Nashik

K. K. Wagh Institute of Engineering Education and Research, Nashik

(An Autonomous Institute from A. Y. 2022-23)

Model Answer

End-Sem Examination-I, Winter 2025

Academic Year: 2025-2026	
Class: F.Y.	Semester: II
Branch Code: FYE	Program:B.Tech
Name of Course:Diffrential Equation and integral calculus	Pattern: 2023
	Course Code: 2300102A

Q1)

$$\cos x \frac{dy}{dx} + y \sin x = \sqrt{y} \sec x$$

$$\frac{dy}{dx} + y \tan x = y^{1/2} \sec^{1/2} x \cdot \sec x .$$

$$\therefore \frac{dy}{dx} + y \tan x = y^{1/2} \sec^{3/2} x .$$

divide by $y^{1/2}$

$$\frac{1}{y^{1/2}} \frac{dy}{dx} + \frac{y}{y^{1/2}} \tan x = \sec^{3/2} x$$

$$\frac{1}{y^{1/2}} \frac{dy}{dx} + y^{1/2} \tan x = \sec^{3/2} x$$

Put $y^{1/2} = u$.

$$\frac{1}{2} y^{-1/2} \frac{dy}{dx} = \frac{du}{dx}$$

$$\therefore \frac{1}{y^{1/2}} \frac{dy}{dx} = 2 \frac{du}{dx}$$

$$\therefore 2 \frac{du}{dx} + (\tan x) u = \sec^{3/2} x .$$

$$\therefore \frac{du}{dx} + \left(\frac{\tan x}{2}\right)u = \frac{\sec^{3/2} x}{2}$$

$$P = \frac{\tan x}{2}, \quad Q = \frac{\sec^{3/2} x}{2}$$

$$\therefore I.F = e^{\int P\,dx}$$

$$= e^{\frac{1}{2}\int \tan x\,dx}$$

$$= e^{\frac{1}{2}\log \sec x}$$

$$= e^{\log \sec^{1/2} x}$$

$$\boxed{I.F = \sec^{1/2} x}$$

$\therefore$ G.S is,

$$u(I.F) = \int Q\,(I.F)\,dx + c$$

$$u \sec^{1/2} x = \int \frac{\sec^{3/2} x}{2} \sec^{1/2} x \, dx + c$$

$$= \frac{1}{2}\int \sec^2 x\,dx + c$$

$$u\sqrt{\sec x} = \frac{1}{2}\tan x + c$$

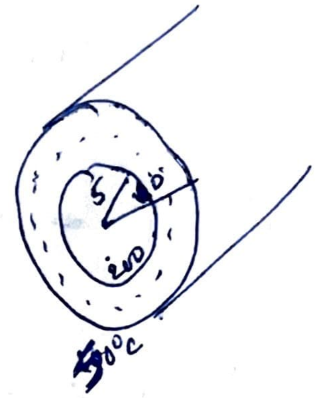
$\oint$ Resubstitute $u = y^{1/2}$

$$\sqrt{y \sec x} = \frac{1}{2}\tan x + c.$$

By fourier's law,

$$q = -KA\frac{dT}{dx}.$$

$$q = -K(2\pi x)\frac{dT}{dx} \quad - [A = 2\pi x]$$



$$\therefore \frac{-q}{2\pi K}\,\frac{dx}{x} = dT.$$

Variables are separated.

$$\int dT = \frac{-q}{2\pi K}\int \frac{dx}{x}$$

$x = 5$ cm, $T = 200°c$
$x = 10$ cm, $T = 50°c$

$$[T]_{200}^{50} = \frac{-q}{2\pi K}[\log x]_{5}^{10}$$

$$50 - 200 = \frac{-q}{2\pi K}[\log 10 - \log 5]$$

$$-150 = \frac{-q}{2\pi K}\log\left(\frac{10}{5}\right)$$

$$\therefore \frac{q}{2\pi K} = \frac{150}{\log 2}$$

$$\therefore q = \frac{150 \times 2 \times 3.14 \times 0.12}{\log 2}$$

$$\boxed{q = 163.8 \text{ cal/sec}}$$

Now, $x = 7.5$ cm, $T = ?$

$$\int_{200}^{T'} dT = \frac{-q}{2\pi K}\int_{5}^{7.5}\frac{dx}{x}$$

$$T - 200 = \frac{-q}{2\pi k}\left[\log 7.5 - \log 5\right]$$

$$\therefore T - 200 = \frac{-q}{2\pi k}\log\left(\frac{7.5}{5}\right)$$

$$\therefore T - 200 = \frac{-150}{\log 2}\log(1.5)$$

$$\therefore T = -\overset{87.74}{\cancel{89.13}} + 200$$

$$\boxed{T = \cancel{110.87^\circ C}}\ 112.23^\circ C$$

Q3)

a)

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1911	12				
		3			
1921	15		2		
		5		0	
1931	20		2		$\boxed{3}$ $\nabla^4 y_n$
		7		$\boxed{3}$ $\nabla^3 y_n$	
1941	27		$\boxed{5}$ $\nabla^2 y_n$		
		$\boxed{12}$ ∇y_n			
$\boxed{1951}$ x_n	$\boxed{39}$ y_n				

$$u = \frac{x - x_n}{h}$$

$$u = \frac{1948 - 1951}{10}$$

$$\boxed{u = -0.3}$$

Using backward Interpolation formula,

$$y = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n$$

$$+ \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 y_n.$$

$$= 39 + (-0.3)(12) + \frac{(-0.3)(-0.3+1)}{2}(5) + \frac{(-0.3)(-0.3+1)(-0.3+2)}{6}(3) + \frac{(-0.3)(-0.3+1)(-0.3+2)(-0.3+3)}{24}(3)$$

$$= 39 - 3.6 - 0.525 - 0.1785 - 0.1204$$

$\boxed{y = 34.57 \approx 35 \text{ Thousand}}$ for $x = 1948$.

Q3) b)

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
		y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
x_0	$\boxed{100}$	$\boxed{10.63}$	$\boxed{2.4}$	$\boxed{-0.39}$	$\boxed{0.15}$	$\boxed{-0.07}$
	150	13.03	2.01	-0.24	0.08	
	200	15.04	1.77	-0.16		
	250	16.81	1.61			
	300	18.42				

$$u = \frac{x - x_0}{h}$$

$$u = \frac{140 - 100}{50}$$

$$\boxed{u = 0.8}$$

$\therefore$ by Newton's forward Interpolation formula.

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

$$y = 10.63 + (0.8)(2.4) + \frac{(0.8)(0.8-1)}{2}(-0.39)$$

$$+ \frac{(0.8)(0.8-1)(0.8-2)}{6}(0.15) + \frac{(0.8)(0.8-1)(0.8-2)(0.8-3)}{24}(-0.0\ldots)$$

$$\doteq 10.63 + 1.92 + 0.0312 + 0.0048 + 0.0012$$

$$\boxed{y = 12.5872} \text{ miles for } x = 140.$$

Q3) c)

i) $(E+2)(E-1)(e^x + x)$, $h = 1$

$\rightarrow (E+2)(E-1)(e^x + x) = (E^2 - E + 2E - 2)(e^x + x)$

$$= (E^2 + E - 2)(e^x + x)$$

$$= E^2(e^x + x) + E(e^x + x) - 2(e^x + x)$$

$$= e^{(x+2h)} + (x+2h) + e^{x+h} + (x+h) - 2e^x - 2x$$

$$= e^x \cdot e^{2h} + \cancel{x} + 2h + e^{x+h} + \cancel{x} + h - 2e^x - \cancel{2x}$$

$$= e^x \cdot e^2 + 2 + e^x \cdot e + 1 - 2e^x - [h = 1]$$

$$\boxed{(E+2)(E-1)(e^x + x) = e^x[e^2 + e - 2] + 3}$$

ii) $E f(x)$, if $f(x) = x^3$, $h = 1$.

$\rightarrow E f(x) = E(x^3)$

$$= (x+h)^3$$

$$= \cancel{x^3 +} (x+1)^3$$

$$\boxed{E f(x) = x^3 + 3x^2 + 3x + 1}$$

d)

i) find $\Delta^n e^x$, $h = 1$

$\rightarrow$ $\Delta e^x = e^{x+h} - e^x$

$= e^{x+1} - e^x$

$= e^x [e - 1]$

$\Delta^2 e^x = \Delta(\Delta e^x)$

$= \Delta[e^x (e-1)]$

$= (e-1)\,\Delta(e^x)$

$= (e-1)\,e^x (e-1)$

$\Delta^2 e^x = (e-1)^2 e^x$

$\Delta^3 e^x = (e-1)^3 e^x$

$\vdots$

$\boxed{\Delta^n e^x = (e-1)^n e^x.}$

ii) P.T. $\delta E^{1/2} = \Delta$.

$\rightarrow$ $\delta E^{1/2} f(x) = \delta\left[f\left(x + \frac{h}{2}\right)\right]$

$= f\left(x + \frac{h}{2} + \frac{h}{2}\right) - f\left(x + \cancel{\frac{h}{2}} - \cancel{\frac{h}{2}}\right)$

$= f(x + h) - f(x)$

$\delta E^{1/2} f(x) = \Delta f(x)$

$\boxed{\therefore\ \delta E^{1/2} = \Delta.}$

Q3] e)

	x_0	x_1	x_2
x	0	1	2
y	1	3	5
	y_0	y_1	y_2

By using Lagrange's Interpolation formula

$$y = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$$

$$= y_0 \left[\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\right] + y_1 \left[\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\right] +$$

$$y_2 \left[\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\right]$$

$$= 1\left[\frac{(x-1)(x-2)}{(0-1)(0-2)}\right] + 3\left[\frac{(x-0)(x-2)}{(1-0)(1-2)}\right] + 5\left[\frac{(x-0)(x-1)}{(2-0)(2-1)}\right]$$

$$= \left[\frac{x^2-2x-x+2}{2}\right] + 3\left[\frac{x^2-2x}{-1}\right] + 5\left[\frac{x^2-x}{2}\right]$$

$$= \left[\frac{x^2-3x+2}{2}\right] - \left[3x^2-6x\right] + \left[\frac{5x^2-5x}{2}\right]$$

$$= \left(\frac{x^2}{2} - \frac{6x^2}{2} + \frac{5x^2}{2}\right) + \left[\frac{-3x+12x-5x}{2}\right] + 1$$

$$\boxed{y = 0 + 2x + 1}$$

$$\left(\frac{dy}{dx}\right)_{x=2} = 2$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.30	y_{-2} 0.1179				
		0.0189			
0.35	y_{-1} 0.1368		−0.0003		
		0.0186		−0.0001	
x_0 $\boxed{0.40}$	y_0 $\boxed{0.1554}$		−0.0004		0.0002
		0.0182		0.0001	
0.45	y_1 0.1736		−0.0003		
		0.0179			
0.50	y_2 0.1915				

$$u = \frac{x - x_0}{h}$$

$$= \frac{0.41 - 0.40}{0.05}$$

$$\boxed{u = 0.2}$$

Using stirling's formula -

$$y = y_0 + \frac{u}{1!}\left(\frac{\Delta y_{-1} + \Delta y_0}{2}\right) + \frac{u^2}{2!}\Delta^2 y_1 + \frac{u(u^2-1)}{3!}\left[\frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2}\right]$$

$$+ \frac{u^2(u^2-1)}{4!}\Delta^4 y_{-2}$$

$$= 0.1554 + (0.2)\left[\frac{0.0186 + 0.0182}{2}\right] + \frac{(0.2)^2}{2}(-0.0004) +$$

$$\frac{(0.2)[(0.2)^2 - 1]}{6}\left[\frac{-0.0001 + 0.0001}{2}\right] + \frac{(0.2)^2[(0.2)^2 - 1]}{24}(0.0002)$$

$$= 0.1554 + 0.0037 + 0.000008 + 0 - 0.00000032$$

$\boxed{y = 0.1591}$ for $x = 0.41$

Q4

a) $\frac{dy}{dx} = \frac{y-x}{y+x}$, $x_0 = 0$, $y_0 = 1$, $h = 0.02$

$x_1 = x_0 + h$

$= 0 + 0.02$

$\boxed{x_1 = 0.02}$

$x_2 = x_1 + h$

$\boxed{x_2 = 0.04}$

$\boxed{x_3 = 0.06}$

$\boxed{x_4 = 0.08}$

$\boxed{x_5 = 0.1}$.

$y_1 = y_0 + hf(x_0, y_0)$

$= 1 + (0.02) f(0, 1)$

$= 1 + (0.02)(1)$

$\boxed{y_1 = 1.02}$

$y_2 = y_1 + hf(x_1, y_1)$

$= 1.02 + (0.02) f(0.02, 1.02)$

$= 1.02 + (0.02)(0.9615)$

$\boxed{y_2 = 1.0392}$

$y_3 = y_2 + hf(x_2, y_2)$

$= 1.0392 + (0.02) f(0.04, 1.0392)$

$= 1.0392 + (0.02)(0.9259)$

$\boxed{y_3 = 1.0577}$

$y_4 = y_3 + hf(x_3, y_3)$.

$= 1.0577 + (0.02) f(0.06, 1.0577)$

$= 1.0577 + (0.02)(0.8926)$

$\boxed{y_4 = 1.0756}$

$y_5 = y_4 + hf(x_4, y_4)$

$= 1.0756 + (0.02) f(0.08, 1.0756)$

$= 1.0756 + (0.02)(0.8615)$

$\boxed{y_5 = 1.0928}$

$$\frac{dy}{dx} = -xy^2, \quad x_0 = 0, \quad y_0 = 2, \quad h = 0.1.$$

$$f(x_0, y_0) = -xy^2$$
$$= 0.$$

$$y_1 = y_0 + hf(x_0, y_0)$$
$$= 2 + 0.$$
$$\boxed{y_1 = 2}$$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1)\right]$$
$$= 2 + \frac{0.1}{2}\left[0 - 0.4\right]$$
$$\boxed{y_1^{(1)} = 1.98}$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$
$$= 2 + \frac{0.1}{2}\left[0 - 0.3920\right]$$
$$\boxed{y_1^{(2)} = 1.9804}$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$
$$= 2 + \frac{0.1}{2}\left[0 - 0.3922\right]$$
$$\boxed{y_1^{(3)} = 1.9804}$$

c) $\frac{dy}{dx} = \frac{1}{x+y}$, $x_0 = 0$, $y_0 = 1$, $h = 0.2$

$K_1 = hf(x_0, y_0)$

$= 0.2 f(0, 1)$

$\boxed{K_1 = 0.2}$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right)$

$= 0.2 f(0.1, 1.1)$

$= (0.2)(0.8333)$

$\boxed{K_2 = 0.1667}$

$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.1667}{2}\right)$

$= 0.2 f(0.1, 1.0834)$

$= (0.2)(0.8450)$

$\boxed{K_3 = 0.1690}$

$K_4 = hf(x_0 + h, y_0 + K_3)$

$= 0.2 f(0 + 0.2, 1 + 0.1690)$

$= 0.2 f(0.2, 1.1690)$

$= (0.2)(0.7305)$

$\boxed{K_4 = 0.1461}$

$$K = \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$= \frac{1}{6}\left[0.2 + 2(0.1667) + 2(0.1690) + 0.1461\right]$$

$$= \frac{1}{6}\left[0.2 + 0.3334 + 0.338 + 0.1461\right]$$

$$= \frac{1}{6}(1.0175)$$

$$\boxed{K = 0.1696}$$

$$y_1 = y_0 + K$$

$$= 1 + 0.1696$$

$\boxed{y_1 = 1.1696}$ at $x_1 = 0.2$.

d) $\frac{dy}{dx} = 1 + y^2$.

$$f_1 = f(x_1, y_1) = f(0.2, 0.2027)$$

$$= 1.0411$$

$$f_2 = f(x_2, y_2) = f(0.4, 0.4228)$$

$$= 1.1788$$

$$f_3 = f(x_3, y_3) = f(0.6, 0.6841)$$

$$= 1.468$$

Using Predictor formula -

$$y_4^P = y_0 + \frac{4h}{3}\left[2f_1 - f_2 + 2f_3\right]$$

$$= 0 + \frac{4(0.2)}{3}\left[2(1.0411) - 1.1788 + 2(1.468)\right]$$

$$y_4^P = 0.2667[2.0822 - 1.1788 + 2.9360]$$

$$= (0.2667)(3.8394)$$

$$\boxed{y_4^P = 1.0240}$$

$$f_4 = f(x_4, y_4^{(P)}) = f(0.8, 1.0240)$$

$$\boxed{f_4 = 2.0486.}$$

Using Corrector formula –

$$y_4^c = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$$

$$= 0.4228 + \frac{0.2}{3}[1.1788 + 4(1.468) + 2.0486]$$

$$= 0.4228 + 0.0667[1.1788 + 5.8720 + 2.0486]$$

$$= 0.4228 + (0.0667)(9.0994)$$

$$\boxed{y_4^c = 1.0297} \text{ at } x_4 = 0.8.$$

e) Drug exposure $= \int_0^9 \text{Concentration } dx$.

$$\text{Total drug exposure} = \frac{h}{2}[(y_0 + y_9) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8)]$$

$$= \frac{1}{2}[(0 + 0.2) + 2(1.5 + 3.2 + 4.1 + 4.0 + 3.2 + 2.1 + 1.2 + 0.6)]$$

$$= \frac{1}{2}[0.2 + 39.8]$$

$$= 20 \text{ mg/L}.$$

$$\text{Volume} = \int_0^{10} \text{Flow of water.}$$

Using Simpson's 1/3 rd Rule.

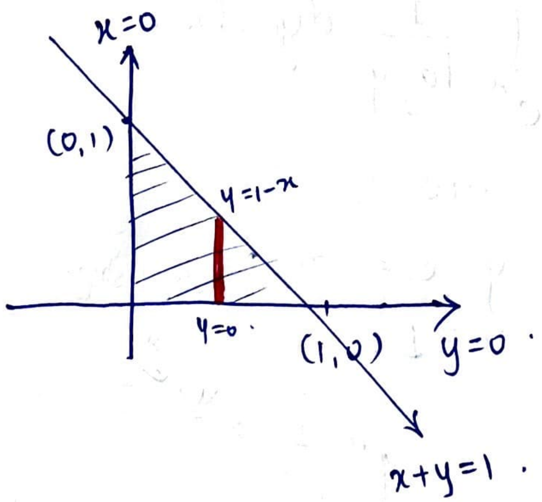
$$\text{Total Volume of water} = \frac{h}{3}\left[(y_0 + y_{10}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)\right]$$

$$= \frac{1}{3}\left\{(0+0) + 4[6 + 20 + 30 + 18 + 4] + 2[14 + 25 + 26 + 10]\right\}$$

$$= \frac{1}{3}\left\{0 + 312 + 150\right\}$$

$= 154$ ~~L/min~~ Liters.

Q5 a)



$x + y = 1$.
$x = 0, \; y = 1$.
$y = 0, \; x = 1$.
$(0,1) \; (1,0)$

limits are,

$y = 0$ to $y = 1 - x$.
$x = 0$, to $x = 1$.

$$I = \int_0^1 \left(\int_0^{1-x} e^{2x+3y} \, dy \right) dx.$$

$$= \int_0^1 \left[\frac{e^{2x+3y}}{3}\right]_0^{1-x} dx$$

$$= \frac{1}{3}\int_0^1 \left[e^{2x+3(1-x)} - e^{2x}\right] dx$$

$$= \frac{1}{3}\int_0^1 \left(e^{3-x} - e^{2x}\right) dx$$

$$= \frac{1}{3}\left[\frac{e^{3-x}}{-1} - \frac{e^{2x}}{2}\right]_0^1$$

$$= \frac{1}{3}\left[\left(\frac{e^2}{-1} - \frac{e^2}{2}\right) - \left(\frac{e^3}{-1} - \frac{e^0}{2}\right)\right]$$

$$= \frac{1}{3}\left[-\frac{3}{2}e^2 + e^3 + \frac{1}{2}\right]$$

$$\boxed{I = -\frac{e^2}{2} + \frac{e^3}{3} + \frac{1}{6}.}$$

b) Given $I = \int_0^1 \int_{e^x}^{e} \frac{1}{\log y}\, dy\, dx.$

Given limits are -

$y = e^x$ to $y = e = 2.7182$ } vertical strip.

$x = 0$ to $x = 1$.

$y = e^x$

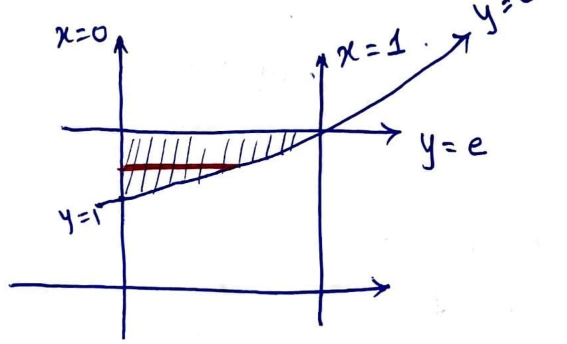
$x = 0$

$y = 1$

$x = 1$

$y = e$

$(0,1)\ (1,e)$.



Consider horizontal strip.

New limits : $x = 0$ to $x = \log y$.

$y = 1$ to $y = e$.

$$I = \int_1^e \left(\int_0^{\log y} \frac{1}{\log y} \right) dx\, dy$$

$$= \int_1^e \frac{1}{\log y} [x]_0^{\log y} dy$$

$$= \int_1^e \frac{1}{\cancel{\log y}} [\cancel{\log y} - 0] dy$$

$$= \int_1^e 1\, dy$$

$$= [y]_1^e$$

$$\boxed{I = e - 1}$$

c) $y^2 = 4x$, & the line $y = x - 8$

$y^2 = 4x$

$(x-8)^2 = 4x$.

$x^2 - 16x + 64 = 4x$

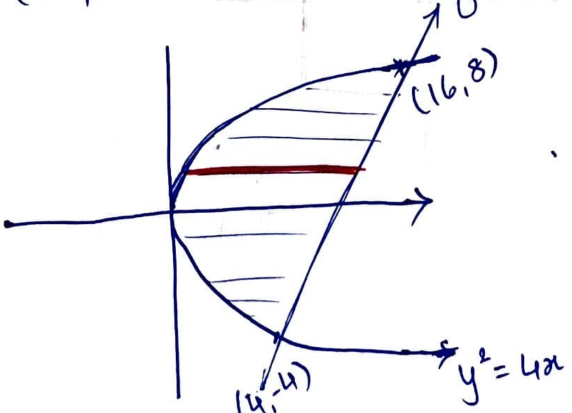
$x^2 - 20x + 64 = 0$

$x = 4, \quad x = 16.$

if $x = 4$, $y = -4$.

if $x = 16$, $y = 8$.

$(4,-4)$ $(16,8)$ are points of intersection.



Consider horizontal strip.

limits are

$x = \frac{y^2}{4}$ to $x = y + 8$

$y = -4$ to $y = 8$

Area $= \iint dx\, dy$

$$= \int_{-4}^{8} \left(\int_{\frac{y^2}{4}}^{y+8} dx \right) dy$$

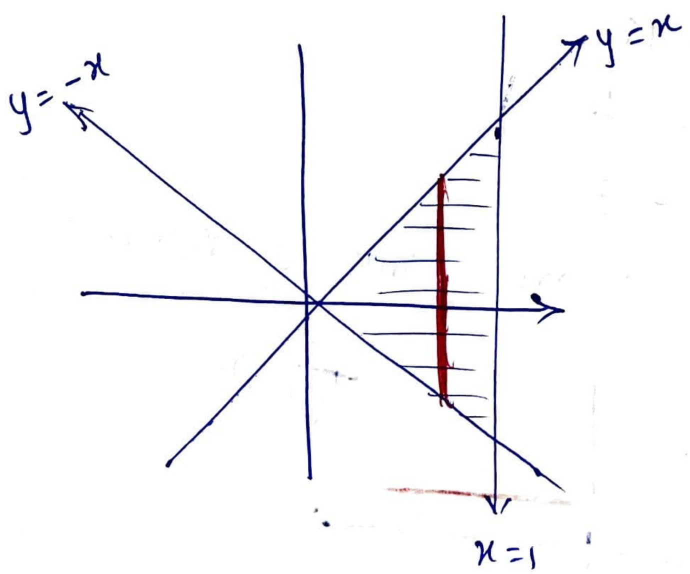
$$= \int_{-4}^{8} \left(y + 8 - \frac{y^2}{4} \right) dy$$

$$= \left[\frac{y^2}{2} + 8y - \frac{y^3}{12} \right]_{-4}^{8}$$

$$= \left[\left(\frac{64}{2} + 64 - \frac{512}{12} \right) - \left(\frac{16}{2} - 32 + \frac{64}{12} \right) \right]$$

$$\boxed{A = 72}$$

d)



Consider vertical strip

limits are

$y = -x$ to $y = x$
$x = 0$ to $x = 1$

C.G lies on X - axis

$\therefore \bar{Y} = 0.$

$\therefore \bar{X} = \frac{N_1}{D}$

$$D = \iint_R dx\, dy$$

$$= \int_0^1 \left(\int_{-x}^{x} dy \right) dx$$

$$= \int_0^1 (x + x)\, dx$$

$$= \int_0^1 2x\, dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^1$$

$$\boxed{D = 1}$$

$$N_1 = \iint_R x\, dx\, dy$$

$$= \int_0^1 \left(\int_{-x}^{x} x\, dy \right) dx$$

$$= \int_0^1 x\,[y]_{-x}^{x}\,dx$$

$$= \int_0^1 x\,[x + x]\,dx$$

$$= \int_0^1 2x^2\,dx$$

$$= 2\left[\frac{x^3}{3}\right]_0^1$$

$$\boxed{N_1 = \frac{2}{3}}$$

$$\therefore \bar{X} = \frac{N_1}{D} = \frac{2}{3}$$

$$\therefore C.G = (\bar{X}, \bar{Y})$$

$$= \left(\frac{2}{3}, 0\right)$$

e) $x = r\sin\theta \cdot \cos\phi$

$y = r\sin\theta \cdot \sin\phi$

$z = r\cos\theta$

$dx\,dy\,dz = r^2 \sin\theta\,dr\,d\theta\,d\phi$

limits are

$r : 0 \rightarrow 1$

$\theta : 0 \rightarrow \pi/2$

$\phi : 0 \rightarrow \pi/2$.

$$I = \int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \frac{\cancel{r^2} \sin\theta\,dr\,d\theta\,d\phi}{\cancel{r^2}}$$

$$= \int_0^{\pi/2} d\phi \int_0^{\pi/2} \sin\theta \, d\theta \cdot \int_0^{1} dr$$

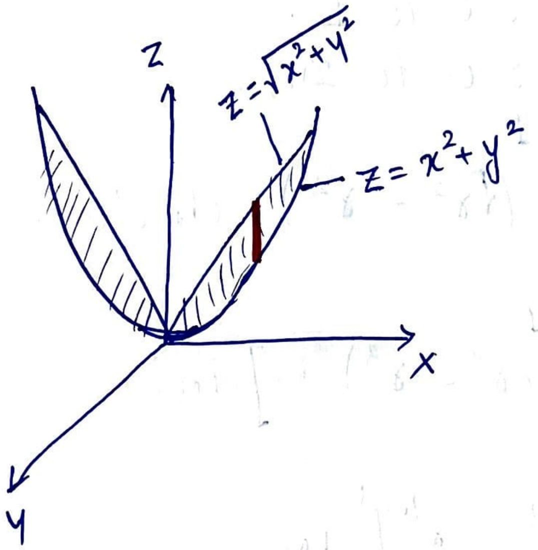
$$= [\phi]_0^{\pi/2} \cdot [-\cos\theta]_0^{\pi/2} [r]_0^1$$

$$= \left(\frac{\pi}{2} - 0\right)\left(-\cos\frac{\pi}{2} + \cos 0\right] [1 - 0]$$

$$= \left(\frac{\pi}{2}\right)(0 + 1)(1)$$

$$\boxed{I = \frac{\pi}{2}}$$

f)



Consider strip parallel to Z-axis

$\therefore$ $Z = x^2 + y^2$ to $Z = \sqrt{x^2 + y^2}$

$$\therefore V = \iint_R \left[\int_{x^2+y^2}^{\sqrt{x^2+y^2}} dz \right] dx \, dy$$

$$= \iint_R \left[\sqrt{x^2+y^2} - (x^2 + y^2) \right] dx \, dy$$

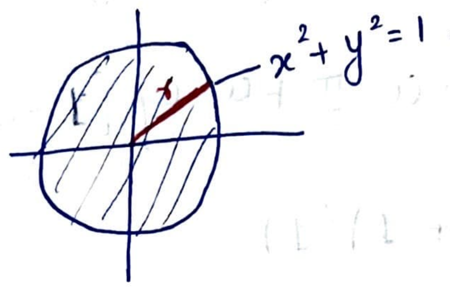
Region of Integration :

Intersection of $Z = x^2 + y^2$ & $Z = \sqrt{x^2 + y^2}$.

$$\sqrt{x^2+y^2} = x^2+y^2$$

$$\therefore x^2+y^2 = 1.$$

Circle centre at $(0,0)$ & radius 1.



Put $x = r\cos\theta$, $y = r\sin\theta$, $dx\,dy = r\,dr\,d\theta$.

$r : 0$ to 1

$\theta : 0$ to 2π.

$$V = \int_0^{2\pi}\int_0^1 \left(\sqrt{r^2} - r^2\right) r\,dr\,d\theta.$$

$$= \int_0^{2\pi}\left[\int_0^1 (r^2 - r^3)\,dr\right] d\theta.$$

$$= \int_0^{2\pi}\left[\frac{r^3}{3} - \frac{r^4}{4}\right]_0^1 d\theta$$

$$= \int_0^{2\pi}\left[\frac{1}{3} - \frac{1}{4}\right] d\theta$$

$$= \frac{1}{12}\int_0^{2\pi} d\theta.$$

$$= \frac{1}{12}\left[\theta\right]_0^{2\pi}$$

$$= \frac{1}{12}\left[2\pi\right]$$

$$\boxed{V = \frac{\pi}{6}}$$