) Poisson equation (d) continuity equation
- 6.16 Maxwell's equation $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ represents
 (a) magnetic vector potential (b) Gauss' law in magnetism
 (c) generalized Ampere's circuital law (d) Biot-Savart law

Answers of Multiple Choice Questions

6.1 (c), 6.2 (a), 6.3 (b), 6.4 (c), 6.5 (c), 6.6 (all), 6.7 (b), 6.8 (b), 6.9 (b), 6.10 (c), 6.11 (c), 6.12 (d), 6.13 (b), 6.14 (c), 6.15 (b), 6.16 (c)

Short and Long Answer Type Questions

Category I

- 6.1 Express Faraday's law of electromagnetic induction in differential form.
- 6.2 Find the average value of Poynting's vector for a plane electromagnetic wave and explain the importance of it.
- 6.3 Prove that $\vec{E} = \sin(y - t)\hat{k}$ and $\vec{B} = \sin(y - t)\hat{i}$ constitute possible electromagnetic wave.
- 6.4 Give the physical significance of Maxwell's equation $\vec{\nabla} \cdot \vec{B} = 0$.
- 6.5 Derive Maxwell's third equation from Faraday's laws of electromagnetic induction.
- 6.6 Explain the utility of Poynting theorem.
- 6.7 Prove that $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.
- 6.8 Use Faraday's law of electromagnetic induction to show that electric field intensity \vec{E} can be expressed as $\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$, where ϕ is a scalar potential.
- 6.9 Write down Maxwell's equations in time varying field and explain the physical significance of each.
- 6.10 Show that $\vec{\nabla} \cdot \vec{E} \times \vec{H} = -\frac{\partial}{\partial t} \left(\frac{1}{2} \mu H^2 + \frac{1}{2} \epsilon E^2 \right) + \vec{E} \cdot \vec{J}$, where the terms have their usual significance.
- 6.11 From Maxwell's field equation identify Gauss' law, Ampere's law and Faraday's law.
- 6.12 Derive the wave equations for an electromagnetic wave and find the velocity of this wave.
- 6.13 A very long cylinder of radius a carries uniformly distributed current I , over the cross-section of the cylinder. If the current is allowed to pass along the cylinder axis, determine the electric field in the conductor. It is given that the magnetic field is situated just outside the conductor and the

Poynting vector can be noted at the surface of the cylinder. You may assume that the conductivity of the material is σ .

- 6.14 The magnetic field in a region of free space is given by $\vec{B} = B_0 \cos(\omega t - kz) \hat{k}$.
- What is the displacement current if there is no free charge?
 - Obtain an equation for E , neglecting the integration constant.
 - Verify that the differential form of Faraday's law of electromagnetic induction is satisfied by E and B .
- 6.15 If ϕ is a scalar potential associated with the electric field \vec{E} and \vec{A} is the vector potential associated with the magnetic field \vec{B} , show that they must satisfy $\nabla^2 \phi + \frac{\partial}{\partial t}(\vec{\nabla} \cdot \vec{A}) = -\frac{\rho}{\epsilon_0}$.
- 6.16 Show that electric field (\vec{E}), magnetic field (\vec{B}) and propagation vector (\hat{k}) are mutually perpendicular.
- 6.17 What do you mean by momentum of an electromagnetic wave? Obtain the expression for momentum carried by electromagnetic waves and also find the corresponding radiation pressure.
- 6.18 Find the displacement current within a parallel plate capacitor in series with a resistor which carries current I . Area of the capacitor plates are A and the dielectric is vacuum. State the law you used in solving the problem.
- 6.19 Give your concept of displacement current using a simple circuit diagram.
- 6.20 What is the relationship among the electric and magnetic field quantities in the static field?
- 6.21 Mention important differences between the conduction current and displacement current.
- 6.22 Clarify the symbols in Maxwell's equations.
- 6.23 Give the statement of third Maxwell's equation and explain its utility.
- 6.24 What was the need to modify the Ampere's circuital law?
- 6.25 Explain clearly that for time-varying fields, the total current density should be equal to the conduction and displacement current densities.
- 6.26 Explain how the wave energy can be determined by the wave amplitude and the energy carried by a wave depends on its amplitude.
- 6.27 On the basis of the definition of Poynting vector which may be defined as the rate of flow of electromagnetic energy per unit area normal to the direction of energy flow, write down the mathematical form and discuss.
- 6.28 Explain the role of electromagnetic potential. Establish a relation among electric field, magnetic scalar potential and vector potential.

Category II

- 6.29 In broad sense, according to your opinion, what is the Physical significance of Maxwell's equations?

- 6.30 Mention the purpose of measuring the displacement current density.
- 6.31 Explain the requirement of Maxwell's modification of Ampere's circuital law.
- 6.32 Give a method for getting the displacement current.
- 6.33 An electrical circuit carrying non-steady current where Ampere's circuital law is not applicable. Give an example of it.
- 6.34 "The displacement current density is the rate of change of electric displacement and the current set-up in a dielectric medium due to the time-varying electric field." – Justify.
- 6.35 "All phenomena of electromagnetism may be explained with the help of four different equations." Discuss critically how the electromagnetic phenomena can be successfully explained with the help of four equations.

Numerical Problems

- 6.1 If $E = E e^{i(ky - \omega t)}$ denotes the electric vector of an electromagnetic field in vacuum then find out the magnetic vector.
- 6.2 Find the value of Poynting vector at the Sun's surface if the radiated power from the Sun is $2.8 \times 10^{26} \text{ W}$ and the radius of the Sun is $7 \times 10^8 \text{ m}$.
[Ans: $4.55 \times 10^7 \text{ W/m}^2$]
- 6.3 The intensity of sunlight reaching the earth's surface is about 1.5 kW/m^2 . Find the strength of the electric and magnetic fields of the incoming sunlight.
[Ans: $752 \text{ V/m}, 1.99 \text{ A/m}$]
- 6.4 A parallel plate capacitor with circular plates of area 0.05 m^2 separated by 5 mm is being charged by a charging current of 0.2 A . Find the displacement current.

KNOW MORE

The physicist James Clerk Maxwell's equations together provide a complete description of the production and interrelation of electric and magnetic fields.

Activity

Maxwell's Equations are the widely accepted and essential tools of both electrical and electronics engineers. Today, these equations are used to design all electrical and electronic equipment from cell phones to satellites, televisions to computers and power stations to washing machines.

It is true that the theory of electromagnetism was built on the discoveries and advances of many scientists and engineers but the pivotal contribution was that of Maxwell who during the second half of the 19th century made the huge conceptual leaps that would enable the great advances in technology over the centuries.

Interesting facts

In the present day context, Maxwell's Equations refer to a set of four relations to describe the properties and interrelations of electric and magnetic fields. These equations can interpret successfully how a compass

needle always points north, how a power station turbine can generate electricity, how hair stands on end when one removes nylon sweater and how a loudspeaker can convert an electric current into sound.

When all the four equations are combined they also describe the transmission of radio waves and the propagation of light.

Analogy

In physics the Yang-Mills theory which is a generalization of Maxwell's unified theory of electromagnetism used to explain the weak and strong force in subatomic particles in geometric structure or quantum field theory called the "mass gap." The theory was introduced in 1954 by Chen Ning Yang and Robert L. Mills who first developed a gauge theory, utilizing Lie groups for describing subatomic interactions.

Yang-Mills theory explains the mass gap or nonzero mass in quantum applications. Evidence for the mass gap has been demonstrated in physical experiments and computer-based mathematical models and it is assumed that the strong force operates only at very small distances within atomic nuclei.

History

Andre-Marie Ampere (1775 – 1836), French physicist: In 1820, a week after Ampere heard of H. C. Oersted's 1806 discovery that a magnetic needle is acted on by a voltaic current, he presented a paper to the Academy containing more complete exposition of that and related phenomena.

Carl Friedrich Gauss (1777-1855), German mathematician: Independently defined Green's theorem, generalized Coulomb's law and formulated separate electrostatic and electro-dynamic laws, including "Gauss's laws", which constitute two of the four "Maxwell's equations."

Michael Faraday (1791-1867), British experimentalist: Discovered that moving a magnet near a loop of wire is responsible for an electrical current to flow. This led to Faraday's law of induction.

James Clerk Maxwell (1831-1879), Scottish physicist: Converted Faraday's physical ideas into mathematical model showing that Faraday's law of induction implies a corresponding "Displacement current" which yields electromagnetic waves. Maxwell's four equations clearly indicated that electromagnetic waves could be generated in the laboratory, a possibility first demonstrated by Heinrich Hertz in 1887 eight years after Maxwell's death.

Timelines

1785: *Coulomb's* law published

1812: *Poisson's* law published

1813: *Gauss' divergence* theorem discovered

1820: *H. C. Oersted* discovered that an electric current creates a magnetic field

1820: *André-Marie Ampère's* work found electrodynamics; *Biot-Savart* law discovered

1826: *Ampère's* law published

1831: *Faraday's* law published

1856: *James Clerk Maxwell* published "On Faraday's lines of force"

1861: *Maxwell* published "On physical lines of force"

1865: *Maxwell* published "A dynamical theory of the electromagnetic field"

1873: *Maxwell* published *Treatise on Electricity and Magnetism*

1888: *Heinrich Hertz* discovered radio waves

1940: *Albert Einstein* popularized the name 'Maxwell's Equations'

1966: *Kane Yee* introduced *finite-difference time domain methods to solve Maxwell's Equations*

References of Historical Significance

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Oliver Heaviside, Electrical Papers, New York and London: Macmillan and Co, 1894.

Applications (Real Life / Industrial)

Sciences of electricity and magnetism and their fusion as electromagnetism evolved through a series of advances. In 1820 Oersted demonstrated in Copenhagen that an electric current can deflect the magnetic needle of a compass.

Within a week Andre Ampère had shown the French Academy of Science in Paris that parallel currents in two wires attract each other, while opposite currents would repel. Soon after, Jean-Baptiste Biot and Philippe Savart demonstrated how the strength of the force falls away with distance to the wire. Ampère's efforts over the next six years founded the field of electrodynamics and his work elegantly combined experiments and theory.

Maxwell himself described this as "one of the most beautiful achievements in science . . . from the brain of the Newton of electricity."

Case Study (Environmental / Sustainability / Social / Ethical Issues)

Maxwell formulated mathematics for Faraday's concepts of induced currents but was unable to interpret these within the fluid model. After a period focused on some other topics like color perception and the dynamics of Saturn's rings, Maxwell returned to the explanation of electromagnetism with a completely new theory.

Maxwell devised a mechanical model which could account for all the known electromagnetic phenomena. This helped him to describe Ampere's, Faraday's, and Gauss' laws but yet it did not properly describe current flow through electrical capacitors.

After the necessary corrections he completed the essential physics of the theory and was published in the final two parts of "On Physical Lines of Force" in 1862. However, his original equations were quite different mathematical form from those used today.

Inquisitiveness and Curiosity Topics

In 20th century, Maxwell's Equations had impact beyond electromagnetism in the discovery of the theory of relativity and in the field equations of quantum mechanics. The dramatic increases in computing power as well as development of numerical finite-difference techniques from the mid 1960s have enabled their widespread everyday use.

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10. <https://nptel.ac.in/courses/115/101/115101005/>
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12. <https://depts.washington.edu/mictech/optics/me557/week2.pdf>

7

Electromagnetic Waves

UNIT SPECIFICS

Through this unit we have discussed the following aspects:

- Maxwell's wave equation;
- Solution of Maxwell's wave equation for free space;
- Electromagnetic wave in a charge free conducting media;
- Skin depth and its physical significance;
- Reflection of electromagnetic waves from a non-conducting medium-vacuum interface for normal incidence;
- Transmission of electromagnetic waves from a non-conducting medium-vacuum interface for normal incidence.

The applications of the topics are discussed for generating further curiosity and creativity as well as improving problem solving capacity.

Besides giving a large number of multiple choice questions as well as questions of short and long answer types marked in two categories following lower and higher order of Bloom's taxonomy, assignments through a number of numerical problems, a list of references and suggested readings are given in the unit so that one can go through them for practice. It is important to note that for getting more information on various topics of interest some QR codes have been provided in different sections which can be scanned for relevant supportive knowledge.

After the end of the unit, based on the content, there is a "Know More" section. This section has been judiciously designed so that the supplementary information provided in this part becomes beneficial for the users of the book. This section mainly highlights the initial activity, examples of some interesting facts, analogy, history of the development of the subject focusing the salient observations and finding, timelines starting from the development of the concerned topics up to the recent time, applications of the subject matter for our day-to-day real life or/and industrial applications on variety of aspects, case study related to environmental, sustainability, social and ethical issues whichever applicable, and finally inquisitiveness and curiosity topics of the unit.

RATIONALE

This unit on Electromagnetic Waves will help our students to get a clear idea about the wave equation and its solution in free space. It will clarify the electromagnetic wave in a charge free conducting media as well as the skin depth and its physical significance. This concept will help to go in a greater detail about the propagation mechanism and to understand and apply properly the properties of reflection and transmission of electromagnetic waves from a non-conducting medium-vacuum interface.

Like the energy, electricity can be static which is also true for magnetism. A varying magnetic field can induce a changing electric field and vice-versa. In this way both of them are interlinked. These varying fields form the so called electromagnetic waves. Electromagnetic waves have a sharp distinction from mechanical waves where they have no need of any medium for propagation. It implies that electromagnetic waves can propagate also through the vacuum of space in addition to air and solid materials. Maxwell who formulated the theory was significantly helped to explain the electromagnetic waves. He explained how the electric and magnetic fields are coupled for forming the electromagnetic waves.

PRE-REQUISITES

Mathematics: Vector Calculus (Class XII)

Physics: Electrostatics, Electromagnetism (Class XII)

UNIT OUTCOMES

List of outcomes of this unit is as follows:

U7-O1: Define Maxwell's wave equation

U7-O2: Explain Maxwell's wave equation solution for free space

U7-O3: Describe electromagnetic wave in a charge free conducting media

U7-O4: Explain skin depth and its physical significance

U7-O5: Explain reflection and transmission of electromagnetic waves from a non-conducting medium-vacuum interface for normal incidence

Unit-7 Outcomes	EXPECTED MAPPING WITH COURSE OUTCOMES (1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation)					
	CO-1	CO-2	CO-3	CO-4	CO-5	CO-6
U7-O1	1	-	1	3	-	3
U7-O2	-	-	1	3	-	3
U7-O3	-	-	1	2	-	3
U7-O4	2	-	-	1	-	3
U7-O5	2	1	-	-	-	3

7.1 INTRODUCTION

A localized disturbance of the electric field and magnetic field can produce an electromagnetic (EM) wave. The wave thus generated will propagate along the outward direction from its origin. If the waves propagate through a homogeneous medium then it spreads out uniformly in every direction from its point of origination.

Far away from the point of origination, this wave appears to have the same amplitude all over a plane perpendicular towards the direction of wave propagation. Such a wave is referred as a *plane wave*. This type of waves satisfies the wave equation in free space or in some homogeneous medium. With this concept of a plane wave we can consider that the entire wave is traveling in a specified direction, rather than spreading out in every direction.

7.2 WAVE EQUATION FOR FREE SPACE

EM wave equation explains the propagation of EM waves either through vacuum or through any medium. It represents a three dimensional form of the equation of the wave.

In free space we have the charge density $\rho = 0$, the current density $\vec{J}_c = 0$ and the conductance $\sigma = 0$.

So, in free space Maxwell's wave equations are given by,

$$\text{i) } \vec{\nabla} \cdot \vec{D} = 0 \text{ or, } \vec{\nabla} \cdot \vec{E} = 0 \quad (7.1) \text{ (a)}$$

$$\text{ii) } \vec{\nabla} \cdot \vec{B} = 0 \quad (7.1) \text{ (b)}$$

$$\text{iii) } \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (7.1) \text{ (c)}$$

$$\text{iv) } \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \text{ or, } \vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \left[\because \vec{D} = \epsilon_0 \vec{E}, \vec{B} = \mu_0 \vec{H} \right] \quad (7.1) \text{ (d)}$$

Taking curl on both sides of Eq. (7.1) (d) we have,

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E}) = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(-\frac{\partial \vec{B}}{\partial t} \right) = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\text{or, } \vec{\nabla} (\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\text{or, } -\nabla^2 \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \quad \left[\because \vec{\nabla} \cdot \vec{B} = 0 \right]$$

$$\text{or, } \nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \quad (7.2)$$

This gives the wave equation for magnetic field in free space.

Again taking curl on both sides of the Eq. (7.1) (c) we have,

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) = -\mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(\frac{\partial \vec{E}}{\partial t} \right) = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\begin{aligned}
\text{or,} \quad & \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \\
\text{or,} \quad & -\nabla^2 \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \left[\because \vec{\nabla} \cdot \vec{E} = 0 \right] \\
\text{or,} \quad & \nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \tag{7.3}
\end{aligned}$$

This gives the wave equation for electric field in free space.

Now, comparing Eqs. (7.2) and (7.3) with the general wave equation, $\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$ we have,

$$v^2 = \frac{1}{\mu_0 \epsilon_0} \tag{7.4}$$

$$\text{or,} \quad v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}} = 3 \times 10^8 \text{ m/s.}$$

Thus, we may conclude that electromagnetic wave propagates with the speed of light in free space, *i.e.*, light is an electromagnetic wave.

Example 7.1 Show that the wave equation of electric field E in free space is given

$$\text{by, } \nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Solution

$$\text{We have,} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\begin{aligned}
\text{So,} \quad \vec{\nabla} \times \vec{\nabla} \times \vec{E} &= -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) = -\mu_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H}) \\
&= -\mu_0 \frac{\partial}{\partial t} \left(\frac{\partial \vec{D}}{\partial t} \right) = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}
\end{aligned}$$

$$\text{or,} \quad \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\text{or,} \quad \nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \left[\because \vec{\nabla} \cdot \vec{E} = 0 \right]$$

which is the required equation.

EXAMPLE 7.1

7.3 TRANSVERSE NATURE OF EM WAVE

EM waves are transverse in nature *i.e.*, the electric and magnetic fields oscillate in a plane perpendicular to the direction of propagation of the wave. Also in an EM wave electric and magnetic fields are perpendicular to each other.

EM wave equations are given by,

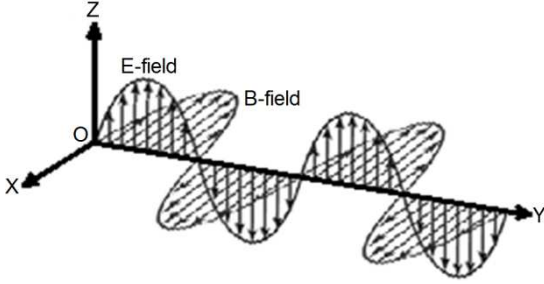


Fig. 7.1: Transverse nature of EM wave

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

and

$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

or,

$$\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

Now, solutions of the above set of equations are given by

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} = \hat{e} E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \quad (7.5)$$

and

$$\vec{H}(\vec{r}, t) = \vec{H}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} = \hat{b} H_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \quad (7.6)$$

Again we have, $\vec{\nabla} \cdot \vec{E} = 0$ and $\vec{\nabla} \cdot \vec{B} = 0$ [for free space]

Thus, taking divergence on both sides of Eq. (7.5) we get,

$$\vec{\nabla} \cdot \vec{E}(\vec{r}, t) = \vec{\nabla} \cdot \left[\hat{e} E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \right] = \vec{\nabla} \cdot \left(E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \right) \hat{e} + E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \vec{\nabla} \cdot \hat{e}$$

$$\text{or,} \quad \vec{\nabla} \cdot \left(E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \right) \hat{e} = 0 \quad \left[\because \vec{\nabla} \cdot \hat{e} = 0 \right]$$

$$\text{or,} \quad i \vec{k} E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \hat{e} = 0$$

$$\text{or,} \quad \vec{k} \cdot \hat{e} = 0 \quad (7.7)$$

Again, taking divergence on both sides of Eq. (7.6) we get,

$$\vec{\nabla} \cdot \vec{H}(\vec{r}, t) = \vec{\nabla} \cdot \left[\hat{b} H_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \right] = \vec{\nabla} \cdot \left(H_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \right) \hat{b} + H_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \vec{\nabla} \cdot \hat{b}$$

$$\text{or,} \quad \vec{\nabla} \cdot \left(H_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \right) \hat{b} = 0 \quad \left[\because \vec{\nabla} \cdot \hat{b} = 0 \right]$$

$$\text{or,} \quad i \vec{k} H_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \hat{b} = 0$$

$$\text{or,} \quad \vec{k} \cdot \hat{b} = 0 \quad (7.8)$$

Thus, from Eqs. (7.7) and (7.8) we can conclude that the E -field and B -field of electromagnetic wave are perpendicular to the propagation vector k . This proves the transverse nature of electromagnetic waves. Also from Maxwell's equation we have,

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\text{or,} \quad \vec{\nabla} \times \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} = -\frac{\partial \vec{B}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}}{\partial t}$$

$$\text{or,} \quad i \vec{k} \times \vec{E} = i \omega \vec{B}$$

or, $\vec{k} \times \vec{E} = \omega \vec{B}$ (7.9)

So, B -field is perpendicular to E -field and the propagation vector \vec{k} .

In terms of magnitude only the relationship between E , B and k will be, $kE = \omega B$

or, $\frac{E}{B} = \frac{\omega}{k} = v$ (7.10)

which gives the relationship between the magnitude of electric and magnetic vectors of an electromagnetic wave. In vacuum, the velocity of the electromagnetic wave will be the speed of light,

i.e., $\frac{E_0}{B_0} = c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ (7.11)

or, $\frac{E_0}{H_0} = \frac{\mu_0}{\sqrt{\epsilon_0 \mu_0}} = \sqrt{\frac{\mu_0}{\epsilon_0}}$ (7.12) (a)

or, $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{4\pi \times 10^{-7} \times 36\pi \times 10^9} = 120\pi = 377\Omega$ (7.12) (b)

where Z_0 is the intrinsic impedance of free space.



EXAMPLE 7.2

Example 7.2 For an electromagnetic wave corresponding to a magnetic field vector of 1 nT magnitude, calculate the electric field vector magnitude.

Solution

Here, $B = 1 \text{ nT} = 10^{-9} \text{ T}$.

Now, we have, $\frac{E}{B} = c$

or, $E = cB = 3 \times 10^8 \times 10^{-9} = 0.3 \text{ V/m}$.

EXAMPLE 7.3

Example 7.3 An electromagnetic wave is travelling along the positive Z-direction in a dielectric medium with relative permittivity 2 and relative permeability 1. Find the speed of the wave and the intrinsic impedance of the medium.

Solution

Given, relative permittivity = 2 and relative permeability = 1.

So, the speed of the electromagnetic wave is

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{3 \times 10^8}{\sqrt{2}} = 2.1 \times 10^8 \text{ m/s}.$$

and, the intrinsic impedance of the medium is

$$z = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{\frac{\mu_r}{\epsilon_r}} = \frac{377}{\sqrt{2}} = 266.62 \Omega.$$

Example 7.4 If the sunlight intensity reaching the earth's surface is about 1300 W/m^2 , find the strength of the electric field and magnetic fields of the incoming sunlight.

Solution

Given $I = P = EH = 1300 \text{ W/m}^2$.

Again, $E/H = 377 \text{ ohm}$.

Thus, $\frac{E}{H} \cdot EH = 377 \times 1300 = 490100 \text{ V}^2/\text{m}^2$

or, $E = \sqrt{490100} \text{ V/m} = 700.07 \text{ V/m}$.

So, $H = \frac{1300}{E} = \frac{1300}{700.07} \text{ A/m} = 1.857 \text{ A/m}$.

These give the strength of the electric and magnetic fields of the incoming sunlight.

EXAMPLE 7.4

7.4 EM WAVE IN NON-CONDUCTING (DIELECTRIC) MEDIA

In any stationary non-conducting medium for an electromagnetic field both dielectric constant and permeability are associated with the point-functions. For a charge free conducting medium the charge density $\rho = 0$ and the conductance $\sigma = 0$, and hence Maxwell's wave equations for a charge free non-conducting (dielectric) media of permittivity ϵ and permeability μ are given by,

$$\text{i) } \vec{\nabla} \cdot \vec{D} = 0 \text{ or, } \vec{\nabla} \cdot \vec{E} = 0 \quad (7.13) \text{ (a)}$$

$$\text{ii) } \vec{\nabla} \cdot \vec{B} = 0 \text{ or, } \vec{\nabla} \cdot \vec{H} = 0 \quad (7.13) \text{ (b)}$$

$$\text{iii) } \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (7.13) \text{ (c)}$$

$$\text{iv) } \vec{\nabla} \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} \quad (7.13) \text{ (d)}$$

Taking curl on both sides of Eq. (7.13) (d) we have,

$$\vec{\nabla} \times \vec{\nabla} \times \vec{H} = \epsilon \frac{\partial \vec{\nabla} \times \vec{E}}{\partial t} = -\mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\text{or, } \vec{\nabla} (\vec{\nabla} \cdot \vec{H}) - \nabla^2 \vec{H} = -\mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\text{or, } -\nabla^2 \vec{H} = -\mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad \left[\because \vec{\nabla} \cdot \vec{H} = 0 \right]$$

$$\text{or, } \nabla^2 \vec{H} = \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad (7.14)$$

This gives the wave equation for magnetic field in a charge free non-conducting medium.

Again, taking curl on both sides of Eq. (7.13) (c) we have,

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{\nabla} \times \vec{H}}{\partial t} = -\mu \frac{\partial}{\partial t} \left(\epsilon \frac{\partial \vec{E}}{\partial t} \right) = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

or,

$$\vec{\nabla} (\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

or,

$$-\nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad \left[\because \vec{\nabla} \cdot \vec{E} = 0 \right]$$

or,

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad (7.15)$$

This gives the wave equation for electric field in a charge free non-conducting medium.

Now, comparing Eqs. (7.14) and (7.15) with the general wave equation, $\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$

we have,

$$v^2 = \frac{1}{\mu \epsilon}$$

or,

$$v = \frac{1}{\sqrt{\mu \epsilon}} < c \quad \left[\because \epsilon > \epsilon_0, \mu > \mu_0 \right] \quad (7.16)$$

Thus, we may conclude that EM wave propagates with a speed less than the speed of light in free space in case of a charge free non-conducting or a perfectly dielectric medium. From Eqs. (7.14) and (7.15) we find that the EM waves will not decay while propagating through a charge free non-conducting medium. Due to this type of behavior this kind of a medium is also known as the *lossless medium*.

Example 7.5 If the relative permittivity and relative permeability of a medium are 5 and 2 respectively. Find the velocity of electromagnetic wave propagation through it.

Solution

Here, $\epsilon_r = 5$ and $\mu_r = 2$.

Velocity of wave propagation will be,

$$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \frac{1}{\sqrt{\mu_r \epsilon_r}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{3 \times 10^8}{\sqrt{5 \times 2}} = 9.5 \times 10^7 \text{ m/s.}$$

EXAMPLE 7.5

7.5 EM WAVE IN CONDUCTING MEDIA

Generally the electric field, inside a conductor is zero and EM waves are reflected by conductors.

For a charge free conducting medium Maxwell's wave equations are given by,

i) $\vec{\nabla} \cdot \vec{D} = 0$ or, $\vec{\nabla} \cdot \vec{E} = 0$ (7.17) (a)

$$\text{ii)} \quad \vec{\nabla} \cdot \vec{B} = 0 \text{ or, } \vec{\nabla} \cdot \vec{H} = 0 \quad (7.17) \text{ (b)}$$

$$\text{iii)} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (7.17) \text{ (c)}$$

$$\text{iv)} \quad \vec{\nabla} \times \vec{H} = \vec{J}_C + \frac{\partial \vec{D}}{\partial t} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad (7.17) \text{ (d)}$$

Taking curl on both sides of Eq. (7.17) (d) we have,

$$\vec{\nabla} \times \vec{\nabla} \times \vec{H} = \vec{\nabla} \times \left(\sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \right) = \sigma \vec{\nabla} \times \vec{E} + \epsilon \frac{\partial \vec{\nabla} \times \vec{E}}{\partial t} = -\mu \sigma \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\text{or,} \quad \vec{\nabla} (\vec{\nabla} \cdot \vec{H}) - \nabla^2 \vec{H} = -\mu \sigma \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\text{or,} \quad -\nabla^2 \vec{H} = -\mu \sigma \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad \left[\because \vec{\nabla} \cdot \vec{H} = 0 \right]$$

$$\text{or,} \quad \nabla^2 \vec{H} - \mu \sigma \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad (7.18)$$

This gives the wave equation for magnetic field in a charge free conducting medium.

Again, taking curl on both sides of Eq. (7.17) (c) we have,

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{\nabla} \times \vec{H}}{\partial t} = -\mu \frac{\partial}{\partial t} \left(\sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \right) = -\mu \sigma \frac{\partial \vec{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\text{or,} \quad \vec{\nabla} (\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \sigma \frac{\partial \vec{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\text{or,} \quad -\nabla^2 \vec{E} = -\mu \sigma \frac{\partial \vec{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad \left[\because \vec{\nabla} \cdot \vec{E} = 0 \right]$$

$$\text{or,} \quad \nabla^2 \vec{E} - \mu \sigma \frac{\partial \vec{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad (7.19)$$

This gives the wave equation for electric field in a charge free conducting medium.

In both the Eqs. (7.18) and (7.19), the terms $-\mu \sigma \frac{\partial \vec{E}}{\partial t}$ and $-\mu \sigma \frac{\partial \vec{H}}{\partial t}$ indicate that the EM waves will decay while propagating through a charge free conducting medium. Due to this type of behavior this kind of a medium is also known as the *lossy medium*.

7.6 ATTENUATION OF EM WAVE IN CONDUCTING MEDIA

Attenuation is the gradual loss of flux intensity through a medium. Attenuation affects the propagation of waves and signals in air, in electrical circuits and in optical fibers.

In case of a charge free conducting medium, the wave equations for electric and magnetic fields are given from Eqs. (7.18) and (7.19) as,

$$\nabla^2 \vec{E} - \mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

and

$$\nabla^2 \vec{H} - \mu\sigma \frac{\partial \vec{H}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

Let, the solutions of the above two equations are given by

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \quad (7.20)$$

and

$$\vec{H}(\vec{r}, t) = \vec{H}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \quad (7.21)$$

Using Eq. (7.20) in Eq. (7.19) we get,

$$\nabla^2 \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} - \mu\sigma \frac{\partial \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}}{\partial t^2} = 0$$

or,

$$-k^2 \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} + i\mu\sigma\omega \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} + \varepsilon\mu\omega^2 \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} = 0$$

or,

$$-k^2 \vec{E} + i\mu\sigma\omega \vec{E} + \varepsilon\mu\omega^2 \vec{E} = 0$$

or,

$$\left(-k^2 + i\mu\sigma\omega + \varepsilon\mu\omega^2\right) \vec{E} = 0$$

or,

$$k^2 = i\mu\sigma\omega + \varepsilon\mu\omega^2 = \varepsilon\mu\omega^2 \left(1 + \frac{i\sigma}{\varepsilon\omega}\right)$$

or,

$$k = \omega\sqrt{\varepsilon\mu} \sqrt{\left(1 + \frac{i\sigma}{\varepsilon\omega}\right)} \quad (7.22)$$

Thus the propagation constant k for a charge free conducting medium is a complex quantity. The relation given by Eq. (7.22) is known as the *dispersion relation* for the charge free conducting medium. In Eq. (7.22), the factor $\frac{\sigma}{\varepsilon\omega}$ is known as the *dissipation factor*. Choosing, $k = \alpha + i\beta$ and

using Eq. (7.22), we get $\alpha + i\beta = \omega\sqrt{\varepsilon\mu} \sqrt{\left(1 + \frac{i\sigma}{\varepsilon\omega}\right)}$

or,

$$(\alpha + i\beta)^2 = \alpha^2 - \beta^2 + 2i\alpha\beta = \varepsilon\mu\omega^2 \left(1 + \frac{i\sigma}{\varepsilon\omega}\right)$$

Comparing the real and imaginary terms on both sides we get, $\alpha^2 - \beta^2 = \varepsilon\mu\omega^2$ and $2\alpha\beta = \sigma\mu\omega$.

Solving for α and β we get,

$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}} \sqrt{\left\{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2\right\}^{1/2} + 1} \quad (7.23)$$

and

$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}} \sqrt{\left\{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2\right\}^{1/2} - 1} \quad (7.24)$$

where α is known as the *attenuation constant* and β is known as the *phase constant*.

In terms of the propagation constant the solutions of Eqs. (7.18) and (7.19) are given by

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{-\beta r} e^{i(\alpha r - \omega t)} \quad (7.25)$$

and

$$\vec{H}(\vec{r}, t) = \vec{H}_0 e^{-\beta r} e^{i(\alpha r - \omega t)} \quad (7.26)$$

where $e^{-\beta r}$ is known as the *attenuation factor* and $e^{i(\alpha r - \omega t)}$ is known as the *phase factor*.

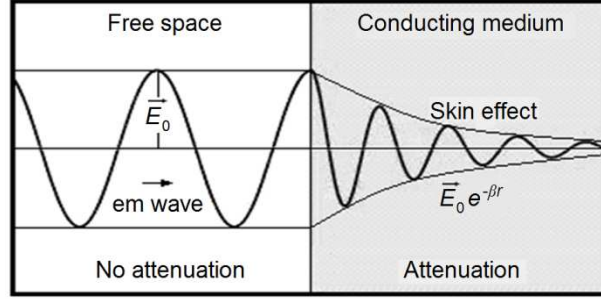


Fig. 7.2: Attenuation of EM wave

7.6.1 Skin Depth

Penetration depth is the measurement of how deep an electromagnetic radiation can penetrate inside a material. It is the depth up to which the intensity of the radiation within the material falls to nearly 37% of its initial value.

On the other hand, the skin depth is the depth up to which the magnetic field penetrates within the metal from the top of its surface. This magnetic field is maximum at the surface of the conductor while it decays in an exponential pattern as it goes towards the other edge.

If the frequency of the conducting medium is not so high, i.e., if $\sigma \gg \varepsilon\omega$, then from Eqs. (7.23) and (7.24) we have,

$$\begin{aligned} \alpha = \beta &= \omega \sqrt{\frac{\varepsilon\mu}{2}} \sqrt{\frac{\sigma}{\varepsilon\omega}} \\ &= \sqrt{\frac{\omega\mu\sigma}{2}} = \frac{1}{\delta} \text{ [say]} \end{aligned} \quad (7.27)$$

where,

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

is known as the skin depth, which decreases with increase in frequency and conductivity of the medium.

The solutions of the wave equations for electric and magnetic fields are given by

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{-r/\delta} e^{i(r/\delta - \omega t)} \quad (7.28)$$

and

$$\vec{H}(\vec{r}, t) = \vec{H}_0 e^{-r/\delta} e^{i(r/\delta - \omega t)} \quad (7.29)$$

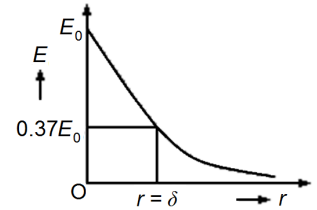


Fig. 7.3: Concept of Skin depth

where the amplitudes are given by $\vec{E}_0 e^{-r/\delta}$ and $\vec{H}_0 e^{-r/\delta}$.

If the value of $r = \delta$, the amplitude of electromagnetic field will be reduced to $1/e$ times (37 %) the value at the surface of the conducting medium.

Physical significance of Skin Depth: i) Large value of skin depth implies less attenuation of electromagnetic waves.

ii) Any high frequency of the wave cannot propagate through the medium. This can be used in *electromagnetic shielding*.



EXAMPLE 7.6

Example 7.6 Find the skin depth for a conducting medium of conductivity 6×10^7 mho/m if the frequency of the electromagnetic wave is 10^4 Hz.

Solution

We have, skin depth $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$

Here, $\sigma = 6 \times 10^7$ mho/m and $\omega = 2\pi f = 2\pi \times 10^4$ Hz.

So, $\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{2\pi \times 10^4 \times 4\pi \times 10^{-7} \times 6 \times 10^7}} = 0.042$ mm.

EXAMPLE 7.7

Example 7.7 If the skin depth for a conducting medium of conductivity 4×10^7 mho/m is 0.02mm, find the frequency of the EM wave. [Take, relative permittivity = relative permeability = 1]

Solution

We have, skin depth $\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f \mu\sigma}}$

So, the frequency of the EM wave is

$$f = \frac{1}{\pi\mu\sigma\delta^2} = \frac{1}{4 \times \pi^2 \times 10^{-7} \times 4 \times 10^7 \times (0.02 \times 10^{-3})^2} = 15.87 \text{ MHz.}$$

7.7 BOUNDARY CONDITIONS FOR EM WAVES

Maxwell's equations constrain the behavior of EM fields strongly at the boundaries of two media having dissimilar properties and certain conditions are satisfied by the field vectors \vec{E} , \vec{D} , \vec{B} and \vec{H} .

These are known as the boundary conditions. All these conditions are discussed below.

Condition for D: At any interface between two media let us consider a small pillbox with negligible height (Fig. 7.4). From first Maxwell's equation we have, $\vec{\nabla} \cdot \vec{D} = \rho$. Taking volume integration on both sides of Maxwell's first equation we get,

$$\begin{aligned}
 & \int_V \vec{\nabla} \cdot \vec{D} dV = \int_V \rho dV \\
 \text{or,} \quad & \int_S \vec{D} \cdot \hat{n} dS = \int_V \rho dV \\
 \text{i.e.,} \quad & \int_{S_1} \vec{D}_1 \cdot \hat{n}_1 dS + \int_{S_2} \vec{D}_2 \cdot \hat{n}_2 dS + \int_{S_3} \vec{D}_1' \cdot \hat{n}_1' dS \\
 & + \int_{S_4} \vec{D}_2' \cdot \hat{n}_2' dS = \int_V \rho dV
 \end{aligned}$$

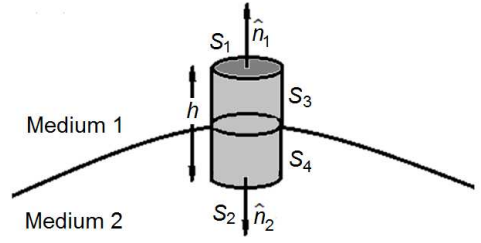


Fig. 7.4: Boundary conditions for D and B

The last two terms in the above expression comes for the walls of the pillbox and so they may be neglected for a very small height h of the pillbox. So, in the limit $h \rightarrow 0$ we have,

$$\lim_{h \rightarrow 0} \left[\int_S \vec{D}_1 \cdot \hat{n}_1 dS + \int_S \vec{D}_2 \cdot \hat{n}_2 dS \right] = \lim_{h \rightarrow 0} \int_V \rho dV = \lim_{h \rightarrow 0} \int_V \sigma S dV \quad [\text{where } \rho = \sigma S]$$

$$\text{or,} \quad \left[\vec{D}_1 \cdot \hat{n} - \vec{D}_2 \cdot \hat{n} \right] S = \sigma S \quad [\because S_1 = S_2 = S; n_1 = n \text{ and } n_2 = -n]$$

$$\text{or,} \quad D_{1n} - D_{2n} = \sigma \quad (7.30)$$

Here, D_{1n} and D_{2n} are the normal components of the electric displacement vector in the two media. From Eq. (7.30), it may be concluded that, the normal components of the electric displacement vector is not continuous at the interface of two media.

Condition for B: From second Maxwell's equation we have, $\vec{\nabla} \cdot \vec{B} = 0$

At any interface between two media let us consider a small pillbox with negligible height (Fig 7.4). Taking volume integration on both sides of second Maxwell's equations we get,

$$\int_V \vec{\nabla} \cdot \vec{B} dV = 0$$

$$\text{or,} \quad \int_S \vec{B} \cdot \hat{n} dS = 0$$

$$\text{i.e.,} \quad \int_{S_1} \vec{B}_1 \cdot \hat{n}_1 dS + \int_{S_2} \vec{B}_2 \cdot \hat{n}_2 dS + \int_{S_3} \vec{B}_1' \cdot \hat{n}_1' dS + \int_{S_4} \vec{B}_2' \cdot \hat{n}_2' dS = 0$$

The last two terms in the above expression comes for the walls of the pillbox and so they may be neglected for a very small height h of the pillbox. So, in the limit $h \rightarrow 0$ we have,

$$\lim_{h \rightarrow 0} \left[\int_S \vec{B}_1 \cdot \hat{n}_1 dS + \int_S \vec{B}_2 \cdot \hat{n}_2 dS \right] = 0$$

$$\text{or,} \quad \left[\vec{B}_1 \cdot \hat{n} - \vec{B}_2 \cdot \hat{n} \right] S = 0 \quad [\because S_1 = S_2 = S; n_1 = n \text{ and } n_2 = -n]$$

$$\text{or,} \quad B_{1n} - B_{2n} = 0$$

$$\text{or,} \quad B_{1n} = B_{2n} \quad (7.31)$$

Here, B_{1n} and B_{2n} are the normal components of the magnetic induction vector in the two media. From Eq. (7.31), it may be concluded that, the normal components of the magnetic induction vector is continuous at the interface of two media.

Condition for E: From third Maxwell's equation we have,

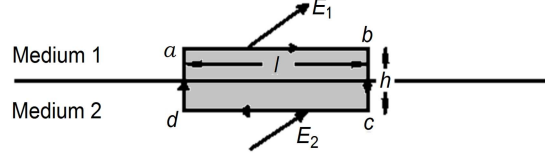


Fig. 7.5: Boundary conditions for E and H

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

At any interface between two media, let us consider a rectangular loop $abcd$ bounding surface S as shown in Fig. 7.5. Taking surface integration on both sides of third Maxwell's equation we get,

$$\int_S (\vec{\nabla} \times \vec{E}) \cdot \hat{n} dS = \oint_{abcd} \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS$$

$$\text{or,} \quad \int_{ab} \vec{E}_1 \cdot d\vec{l} + \int_{cd} \vec{E}_2 \cdot d\vec{l} + \int_{bc} \vec{E}_1' \cdot d\vec{l} + \int_{da} \vec{E}_2' \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \hat{n} dS$$

The contribution for the last two terms in the left hand side will cancel each other and if for the loop $h \rightarrow 0$ the surface integration on the right hand side tends to zero provided $\frac{\partial \vec{B}}{\partial t}$ is finite everywhere.

$$\text{So,} \quad \int_{ab} \vec{E}_1 \cdot d\vec{l} + \int_{cd} \vec{E}_2 \cdot d\vec{l} = 0$$

$$\text{or,} \quad (E_{1t} - E_{2t})l = 0$$

$$\text{or,} \quad E_{1t} = E_{2t} \quad (7.32)$$

Here, E_{1t} and E_{2t} are the tangential components of the electric field vector in the two media. From Eq. (7.32), it may be concluded that, the tangential components of the electric field vector is continuous at the interface of two media.

Condition for H: From fourth Maxwell's equation we have, $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

At any interface between two media, let us consider a rectangular loop $abcd$ bounding surface S as shown in Fig. 7.5. Taking surface integration on both sides of fourth Maxwell's equation we get,

$$\int_S (\vec{\nabla} \times \vec{H}) \cdot \hat{n} dS = \oint_{abcd} \vec{H} \cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot \hat{n} dS$$

$$\text{or,} \quad \int_{ab} \vec{H}_1 \cdot d\vec{l} + \int_{cd} \vec{H}_2 \cdot d\vec{l} + \int_{bc} \vec{H}_1' \cdot d\vec{l} + \int_{da} \vec{H}_2' \cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot \hat{n} dS$$

The contribution for the last two terms in the left hand side will cancel each other.

If for the loop $h \rightarrow 0$ the surface integration on the right hand side tends to $\int_S \vec{J} \cdot \hat{n} dS$ provided $\frac{\partial \vec{D}}{\partial t}$

is bounded everywhere.

$$\text{So,} \quad \int_{ab} \vec{H}_1 \cdot d\vec{l} + \int_{cd} \vec{H}_2 \cdot d\vec{l} = \lim_{h \rightarrow 0} \int_S \vec{J} \cdot \hat{n} dS = J_{s\perp} l$$

$$\text{or,} \quad (H_{1t} - H_{2t})l = J_{s\perp} l$$

$$\text{or,} \quad H_{1t} - H_{2t} = J_{s\perp} \quad (7.33)$$

Here, H_{1t} and H_{2t} are the tangential components of the magnetic field intensity in the two media. From Eq. (7.33), it may be concluded that, the tangential components of the magnetic field intensity is not continuous at the interface of two media. Here $J_{s\perp}$ is the surface current density and is equal to zero unless the conductivity is infinite. So, for finite conductivity,

$$H_{1t} = H_{2t} \quad (7.34)$$

7.8 REFLECTION AND TRANSMISSION OF EM WAVES

The reflection and transmission of EM waves by a moving dielectric medium can be observed theoretically to find the reflection and transmission coefficients. It may be noted that the reflected wave moves away from the boundary but in the same medium as the incident wave while the transmitted wave moves away from the boundary on the other side of the boundary from the incident wave.

Consider plane polarized waves given by $\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$ and $\vec{B} = \vec{B}_0 e^{i(kz - \omega t)}$ are propagating along the Z-direction. Now, from Maxwell's equation $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ we have

$$ik\hat{e}_z \times \vec{E}_0 = i\omega B_0 \hat{e}_y$$

$$\text{Here we assume } \vec{E}_0 = E_0 \hat{e}_x \text{ and } \vec{B}_0 = \frac{kE_0}{\omega} \hat{e}_y$$

Here E_0 and B_0 are taken as complex quantities and are related to the corresponding real quantities as,

$$E_0 = |E_0| e^{i\delta}$$

The complex impedance is given by the ratio of complex amplitudes as

$$Z = \frac{E_0}{H_0} = \frac{\mu E_0}{B_0} \quad (7.35)$$

which allows a phase shift between E and H .

\hat{e}_z Now consider a plane polarized wave propagating in the Z-direction normal incidence to an interface \vec{E}_i . Generally medium 1 has complex impedance $Z = Z_1$ and medium 2 has complex impedance $Z = Z_2$. We take coordinates \hat{e}_x along \vec{E}_i ; \hat{e}_y along \vec{H}_i and \hat{e}_z along \vec{k}_1 . We place the boundary at $Z = 0$ so that the X-Y plane is the interface between the two media.

It is simplest to start by considering two dielectric media where we have seen that

$$Z_i = v_i \mu_i \quad (7.36)$$

is real and there is no phase lag between \vec{E} and \vec{H} .

$$\vec{E}_i = E_i \hat{e}_x e^{i(k_1 z - \omega t)}, \quad \vec{H}_i = \frac{E_i}{\mu_1 v_1} \hat{e}_y e^{i(k_1 z - \omega t)} \quad (7.37) \text{ (a)}$$

Also we can take the amplitude E_i to be real. Likewise for transmitted and reflected waves we can write:

$$\vec{E}_t = E_t \hat{e}_x e^{i(k_2 z - \omega t)}, \quad \vec{H}_t = \frac{E_t}{\mu_2 v_2} \hat{e}_y e^{i(k_2 z - \omega t)} \quad (7.37) \text{ (b)}$$

$$\text{and} \quad \vec{E}_r = E_r \hat{e}_x e^{i(-k_2 z - \omega t)}, \quad \vec{H}_r = -\frac{E_r}{\mu_1 v_1} \hat{e}_y e^{i(-k_2 z - \omega t)} \quad (7.37) \text{ (c)}$$

Now, \hat{e}_x and \hat{e}_y are tangential to the interface and tangential components of \vec{E} and \vec{H} are continuous. So, the continuity conditions become

$$\vec{E}_{\text{tan}} = E_x \text{ is continuous : } E_i + E_r = E_t \quad (7.38) \text{ (a)}$$

$$\text{and } \vec{H}_{\text{tan}} = H_y \text{ is continuous : } \frac{E_i}{\mu_1 v_1} - \frac{E_r}{\mu_1 v_1} = \frac{E_t}{\mu_2 v_2} \quad (7.38) \text{ (b)}$$

Solving the above two expressions we get, amplitude transmission coefficient

$$t = \frac{E_t}{E_i} = \frac{2}{1 + \beta} \quad (7.39)$$

and amplitude reflection coefficient

$$r = \frac{E_r}{E_i} = \frac{1 - \beta}{1 + \beta} \quad (7.40)$$

$$\text{where} \quad \beta = \frac{\mu_1 v_1}{\mu_2 v_2} \quad (7.41)$$

Now if the permeability $\mu_i = \mu_0$ (non-magnetic media) we find

$$r = \frac{v_2 - v_1}{v_2 + v_1} = \frac{n_1 - n_2}{n_1 + n_2} \quad (7.42)$$

$$\text{and} \quad t = \frac{2v_2}{v_2 + v_1} = \frac{2n_1}{n_1 + n_2} \quad (7.43)$$

So the reflected wave is in phase if $v_2 > v_1$ but out of phase if $v_2 < v_1$. If $v_2 = v_1$ (same for the two media) there is no reflected wave as expected.

Energy flow: The Poynting vector is $\vec{P} = \vec{E} \times \vec{H} = \frac{\vec{E} \times \vec{B}}{\mu}$

So, the energy flux per unit volume averaged over one complete period of the wave is given by

$$\left\langle \vec{P} \right\rangle = \frac{\left\langle \vec{E} \times \vec{B} \right\rangle}{\mu} = \frac{E_0^2}{2\mu\nu} = \frac{\epsilon\nu E_0^2}{2} \quad (7.44)$$

Now, R the ratio of reflected to incident intensity and T the ratio of transmitted to incident intensity

$$\therefore R = r^2 = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2 \quad (7.45)$$

and
$$T = \frac{\epsilon_2 \nu_2}{\epsilon_1 \nu_1} t^2 = \frac{4n_1 n_2}{(n_1 + n_2)^2} \quad (7.46)$$

So, that $R + T = 1$; leading to energy conservation.

For complex impedance there may be phase lag between \vec{E} and \vec{H} .

So,
$$\vec{E}_i = E_i \hat{e}_x e^{i(k_1 z - \omega t)}, \quad \vec{H}_i = \frac{E_i}{Z_1} \hat{e}_y e^{i(k_1 z - \omega t)}$$

$$\vec{E}_t = E_t \hat{e}_x e^{i(k_2 z - \omega t)}, \quad \vec{H}_t = \frac{E_t}{Z_2} \hat{e}_y e^{i(k_2 z - \omega t)}$$

and
$$\vec{E}_r = E_r \hat{e}_x e^{i(-k_2 z - \omega t)}, \quad \vec{H}_r = -\frac{E_r}{Z_1} \hat{e}_y e^{i(-k_2 z - \omega t)}$$

Assume that there no surface current or charges and then continuity conditions reduce to

$$\vec{E}_{\tan} = E_x \text{ is continuous : } E_i + E_r = E_t$$

and
$$\vec{H}_{\tan} = H_y \text{ is continuous : } \frac{E_i}{Z_1} - \frac{E_r}{Z_1} = \frac{E_t}{Z_2}$$

So, amplitude transmission coefficient

$$t = \frac{E_t}{E_i} = \frac{2Z_2}{Z_1 + Z_2} \quad (7.47)$$

and amplitude reflection coefficient

$$r = \frac{E_r}{E_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad (7.48)$$

These are now complex quantities.

UNIT SUMMARY

- **Maxwell's wave equations and its solution for free space**

$$\text{E-wave equation } \nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\text{H-wave equation } \nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\text{Propagation velocity } v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}$$

- **Transverse nature of electromagnetic wave**

$$\vec{k} \cdot \hat{e} = 0 \qquad \vec{k} \cdot \hat{b} = 0$$

- **EM waves in a charge free non-conducting (dielectric) media**

$$\text{E-wave equation } \nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\text{H-wave equation } \nabla^2 \vec{H} = \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\text{Propagation velocity } v = \frac{1}{\sqrt{\mu \epsilon}} < c \quad [\because \epsilon > \epsilon_0, \mu > \mu_0]$$

- **EM waves in a charge free conducting media**

$$\text{E-wave equation } \nabla^2 \vec{E} - \mu \sigma \frac{\partial \vec{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\text{H-wave equation } \nabla^2 \vec{H} - \mu \sigma \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

- **Attenuation of EM wave in a charge free conducting media**

$$\text{Propagation constant } \gamma = \omega \sqrt{\epsilon \mu} \sqrt{1 + \frac{i\sigma}{\epsilon \omega}}$$

[Dispersion relation]

$$\text{Attenuation constant } \alpha = \omega \sqrt{\frac{\epsilon \mu}{2}} \sqrt{\left\{ 1 + \left(\frac{\sigma}{\epsilon \omega} \right)^2 \right\}^{1/2} + 1}$$

$$\text{Phase constant } \beta = \omega \sqrt{\frac{\epsilon \mu}{2}} \sqrt{\left\{ 1 + \left(\frac{\sigma}{\epsilon \omega} \right)^2 \right\}^{1/2} - 1}$$

$$\text{Penetration depth or skin depth } \delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

• **Boundary conditions for electromagnetic waves at the interface of two media**

$$D_{1n} - D_{2n} = \sigma$$

$$B_{1n} = B_{2n}$$

$$E_{1t} = E_{2t}$$

$$H_{1t} - H_{2t} = J_{s\perp}$$

• **Reflection and transmission of electromagnetic waves from a non-conducting medium-vacuum interface for normal incidence**

$$\text{Amplitude transmission coefficient } t = \frac{E_t}{E_i} = \frac{2Z_2}{Z_1 + Z_2}$$

$$\text{Amplitude reflection coefficient } r = \frac{E_r}{E_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

EXERCISES

Multiple Choice Questions

- 7.1 The velocity of electromagnetic wave in free space is
 (a) equal to velocity of light (b) greater than the velocity of light
 (c) less than the velocity of light (d) zero
- 7.2 In a homogeneous, isotropic, conducting medium of permittivity ϵ , permeability μ and conductivity σ the skin depth δ is
 (a) $\sqrt{\frac{1}{\omega\mu\sigma}}$ (b) $\sqrt{\frac{2}{\omega\mu\sigma}}$ (c) $\sqrt{\frac{\mu}{\omega\sigma}}$ (d) $\sqrt{\frac{2\mu}{\omega\sigma}}$
- 7.3 In a plane *e.m.* wave
 (a) $\vec{E} \times \vec{B} = 0$ (b) $\vec{E} \parallel \vec{B}$ (c) $\vec{E} \perp \vec{B}$ (d) $\vec{k} \times \vec{B} = 0$
- 7.4 The dimension of $1/\mu_0\epsilon_0$ is
 (a) $L^{-2}T^{-2}$ (b) $L^{-2}T^2$ (c) LT^{-1} (d) L^2T^{-2}
- 7.5 Depth of penetration of a wave in a lossy dielectric increases with increase in
 (a) conductivity (b) wavelength (c) permeability (d) permittivity
- 7.6 If E_0 and B_0 be the amplitude of electric field and magnetic field associated with an electromagnetic wave propagating in space, then E_0/B_0 is
 (a) $\sqrt{\frac{\mu_0}{\epsilon_0}}$ (b) $\sqrt{\frac{\epsilon_0}{\mu_0}}$ (c) $\sqrt{\epsilon_0\mu_0}$ (d) $\sqrt{\frac{1}{\epsilon_0\mu_0}}$
- 7.7 Intrinsic impedance in free space is
 (a) 0 ohm (b) 370 ohm (c) 377 ohm (d) none of these
- 7.8 The velocity of electromagnetic wave is obtained from
 (a) EB (b) $E - B$ (c) E (d) E/B

- 7.9 The intrinsic wave impedance of a medium with permeability μ and permittivity ε is
 (a) $\sqrt{\frac{\mu}{\varepsilon}}$ (b) $\sqrt{\frac{\varepsilon}{\mu}}$ (c) $\sqrt{\frac{1}{\varepsilon\mu}}$ (d) $\sqrt{\varepsilon\mu}$
- 7.10 For a good plane conductor, skin depth varies
 (a) directly as square root of frequency (b) inversely as square root of frequency
 (c) directly as a function of frequency (d) inversely with frequency
- 7.11 If \vec{E} and \vec{B} represents the electric field and magnetic field respectively of an electromagnetic waves travelling in vacuum with propagation vector \hat{k} then which condition is correct?
 (a) $\hat{k} \cdot \vec{E} = 0$ (b) $\hat{k} \times \vec{E} = 0$ (c) $\vec{B} \times \vec{E} = 0$ (d) $\hat{k} \times \vec{E} = -\vec{B}$
- 7.12 Electromagnetic wave is propagated through a region of vacuum which does not contain any charge or current. If the electric vector is given by $\vec{E} = E_0 \exp i(kx - \omega t) \hat{j}$ then the magnetic vector is
 (a) in X direction (b) in Y direction
 (c) in Z direction (d) rotating uniformly in the X-Y plane
- 7.13 The electromagnetic wave is called transverse because
 (a) the electric field and magnetic field are perpendicular to each other
 (b) the electric field is perpendicular to the direction of propagation
 (c) the magnetic field is perpendicular to the direction of propagation
 (d) both the electric and magnetic fields are perpendicular to the direction of propagation
- 7.14 In an electromagnetic wave in free space, the electric and magnetic fields are
 (a) parallel to each other (b) perpendicular to each other
 (c) inclined at an angle (d) inclined at an obtuse angle
- 7.15 Dimension of $\mu_0 \varepsilon_0$ (symbols with their usual significance) is
 (a) $L^{-2}T^{-2}$ (b) LT^{-1} (c) $L^{-2}T^{-1}$ (d) $L^{-2}T^2$
- 7.16 Skin depth for a conductor in reference to electromagnetic wave varies
 (a) inversely as frequency (b) directly as frequency
 (c) inversely as square root of frequency (d) directly as square root of frequency
- 7.17 If the frequency of the incident wave increase by a factor of 4 the depth to which a wave penetrates a conducting material
 (a) increases by a factor of 2 (b) increases by a factor of 4
 (c) decreases by a factor of 2 (d) decreases by a factor 4
- 7.18 The direction of propagation of electromagnetic wave is along the direction of
 (a) \vec{E} (b) \vec{B} (c) $\vec{E} \times \vec{B}$ (d) $\vec{B} \times \vec{E}$

Answers of Multiple Choice Questions

7.1 (a), 7.2 (b), 7.3 (c), 7.4 (d), 7.5 (b), 7.6 (d), 7.7 (c), 7.8 (d), 7.9 (a), 7.10 (b), 7.11 (a), 7.12 (c), 7.13 (d), 7.14 (a), 7.15 (d), 7.16 (c), 7.17 (c), 7.18 (c)

Short and Long Answer Type Questions

Category I

- 7.1 Write down Maxwell's equations for a charge free non-conducting medium. Hence, show that the speed of electromagnetic wave in the medium is less than the velocity of light in free space.
- 7.2 Starting from Maxwell's equation in a charge free conducting media arrive at the concept of skin depth.
- 7.3 Write down the Maxwell's equations of an electromagnetic field. Hence, obtain the wave equation for electric field in free space.
- 7.4 Prove that \vec{E} , \vec{B} and \vec{k} are mutually perpendicular to each other.
- 7.5 Explain how velocity of light depends on the properties of the medium.
- 7.6 Prove that in vacuum both electric and magnetic vectors obey wave equation. Assuming a plane wave solution show that electric field is always orthogonal to the magnetic field.
- 7.7 Show that for free space, the electromagnetic wave equation for \vec{E} is $\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$, where symbols have their usual significance.
- 7.8 What is meant by transverse nature of electromagnetic wave?
- 7.9 How can you conclude that the E-field and B-field of electromagnetic wave is perpendicular to the propagation vector k ?
- 7.10 What is attenuation and how attenuation affects the propagation of waves and signals?
- 7.11 Show mathematically that the propagation constant k for a charge free conducting medium is a complex quantity.
- 7.12 Obtain the relation between propagation constant with the attenuation constant and phase constant.
- 7.13 Prove that electromagnetic wave moves with the velocity of light in free space.
- 7.14 Obtain the boundary conditions for EM waves.

Category II

- 7.15 From wave equation for electric field in a charge free non-conducting medium, what type of conclusions can be drawn?
- 7.16 What will be the form of Maxwell's equations in case of dielectric medium and good conductors?
- 7.17 Justify the statement that electromagnetic wave equation describes the propagation of electromagnetic waves.
- 7.18 For what type of behavior non conducting medium is also known as the lossless medium?
- 7.19 From the wave equation for magnetic field in a charge free conducting medium, make your critical comments.
- 7.20 There is a basic difference between penetration depth and skin depth. Explain it clearly.
- 7.21 For the propagation of EM waves in a good conductor find the skin depth in terms of signal frequency.

Numerical Problems

- 7.1 If the relative permittivity and permeability of a medium are 50 and 2 respectively, find the velocity of electromagnetic wave propagation through it. [Ans: 3×10^7 m/s]
- 7.2 If the magnitude of the magnetic field vector of an *em* wave is 1 μ T, find the corresponding magnitude of the electric field vector. [Ans: 300 V/m]

KNOW MORE

It is said that the greatest theoretical achievement of physics in the 19th century was the Maxwell's equations and the related discovery of electromagnetic waves. Electric forces in nature are of two kinds, viz., the electric attraction or repulsion between (+) and (-) electric charges. There is also an attraction and repulsion between parallel electric currents and so electric current and charge are related!

We could have just as well based the unit of current on the unit of charge, *e.g.*, the current in which one unit of charge passes each second through any cross section of the wire. This definition turns out to be quite different and if meters and seconds are used in all definitions, the ratio of the two units of current turns out to be the speed of light, 3×10^8 meters per second.

Activity

The wave nature of light exhibits different colors to be reflected differently by a surface ruled in fine parallel scratches. This is why a compact laser disk, for music or computer use, shimmers in all colors of the rainbow.

Similarly, the orderly rows of atoms in a crystal form parallel lines but spaced closely and they turned out to have the same effect on X-rays revealing that X-rays, like light, are electromagnetic waves of shorter wavelength. It was also found that inside vacuum tube beams of electrons in a magnetic field could become unstable and emit waves larger than light. The magnetron tube where this occurred was a top-secret radar device in World War II, and it later made the microwave oven possible.

Interesting facts

Maxwell transformed the well known Faraday's "lines of force" into mathematical forms that we are using today. EM fields are utilized in many disciplines across mathematics. The first mention of a field in Maxwell's "A Treatise on Electricity and Magnetism" was of an electric field which he described as "the portion of space in the neighborhood of electrified bodies, considered with reference to electric phenomena" (Maxwell, 1892, pp. 47-48). It may be occupied by air or other bodies wherefrom we have withdrawn every substance to act upon with the means at our disposal. If an electrified body is kept at any part of an electric field generally it produces a sensible disturbance in the electrification of the other bodies.

Analogy

Electromagnetic waves not only led to radio and television, but today it has major role to a huge electronic industry. EM waves are also generated in space by unstable electron beams in the magnetosphere, as well as within the Sun and in the far-away universe, suggesting us about energetic particles in distant space or telling us with unresolved mysteries.

History

As far as the history of electromagnetic waves is concerned the important milestones are just noted below without much elaborating:

- 1770-90 *Cavendish* and *Coulomb* established foundations of electrostatics
- 1820 *Oersted* made connection between flowing charge and magnetism
- 1820s *Ampere* identified currents as the source of all magnetism, even for permanent magnets
- 1831 *Faraday* as well as *Henry* discovered that time varying magnetic fields serve as sources for electric fields
- 1864 *Maxwell* put it all together
- 1887 *Hertz* demonstrated the existence of electromagnetic radiation

Timelines

- 600 BCE: Sparking Amber in Ancient Greece
- 221–206 BCE: Chinese Lodestone Compass
- 1600: Gilbert and the Lodestone
- 1752: Franklin's Kite Experiments
- 1785: Coulomb's Law
- 1789: Galvanic Electricity
- 1790: Voltaic Electricity
- 1820: Magnetic Fields

Applications (Real Life / Industrial)

'A Treatise on Electricity and Magnetism', published in 1873, showed the transformation of Maxwell's complete theory of electromagnetism in twelve equations summarizing an entire field of research. Oliver Heaviside condensed Maxwell's theory into the four equations which are well known today. Maxwell's theory of electromagnetism was revolutionary as it totally changed the study of electromagnetism in the right direction. When Hertz discovered electromagnetic waves in space, the wireless telegraph was invented and fast communication was possible across oceans for the first time. Maxwell's theory further made long-distance electrical wires effective and we achieved an enormous amount of technology from Maxwell's theory of electromagnetism.

Case Study (Environmental / Sustainability / Social / Ethical Issues)

While developing the equations Maxwell discovered that light is similar to electricity and magnetism. During the mid 19th century, the concept that the principle of free trade would bring peace and prosperity began to percolate through European society and, in fact, for some time, the dominating economic philosophy had been mercantilism. This was a doctrine of strict protection of a state's resources, high tariffs on trade, and colonial domination. These thoughts of the benefits of free trade in the intellectual culture had a place in the scientific culture also. Instead of working independently, scientists started to find the benefits of developing upon their peers' ideas and elaborating the relative merit of scientific theories as a community (Turner, 1980).

It was this idea of free trade that made Maxwell to share his theory with Europe to peacefully offer a comparison to the leading theory (Forfar, 1995). However, the people who were in the public forefront at the time including politicians and businessmen dismissed the idea that scientific work had anything to do

with their work. By the late 19th century, Germany became an industrial threat to Great Britain's economic dominance and was slowly becoming a military threat. Germany's public funding and recognition of their scientists' research had made German science flourish in the 19th century and was influential and prosperous than British science (Turner, 1980).

Inquisitiveness and Curiosity Topics

The leading theory of electromagnetism, "action at a distance," was developed in Germany. It was the German scientists who looked down upon and refused to acknowledge Maxwell's theory. Frustrated with the lack of response of the scientific community to his paper "On Faraday's Lines of Force," Maxwell transformed his comprehensive theory into twelve equations, a great feat even for today's pace of scientific progress (Forfar, 1995). However, the German scientists still refused to relinquish the monopoly on electromagnetism and simply attempted to dismiss Maxwell's theory as something that just works out on paper (Turner, 1980). However, Oliver Heaviside's editing of Maxwell's equations started to turn the tide, though it was not until Heinrich Hertz, a German physicist, demonstrated in 1887 the existence of a field of electromagnetism by identifying electromagnetic waves in space that German scientists began to accept Maxwell's theory (Hunt, 1983).

References for Case study; Inquisitiveness and Curiosity Topics

Forfar, D. O. (1995, July). *James Clerk Maxwell: Maker of Waves*. Retrieved November 30, 2011, from James Clerk Maxwell Foundation: http://www.clerkmaxwellfoundation.org/Maker_of_Waves.pdf

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10. <https://nptel.ac.in/courses/115/101/115101005/>
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TABLE OF PHYSICAL CONSTANTS

Table I: Molar Susceptibility of Different Materials at 20°C

Helium	-2.38×10^{-11}
Xenon	-5.71×10^{-10}
Oxygen	$+4.3 \times 10^{-8}$
Nitrogen	-1.56×10^{-10}
Water	-1.631×10^{-10}
Bismuth	-3.55×10^{-9}

Table II: Permeability and Relative Permeability of Different Materials

Medium	Permeability, μ (H/m)	Relative permeability, max., μ/μ_0
Vacuum	$4\pi \times 10^{-7}$	1
Water	1.256627×10^{-6}	0.999992
Hydrogen	1.2566371×10^{-6}	1
Wood	$1.25663760 \times 10^{-6}$	1.00000043
Air	$1.25663753 \times 10^{-6}$	1.00000037
Bismuth	1.25643×10^{-6}	0.999834
Carbon steel	1.26×10^{-4}	100
Copper	1.256629×10^{-6}	0.999994
Nickel	$1.26 \times 10^{-4} - 7.54 \times 10^{-4}$	100 - 600
Platinum	1.256970×10^{-6}	1.000265
Sapphire	1.2566368×10^{-6}	0.99999976
Electrical steel	5.0×10^{-3}	4000
Ferrite (nickel zinc)	$1.26 \times 10^{-5} - 2.89 \times 10^{-3}$	10 - 2300
Ferrite (manganese zinc)	$4.4 \times 10^{-4} - 2.51 \times 10^{-2}$	350 - 20000
Ferrite (cobalt nickel zinc)	$5.03 \times 10^{-5} - 1.57 \times 10^{-4}$	40 - 125
Cobalt iron	2.3×10^{-2}	18000
Austenitic stainless steel	$1.260 \times 10^{-6} - 8.8 \times 10^{-6}$	1.003 - 1.05
Teflon	1.2567×10^{-6}	1
Silicon iron powder compound	$2.39 \times 10^{-5} - 1.13 \times 10^{-4}$	19 - 90
Permalloy	1.25×10^{-1}	100000

Table III: Dielectric Constant of Different Materials

Medium	Dielectric constant
Vacuum	1
Polyimide	3.4
Polypropylene	2.2 -2.36
Polystyrene	2.4 - 2.7
Carbon disulfide	2.6
Mylar	3.1
Electro-active polymers	2 - 12
Mica	3 - 6
Silicon dioxide	3.9
Concrete	4.5
Pyrex	4.7
Neoprene	6.7
Rubber	7
Diamond	5.5 - 10
Salt	3 - 15
Graphite	10 - 15
Silicone rubber	2.9 - 4
Silicon	11.68
GaAs	12.4
Glass	3.7 - 10
Methanol	30
Ethylene glycol	37
Furfural	42
Titanium dioxide	86 - 173
Strontium titanate	310
Barium strontium titanate	500
Lead zirconate titanate	500 - 6000
Calcium copper titanate	> 250000

APPENDICES

APPENDIX-A: Suggestive Template for Practicals

- **Aim**
Explain briefly about the aim of the experiment.
- **Relevance**
Explain about the relevancy of the experiment in your own words.
- **Requirements**
List out all the required apparatus along with their proper specifications.
- **Procedure, Observations and Inference**
Explain the procedure of the experiment step-wise and note the observations properly. On the basis of observations certain inference is to be made. You can use a table similar to that given below:

Step No.	Procedure	Observation	Inference
1			
2			
3			

- **Video / animation**
If possible, you can go through some video/animation to visualize the steps physically.
- **Calculations**
Properly calculate all the required physical quantities essential for your experiment.
- **Result and Discussion (Error measurement)**
Obtain the final result and discuss about it with proper considerations of errors which can be introduced during your experiment.
- **Conclusions**
Finally give your conclusion based on the obtained results.
- **Validation of the topics in Experiment**
Try to validate the result of the experiment in real life scenario.
- **Use of ICT**
You can also study using the available online resources. These are useful as there is no time constraint at all. Some of which are listed (not limited to) below:
<https://swayam.gov.in/>
<https://nptel.ac.in/>
<https://www.swayamprabha.gov.in/>

Note for Instructor and Lab-Technicians

Some general and specific instructions are listed separately [see Annexure V] for laboratory preparation, maintenance, safety aspects, etc. Laboratory Instructor and Lab-Technicians can follow those instructions properly to run the laboratory smoothly without any hazard.

APPENDIX-B: Indicative Evaluation Guidelines for Practicals / Projects / Activities in Group

Process Related Skills

Criteria and Level	Developing	Competent	Proficient
Handling the Set-up			
Recording of Data			
Time management			
Team Work			
Individual Work			
Safety Precautions			

Product Related Skills

Criteria and Level	Developing	Competent	Proficient
Content			
Research/Survey			
Use of latest Technology			
Stays on Topic			
Preparedness			
Confidence of Presentation			
ICT Usage including ppt Making Skill			
Time Management			
Group Efforts			
Individual Efforts			

APPENDIX-C: Assessments Aligned to Bloom's Level

- Bloom's Taxonomy – It has been coupled into following two categories for development of Questions for this Book as given below:

Category I Questions	Category II Questions <i>- Higher Order Thinking Skills</i>
Bloom's Level 1: Remember Bloom's Level 2: Understand Bloom's Level 3: Apply	Bloom's Level 4: Analyse Bloom's Level 5: Evaluate Bloom's Level 6: Create

APPENDIX-D: Records for Practical

Sl No.	Page No.	Name of the Experiment	Date			Marks	Signature
			Actual	Repeat	Remarks		
1.		Determination of dielectric constant of a given dielectric material					
2.		Study of Faraday's law and Lenz's law of electromagnetic induction					
3.		To calculate energy loss of a magnetic material using B-H curve					
4.		Measurement of Hall co-efficient and carrier concentration of a semiconducting sample using Hall effect experiment					
5.		Study of resonance condition of a series LCR circuit					
6.		Determination of specific charge (e/m) of electron by J. J. Thomson's method					
7.		Study of the variation of magnetic field using Helmholtz coils					

ANNEXURES

Annexure- I: Some important second order partial differential equations

(i) Electrostatic potential: The electrostatic potential V due to a charge distribution is a function of the space variables x , y and z . The equation that determines V is the well-known partial differential equation.

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) V = 4\pi\rho$$

where $\rho = \rho(x, y, z)$, is a given volume-distribution of charge.

The equation $\nabla^2 V = 4\pi\rho$ is the Poisson's equation of electrostatics.

If however the region is charge free, $\rho = 0$.

$$\therefore \nabla^2 V = 0$$

This is the Laplace's equation of electrostatics and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \sum \frac{\partial^2}{\partial x^2}$$

is the Laplacian operator or simply the Laplacian.

Propagation of waves: When a quantity propagates in the form of a wave, it is a function of x , y , z (the space coordinates) and t (the time variable).

$$\therefore \psi = \psi(x, y, z, t)$$

The equation satisfied by ψ is given by
$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) = c^2 \nabla^2 \psi$$

where c is the velocity of propagation of the wave. This is known as wave equation.

For elastic waves, ψ is a component of displacement but for electromagnetic waves it is a component of electric or magnetic intensity.

(ii) Maxwell's field equations: These constitute a set of differential equations involving the electric and magnetic intensity vector E and H and are of fundamental importance in all problems of electromagnetism.

$$\begin{aligned} \vec{\nabla} \times \vec{H} &= \vec{J} & \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{D} &= \rho & \vec{\nabla} \cdot \vec{B} &= 0 \end{aligned}$$

Annexure- II: Solution of Laplace's equation

Three dimensional Cartesian form: Let us take

$$u(x, y, z) = X(x)Y(y)Z(z)$$

as the solution of three dimensional (Cartesian) Laplace's equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

Substituting $u(x, y, z)$ in the above equation and dividing throughout by XYZ , we obtain

$$\frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} + \frac{1}{Z} \frac{d^2 Z}{dz^2} = 0$$

This relation is of the form

$$f_1(x) + f_2(y) + f_3(z) = 0$$

Since x, y, z are independent, this will not be true unless f_1, f_2, f_3 are constant functions. Taking these constants to be $k^2, l^2, -(k^2 + l^2)$ respectively, we get the equations

$$\frac{d^2 X}{dx^2} - k^2 X = 0$$

$$\frac{d^2 Y}{dy^2} - l^2 Y = 0$$

$$\frac{d^2 Z}{dz^2} + (k^2 + l^2) Z = 0$$

where k^2, l^2 may be real or complex.

These have the respective solutions

$$X = Ae^{kx} + Be^{-kx}$$

$$Y = Ce^{ly} + De^{-ly}$$

$$Z = E \cos \sqrt{k^2 + l^2} z + F \sin \sqrt{k^2 + l^2} z$$


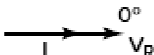

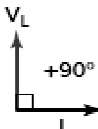
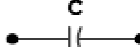
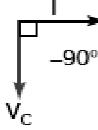

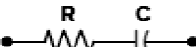
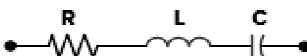
$$\therefore u = XYZ = (Ae^{kx} + Be^{-kx})(Ce^{ly} + De^{-ly})(E \cos \sqrt{k^2 + l^2} z + F \sin \sqrt{k^2 + l^2} z)$$

Annexure- III: Series *RLC* Circuit Phase Angle

The circuit's phase angle θ (ϕ) is always the angle that separates the circuit's current and the applied voltage source, as summarized below.

- A series *RLC* circuit will be inductive and have a positive phase angle when the inductive reactance and resulting voltage across the inductor is greater than the capacitive reactance and the resulting voltage across the capacitor.
- A series *RLC* circuit will be capacitive and have a negative phase angle when the capacitive reactance and resulting voltage across the capacitor is greater than the inductive reactance and the resulting voltage across the inductor.

Table A.1: Impedance value and phase angle for different circuit elements

Circuit Elements	Impedance Z	Phase Angle ϕ
	$Z = R$	
	$Z = X_L$	
	$Z = X_C$	
	$Z = \sqrt{R^2 + X_L^2}$	Positive, between 0° and 90°
	$Z = \sqrt{R^2 + X_C^2}$	Negative, between -90° and 0°
	$Z = \sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Annexure- IV: Legendre polynomials

The Legendre polynomials $P_n(u)$ of order n can be defined by the following generating function:

$$\frac{1}{\sqrt{1+h^2-2hu}} = \sum_{n=0}^{\infty} h^n P_n(u)$$

where $|h| < 1, |u| < 1$

These are the first 5 Legendre polynomials:

$$P_0(u) = 1$$

$$P_1(u) = u$$

$$P_2(u) = \frac{3u^2 - 1}{2}$$

$$P_3(u) = \frac{5u^3 - 3u}{2}$$

$$P_4(u) = \frac{35u^4 - 30u^2 + 3}{8}$$

In some Legendre polynomial formulae also the $n = -1$ index can occur. In order that the formulae are valid for this index, one has to use the convention $P_{-1}(u) = P'_{-1}(u) = P_{-1}(u) = 0$

where $P'_n = P'_n(u)$ and $P''_n = P''_n(u)$ denote the first and second derivatives of the Legendre polynomial $P_n = P_n(u)$

Special values of the Legendre polynomials are the following:

$$P_n(1) = 1$$

$$P_n(-1) = (-1)^n$$

$$P_{2n+1}(0) = 0$$

$$P_{2n}(0) = (-1)^n \frac{2n!}{(2^n n!)^2}$$

$$P'_n(\pm 1) = (\pm 1)^{n+1} \frac{n(n+1)}{2}$$

The following 2 recurrence relations are recommended for the fast computation of the Legendre polynomials and their first derivatives (for $n > 1$):

$$P_n = 2uP_{n-1} - P_{n-2} - (uP_{n-1} - P_{n-2}) / n$$

$$P'_n = 2uP'_{n-1} - P'_{n-2} + (uP'_{n-1} - P'_{n-2}) / (n-1)$$

with $P'_0(u) = 0, P'_1(u) = 1$

The following recurrence relation is valid for arbitrary higher derivatives

$$P_n^{(m)} = d^m P_n / du^m \quad (\text{for } n > m)$$

$$P_n^{(m)} = 2uP_{n-1}^{(m)} - P_{n-2}^{(m)} + \frac{2m-1}{n-m} \left(uP_{n-1}^{(m)} - P_{n-2}^{(m)} \right)$$

Recurrence relations are extremely useful for the analytical and numerical investigations connected with Legendre polynomials.

There are several other mixed recurrence relations that contain both P_n and P'_n

$$nP_n = uP'_n - P'_{n-1}$$

$$(n+1)P_n = P'_{n+1} - uP'_n$$

$$(2n+1)P_n = P'_{n+1} - P'_{n-1}$$

$$(1-u^2)P'_n = n(P_{n-1} - uP_n)$$

$$(1-u^2)P'_n = (n+1)(uP_n - P_{n+1})$$

By differentiating the above equations over u , we obtain useful relations containing the second derivatives P''_n

$$(1-u^2)P''_n = (n+2)uP'_n - nP'_{n+1}$$

$$(1-u^2)P''_n = (n+1)P'_{n-1} - (n-1)uP'_n$$

$$(1-u^2)P''_n = 2P'_{n-1} - n(n-1)P_n$$

$$(1-u^2)P''_n = 2P'_{n+1} - (n+1)(n+2)P_n$$

Annexure- V: Different type of errors in measurements

Measurement and its importance in Science and Technology

Measurements represent quantities relating to a real time system by making use of numerical values. Requirements for accurate measurements are:

- i) Apparatus must be accurate,
- ii) Method used ought to be provable and
- iii) Standard used should be defined correctly.

In science and technology, advancement is of little significance without the availability of actual measured values with practical proofs. A device to determine variable is known as an instrument. It serves an aid for humans to measure values of unknown quantities. An instrument can be electronic, mechanical or electrical. Based on the degree of variation of the measured quantity with respect to time, an instrument can have static or dynamic characteristics.

Errors in Measurement

If ideal conditions are applied for measuring any parameter, the average deviations of different factors tend to be zero. Average of these infinite numbers of measured values is known as True Value. But this is hypothetical as the negative and positive deviations do not cancel each other in practice. The measured value obtained under ideal conditions is considered as the true value or the best-measured value. A difference between the actual value and the true value is called an Error.

Types of Error

Systematic errors are broadly of three categories;

- i) Instrumental Error,
- ii) Environmental Error and
- iii) Observational Error.

Instrumental errors occur owing to shortcomings in the instruments, improper use of instruments or loading effect of the instrument. Improper construction, calibration or operation of an instrument might result in some inherent errors. Environmental errors occur due to external ambient conditions of the instrument which include changes in temperature, humidity, availability of dust, vibrations or effects of external magnetic or electrostatic fields. Random errors occur due to some small factors which fluctuate from one measurement to another.

The manufacturer of any instrument defines a certain accuracy that depends on the type of material and the effort to manufacture the instrument. This accuracy is defined within a certain deviations from the nominal value. The limits of these deviations are termed as Limiting Errors. The ratio of error to the specified nominal value is called relative Limiting Error. Manual errors in reading an instrument or recording any measurement are known as Gross errors which occur during the experiments. The errors in any scientific measurement may happen from different sources which are presented in Fig. A.1. Observational errors may occur due to the fault study of the instrument reading for many sources. Environmental errors, on the other hand, happen due to the outside situation of the measuring instruments, mostly due to the temperature result, force, moisture, dirt, vibration.

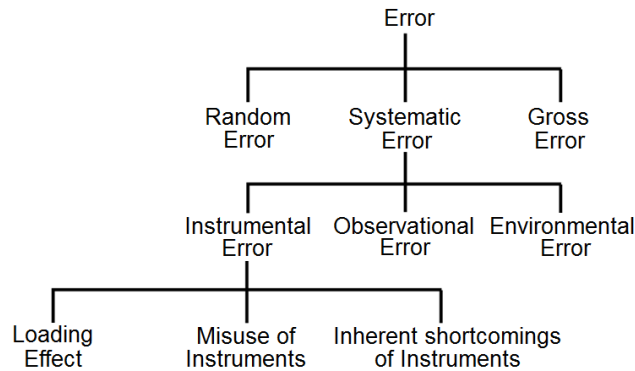


Fig. A.1: Error in measurements

There is an inherent limitation of devices due to their mechanical arrangement.

Calculation of Error in Measurement

These errors are categorized into three types:

- i) absolute error,
- ii) relative error, and
- iii) percentage error.

The absolute error is defined as the variation between the values of actual and measured quantities.

If we denote the measured value as V_A , and the exact value as V_E , then we have

$$\text{Absolute error} = |V_A - V_E|$$

$$\begin{aligned} \text{Relative Error} &= \frac{\text{Absolute error}}{\text{Actual error}} \\ &= \frac{|V_A - V_E|}{V_E} \end{aligned}$$

$$\text{Percentage error (\%)} = \frac{|V_A - V_E|}{V_E} \times 100$$

Example: A length was calculated as 6.8 cm but the absolute length was 6.74 cm. Find the absolute, relative and percentage errors.

Ans: Given that $V_A = 6.8$ cm and $V_E = 6.74$ cm

$$\text{Absolute error} = |V_A - V_E| = |6.8 - 6.74| = 0.06 \text{ cm}$$

$$\text{Relative Error} = \frac{|V_A - V_E|}{V_E} = \frac{0.06}{6.74} = 0.0089$$

$$\text{Percentage error (\%)} = \frac{|V_A - V_E|}{V_E} \times 100 = \frac{0.06}{6.74} \times 100 = 0.89\%$$

Arithmetic Mean Value

To minimize random errors, the measurements are repeated and the average value is taken as the correct value of the measured quantity. The arithmetic mean value would be very close to the most accurate reading. If the number of observation is taken 'n' times, the random error reduces to $\frac{1}{n}$ times.

Let $a_1, a_2, a_3, \dots, a_n$ are the n different measured readings of a physical quantity. The most accurate value is its arithmetic mean value which can be obtained from,

$$a_{\text{mean}} = \frac{a_1 + a_2 + \dots + a_n}{n} = \frac{1}{n} \sum_{i=1}^n a_i$$

Absolute Error

The magnitude of the difference between the true value of the quantity and the measured value is known as the absolute error in the measurement. Since the true value of the quantity is not known, the arithmetic mean of the measured values is taken as the true value.

If a_1, a_2, \dots are the measured values of a certain quantity, the errors $\Delta a_1, \Delta a_2, \dots$ in the measurements are

$$\Delta a_1 = a_{\text{mean}} - a_1$$

$$\Delta a_2 = a_{\text{mean}} - a_2$$

$$\Delta a_3 = a_{\text{mean}} - a_3$$

$$\Delta a_4 = a_{\text{mean}} - a_4 \quad \text{and so on}$$

The arithmetic mean of all the absolute errors is considered as the final absolute error in the measurement and is called mean absolute error. The value obtained in a single measurement may be in the range: $a_{\text{mean}} \pm \Delta a_{\text{mean}}$

Relative Error

The ratio of the absolute error to the true value of the measured quantity is known as the relative error or fractional error. As the arithmetic mean value is taken as the true value, the relative error can be expressed as,

$$\text{Relative error, } \delta a = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$$

Percentage Error

It is the relative error when expressed in percentage. Thus, we have,

$$\text{Percentage error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$$

Example: Using a screw gauge successive measured readings of the radius of a wire are 1.21 mm, 1.19 mm, 1.20 mm, 1.18 mm and 1.17 mm respectively. Determine the absolute errors and the relative error in the measurement.

Ans: Arithmetic mean value is, $a_{\text{mean}} = \frac{1.21 + 1.19 + 1.20 + 1.18 + 1.17}{5} = 1.19 \text{ mm}$

Table A.2: Error in measurements

Difference between a_{mean} and measured value in mm	Magnitude of error in mm
$1.21 - 1.19 = 0.02$	0.02
$1.19 - 1.19 = 0.00$	0.00
$1.20 - 1.19 = 0.01$	0.01
$1.18 - 1.19 = -0.01$	0.01
$1.17 - 1.19 = -0.02$	0.02

The arithmetic mean of the absolute errors is, $\Delta a_{\text{mean}} = \frac{0.02 + 0.00 + 0.01 + 0.01 + 0.02}{5} = 0.016 \text{ mm}$

$$\text{Relative error, } \delta a = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} = \frac{0.016}{1.19} = 0.0134$$

$$\text{Percentage error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\% = \pm 1.34\%$$

Combination of errors

If a quantity is obtained by combining a few measurements, the errors in those measurements can be combined in some way or other.

Error in the sum of the quantities

We consider two quantities A and B which have measured values $A \pm \Delta A$ and $B \pm \Delta B$ respectively; ΔA and ΔB are the absolute errors in the measurements.

To determine the error ΔZ which may occur in the sum $Z = A + B$, we consider

$$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B) = A + B \pm \Delta A \pm \Delta B$$

Maximum possible error in the value of Z is,

$$\Delta Z = \Delta A + \Delta B.$$

Hence when two quantities are added, the absolute error in the result is the sum of the absolute errors in the measured quantities.

Error in the difference of the quantities

We consider two quantities, A and B which have measured values $A \pm \Delta A$ and $B \pm \Delta B$ respectively; ΔA and ΔB are the absolute errors in their measurements.

To determine the error ΔZ which may occur in the difference $Z = A - B$, we consider

$$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B) = A - B \pm \Delta A \pm \Delta B$$

Hence the maximum possible error in the value of Z is given by

$$\Delta Z = \Delta A + \Delta B.$$

So when two quantities are subtracted, the absolute error in the result is the sum of the absolute errors in the measured quantities.

Error in the product of the quantities

We consider two quantities, A and B which have measured values $A \pm \Delta A$ and $B \pm \Delta B$ respectively; ΔA and ΔB are the absolute errors in their measurements.

To get the error ΔZ which may occur in the product $Z = AB$, we consider

$$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B) = AB \pm A\Delta B \pm B\Delta A \pm \Delta A\Delta B$$

Dividing left side by Z and right side by AB we have,

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \frac{\Delta A\Delta B}{AB}$$

The maximum fractional error in Z is,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$$

(As ΔA and ΔB are small, their products $\frac{\Delta A\Delta B}{AB}$ are too small and can be ignored)

Hence when two quantities are multiplied, the fractional error in the result is the sum of the fractional errors in the measured quantities.

Error in the quotient of the quantities

We consider two quantities, A and B which have measured values $A \pm \Delta A$ and $B \pm \Delta B$ respectively; ΔA and ΔB are the absolute errors in their measurements.

To find the error ΔZ that may occur in the quotient $Z = \frac{A}{B}$, we consider

$$Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B}$$

In a similar way, solving we have, the maximum fractional error in Z as,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$$

Hence when two quantities are divided, the fractional error in the result is the sum of the fractional errors in the measured quantities.

It shows that the maximum percentage error in Z is the sum of the maximum percentage error in A and maximum percentage error in B i.e.,

$$\frac{\Delta Z}{Z} \times 100 = \frac{\Delta A}{A} \times 100 + \frac{\Delta B}{B} \times 100$$

Error when a quantity is raised to a power

The error ΔZ which may occur when a quantity is raised to its n^{th} power is n times the fractional error in the quantity itself *i.e.*, if $Z = A^n$,

$$\frac{\Delta Z}{Z} = n \frac{\Delta A}{A}$$

The maximum percentage error in Z can be written as,

$$\frac{\Delta Z}{Z} \times 100 = n \times \frac{\Delta A}{A}$$

Systematic Errors

Systematic errors occur due to fault in the measuring device. They are also called as Zero Error – a positive or negative error. These errors can be detached after correction of the measurement device. These errors are classified into different categories.

Systematic errors are categorized as:

- Instrumental error
- Environmental error
- Observational error
- Theoretical error

Parallax Error

This error occurs due to wrong observations of reading in the instruments. The wrong observations may be due to parallax. To minimize the parallax error highly accurate meters provided with mirror scales are essential.

Estimating Random Errors

There are many ways to make an estimate of the random error in a particular measurement. The simplest way is to make a series of measurements of a given quantity (say, x) and calculate the mean and standard deviation (\bar{x} and σ_x) from this data.

The mean \bar{x} is defined as,

$$\bar{x} = \frac{1}{N} \sum_{i=1}^n x_i$$

where, x_i is the result of the i^{th} measurements, N is the number of measurements

The standard variation is given by,

$$\sigma_x = \frac{1}{N} \sum_{i=1}^n (x_i - \bar{x})^2$$

Annexure-VI: Some general and specific instructions when working in the laboratory

General Instructions

1. In the laboratory, work quietly and cautiously. Remember the main purpose of doing any experiment is to make faithful measurements.
2. Always share equally all the steps of the work with your partner.
3. Presentations of data in tabular form, graphs and calculations should be done correctly and sincerely.
4. Be always honest at the time of recording and representing the experimental data.
5. It is very important to keep in mind that never make up readings or doctor them to get a better fit of the graph as per theory. If any reading appears incorrect, you have to repeat the measurement again and again to find the source of error.
6. At the time of drawing the graph all the data obtained from experiment are to be properly plotted.
7. It is a fact that the objective of the laboratory is learning and also a verification of the knowledge that you have gathered. The experiments are designed properly for the purpose of illustrating different phenomena in all the important areas of Physics.
8. By doing the experiment with your own interest only it is possible to be familiar with all the fine points and to expose you to measuring instruments.
9. Always perform the experiment with an attitude of learning and with your interest to verify the theoretical knowledge that you have gathered.
10. Be very particular to arrive in time in the laboratory and always with proper preparation with a clear knowledge about the experiment.

Specific Instructions

1. When working in the laboratory for collecting data of your experiment, it is important to note all the measured data neatly in the notebook.
2. The recorded data entered in the notebook have to confirm by your instructor before leaving the laboratory.
3. All the students doing the same experiment have to maintain individual copy of the recorded data. The laboratory notebook is required to bring in the laboratory regularly when you come for doing the experiment.
4. Graphs are to be drawn properly at the end of each of experiment.
5. For this you need to know how to optimize on usage of graph paper. Remember all the repeated data are to be accommodated on a single graph sheet.
6. Graphs are to be labeled properly along with the axes showing the corresponding units.
7. During the working hours in the laboratory you are supposed to fully utilize the duration and do not leave the laboratory before the completion of the working hours. If you finish early, you may spend the remaining time to complete the calculations and graphs drawing and for that in the laboratory you are supposed to come equipped with calculators, pencils and scale.

REFERENCES FOR FURTHER LEARNING

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CO AND PO ATTAINMENT TABLE

Course outcomes (COs) for this course can be mapped with the programme outcomes (POs) after the completion of the course and a correlation can be made for the attainment of POs to analyze the gap. After proper analysis of the gap in the attainment of POs necessary measures can be taken to overcome the gaps.

Table for CO and PO attainment

Course Outcomes	Attainment of Programme Outcomes (1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation)											
	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7	PO-8	PO-9	PO-10	PO-11	PO-12
CO-1												
CO-2												
CO-3												
CO-4												
CO-5												
CO-6												

The data filled in the above table can be used for gap analysis.

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