



Model Answer
End-Sem Examination-I, Winter 2025

Academic Year: 2025-2026	Semester: I
Class: F.Y.	Program: B.Tech
Branch Code: FYE	Pattern: 2022
Name of Course: Applied Mathematics-II	Course Code: FYE221002

$$Q1) \cos x \frac{dy}{dx} + y \sin x = \sqrt{y} \sec x$$

$$\frac{dy}{dx} + y \tan x = y^{1/2} \sec^{1/2} x \cdot \sec x$$

$$\therefore \frac{dy}{dx} + y \tan x = y^{1/2} \sec^{3/2} x$$

divide by $y^{1/2}$

$$\frac{1}{y^{1/2}} \frac{dy}{dx} + \frac{y}{y^{1/2}} \tan x = \sec^{3/2} x$$

$$\frac{1}{y^{1/2}} \frac{dy}{dx} + y^{1/2} \tan x = \sec^{3/2} x$$

put $y^{1/2} = u$

$$\frac{1}{2} y^{-1/2} \frac{dy}{dx} = \frac{du}{dx}$$

$$\therefore \frac{1}{y^{1/2}} \frac{dy}{dx} = \frac{2 du}{dx}$$

$$\therefore 2 \frac{du}{dx} + (\tan x) u = \sec^{3/2} x$$

$$\therefore \frac{dy}{dx} + \left(\frac{\tan x}{2}\right)u = \frac{\sec^{3/2} x}{2}$$

$$P = \frac{\tan x}{2}, \quad Q = \frac{\sec^{3/2} x}{2}$$

$$\begin{aligned}\therefore \text{I.F.} &= e^{\int P dx} \\ &= e^{\frac{1}{2} \int \tan x dx} \\ &= e^{\frac{1}{2} \log \sec x} \\ &= e^{\log \sec^{1/2} x} \\ &= \sec^{1/2} x\end{aligned}$$

$$\boxed{\text{I.F.} = \sec^{1/2} x}$$

\therefore G.S is,

$$\begin{aligned}u(\text{I.F.}) &= \int Q(\text{I.F.}) dx + C \\ u \sec^{1/2} x &= \int \frac{\sec^{3/2} x}{2} \sec^{1/2} x dx + C \\ &= \frac{1}{2} \int \sec^2 x dx + C\end{aligned}$$

$$u \sqrt{\sec x} = \frac{1}{2} \tan x + C$$

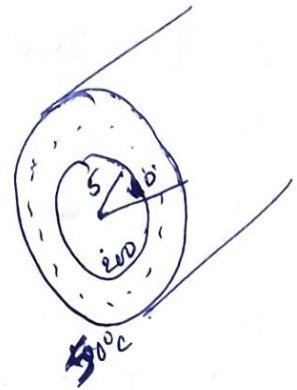
Resubstitute $u = y^{1/2}$

$$\sqrt{y} \sec x = \frac{1}{2} \tan x + C$$

By Fourier's law,

$$q = -KA \frac{dT}{dx}$$

$$q = -K(2\pi r) \frac{dT}{dx} \quad - [A = 2\pi r]$$



$$\therefore -\frac{q}{2\pi K} \frac{dx}{x} = dT$$

Variables are separated.

$$\int dT = -\frac{q}{2\pi K} \int \frac{dx}{x}$$

$$x = 5 \text{ cm}, T = 200^\circ \text{C}$$

$$x = 10 \text{ cm}, T = 50^\circ \text{C}$$

$$[T]_{200}^{50} = -\frac{q}{2\pi K} [\log x]_5^{10}$$

$$50 - 200 = -\frac{q}{2\pi K} [\log 10 - \log 5]$$

$$-150 = -\frac{q}{2\pi K} \log \left(\frac{10}{5}\right)$$

$$\therefore \frac{q}{2\pi K} = \frac{150}{\log 2}$$

$$\therefore q = \frac{150 \times 2 \times 3.14 \times 0.12}{\log 2}$$

$$\boxed{q = 163.8 \text{ cal/sec}}$$

Now, $x = 7.5 \text{ cm}$, $T = ?$

$$\int_{200}^T dT = -\frac{q}{2\pi K} \int_5^{7.5} \frac{dx}{x}$$

$$T - 200 = \frac{-9}{2\pi k} [\log 7.5 - \log 5]$$

$$\therefore T - 200 = \frac{-9}{2\pi k} \log \left(\frac{7.5}{5} \right)$$

$$\therefore T - 200 = \frac{-150}{\log 2}$$

$$\therefore T = -87.74 + 200$$

$$\boxed{T = 112.23^\circ\text{C}}$$

Q3)

a)

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1911	12	3	2	0	
1921	15	5	2		$\boxed{3}$
1931	20	7		$\boxed{3}$	$\nabla^4 y_n$
1941	27	$\boxed{12}$	$\boxed{5}$	$\nabla^3 y_n$	
$\boxed{1951}$	$\boxed{39}$	∇y_n	$\nabla^2 y_n$		

$$u = \frac{x - x_n}{h}$$

$$u = \frac{1948 - 1951}{10}$$

$$\boxed{u = -0.3}$$

Using backward interpolation formula,

$$y = y_n + u \nabla y_n + \frac{u(u+1)}{2!} \nabla^2 y_n + \frac{u(u+1)(u+2)}{3!} \nabla^3 y_n + \frac{u(u+1)(u+2)(u+3)}{4!} \nabla^4 y_n$$

$$= 39 + (-0.3)(12) + \frac{(-0.3)(-0.3+1)(5)}{2} +$$

$$\frac{(-0.3)(-0.3+1)(-0.3+2)(3)}{6} +$$

$$\frac{(-0.3)(-0.3+1)(-0.3+2)(-0.3+3)(3)}{24}$$

$$= 39 - 3.6 - 0.525 - 0.1785 - 0.1204$$

$$y = 34.57 \approx 35 \text{ Thousand for } x = 1948.$$

Q3) b)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
100	10.63	2.4	-0.39	0.15	-0.07
150	13.03	2.01	-0.24	0.08	
200	15.04	1.77	-0.16		
250	16.81	1.61			
300	18.42				

$$u = \frac{x - x_0}{h}$$

$$u = \frac{140 - 100}{50}$$

$$u = 0.8$$

\therefore by Newton's forward Interpolation formula.

$$y = y_0 + u \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!} \Delta^4 y_0 + \dots$$

$$y = 10.63 + (0.8)(2.4) + \frac{(0.8)(0.8-1)(-0.39)}{2}$$

$$+ \frac{(0.8)(0.8-1)(0.8-2)(0.15)}{6} + \frac{(0.8)(0.8-1)(0.8-2)(0.8-3)(0.05)}{24}$$

$$= 10.63 + 1.92 + 0.0312 + 0.0048 + 0.0012$$

$$y = 12.5872 \text{ miles for } x = 140$$

Q3) c) i) $(E+2)(E-1)(e^x + x)$, $h=1$

$$\begin{aligned} \rightarrow (E+2)(E-1)(e^x + x) &= (E^2 - E + 2E - 2)(e^x + x) \\ &= (E^2 + E - 2)(e^x + x) \\ &= E^2(e^x + x) + E(e^x + x) - 2(e^x + x) \\ &= e^{x+2h} + (x+2h)e^{x+h} + e^{x+h} + (x+h) - 2e^x - 2x \\ &= e^x \cdot e^{2h} + x + 2h + e^{x+h} + x + h - 2e^x - 2x \\ &= e^x \cdot e^2 + 2 + e^x \cdot e + 1 - 2e^x - [h=1] \end{aligned}$$

$$(E+2)(E-1)(e^x + x) = e^x [e^2 + e - 2] + 3$$

ii) $E f(x)$, if $f(x) = x^3$, $h=1$

$$\begin{aligned} \rightarrow E f(x) &= E(x^3) \\ &= (x+h)^3 \\ &= \cancel{x^3} + (x+1)^3 \end{aligned}$$

$$E f(x) = x^3 + 3x^2 + 3x + 1$$

d)

i) Find $\Delta^n e^x$, $h=1$

$$\begin{aligned}\rightarrow \Delta e^x &= e^{x+h} - e^x \\ &= e^{x+1} - e^x \\ &= e^x [e-1]\end{aligned}$$

$$\begin{aligned}\Delta^2 e^x &= \Delta(\Delta e^x) \\ &= \Delta[e^x (e-1)] \\ &= (e-1) \Delta(e^x) \\ &= (e-1) e^x (e-1)\end{aligned}$$

$$\Delta^2 e^x = (e-1)^2 e^x$$

$$\Delta^3 e^x = (e-1)^3 e^x$$

⋮

$$\boxed{\Delta^n e^x = (e-1)^n e^x}$$

ii) P.T. $\delta E^{1/2} = \Delta$

$$\begin{aligned}\rightarrow \delta E^{1/2} f(x) &= \delta \left[f\left(x + \frac{h}{2}\right) \right] \\ &= f\left(x + \frac{h}{2} + \frac{h}{2}\right) - f\left(x + \frac{h}{2} - \frac{h}{2}\right) \\ &= f(x+h) - f(x)\end{aligned}$$

$$\delta E^{1/2} f(x) = \Delta f(x)$$

$$\boxed{\therefore \delta E^{1/2} = \Delta}$$

Q3) e)

	x_0	x_1	x_2
x	0	1	2
y	1	3	5
	y_0	y_1	y_2

By using Lagrange's Interpolation formula -

$$y = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$$

$$= y_0 \left[\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \right] + y_1 \left[\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \right] +$$

$$y_2 \left[\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \right]$$

$$= 1 \left[\frac{(x-1)(x-2)}{(0-1)(0-2)} \right] + 3 \left[\frac{(x-0)(x-2)}{(1-0)(1-2)} \right] + 5 \left[\frac{(x-0)(x-1)}{(2-0)(2-1)} \right]$$

$$= \left[\frac{x^2 - 2x - x + 2}{2} \right] + 3 \left[\frac{x^2 - 2x}{-1} \right] + 5 \left[\frac{x^2 - x}{2} \right]$$

$$= \left[\frac{x^2 - 3x + 2}{2} \right] - [3x^2 - 6x] + \left[\frac{5x^2 - 5x}{2} \right]$$

$$= \left(\frac{x^2}{2} - \frac{6x^2}{2} + \frac{5x^2}{2} \right) + \left[\frac{-3x + 12x - 5x}{2} \right] + 1$$

$$\boxed{y = 0 + 2x + 1}$$

$$\left(\frac{dy}{dx} \right)_{x=2} = 2$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.30	y_{-2} 0.1179	0.0189			
0.35	y_{-1} 0.1368		-0.0003		
x_0 0.40	y_0 0.1554	0.0186 0.0182	-0.0004	-0.0001 0.0001	0.0002
0.45	y_1 0.1736		-0.0003		
0.50	y_2 0.1915	0.0179			

$$u = \frac{x - x_0}{h}$$

$$= \frac{0.41 - 0.40}{0.05}$$

$$u = 0.2$$

Using Stirling's formula -

$$y = y_0 + \frac{u}{1!} \left(\frac{\Delta y_{-1} + \Delta y_0}{2} \right) + \frac{u^2}{2!} \Delta^2 y_0 + \frac{u(u^2-1)}{3!} \left[\frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2} \right]$$

$$+ \frac{u^2(u^2-1)}{4!} \Delta^4 y_{-2}$$

$$= 0.1554 + (0.2) \left[\frac{0.0186 + 0.0182}{2} \right] + \frac{(0.2)^2}{2} (-0.0004) +$$

$$\frac{(0.2)[(0.2)^2-1]}{6} \left[\frac{-0.0001 + 0.0001}{2} \right] + \frac{(0.2)^2[(0.2)^2-1]}{24} (0.0002)$$

$$= 0.1554 + 0.0037 + 0.000008 + 0 = 0.00000032$$

$$y = 0.1591 \quad \text{for } x = 0.41$$

Q4

$$a) \frac{dy}{dx} = \frac{y-x}{y+x}, \quad x_0 = 0, \quad y_0 = 1, \quad h = 0.02$$

$$x_1 = x_0 + h$$

$$= 0 + 0.02$$

$$\boxed{x_1 = 0.02}$$

$$x_2 = x_1 + h$$

$$\boxed{x_2 = 0.04}$$

$$\boxed{x_3 = 0.06}$$

$$\boxed{x_4 = 0.08}$$

$$\boxed{x_5 = 0.1}$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + (0.02)f(0, 1)$$

$$= 1 + (0.02)(1)$$

$$\boxed{y_1 = 1.02}$$

$$y_2 = y_1 + hf(x_1, y_1)$$

$$= 1.02 + (0.02)f(0.02, 1.02)$$

$$= 1.02 + (0.02)(0.9615)$$

$$\boxed{y_2 = 1.0392}$$

$$y_3 = y_2 + hf(x_2, y_2)$$

$$= 1.0392 + (0.02)f(0.04, 1.0392)$$

$$= 1.0392 + (0.02)(0.9259)$$

$$\boxed{y_3 = 1.0577}$$

$$y_4 = y_3 + hf(x_3, y_3)$$

$$= 1.0577 + (0.02)f(0.06, 1.0577)$$

$$= 1.0577 + (0.02)(0.8926)$$

$$\boxed{y_4 = 1.0756}$$

$$y_5 = y_4 + hf(x_4, y_4)$$

$$= 1.0756 + (0.02)f(0.08, 1.0756)$$

$$= 1.0756 + (0.02)(0.8615)$$

$$\boxed{y_5 = 1.0928}$$

 $\frac{dy}{dx}$

$$\frac{dy}{dx} = -xy^2, \quad x_0 = 0, \quad y_0 = 2, \quad h = 0.1.$$

$$f(x_0, y_0) = -xy^2 \\ = 0.$$

$$y_1 = y_0 + hf(x_0, y_0) \\ = 2 + 0.$$

$$\boxed{y_1 = 2}$$

$$y_1^{(1)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1)] \\ = 2 + \frac{0.1}{2} [0 - 0.4]$$

$$\boxed{y_1^{(1)} = 1.98}$$

$$y_1^{(2)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(1)})] \\ = 2 + \frac{0.1}{2} [0 - 0.3920]$$

$$\boxed{y_1^{(2)} = 1.9804}$$

$$y_1^{(3)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(2)})] \\ = 2 + \frac{0.1}{2} [0 - 0.3922]$$

$$\boxed{y_1^{(3)} = 1.9804}$$

$$c) \quad c) \quad \frac{dy}{dx} = \frac{1}{x+y}, \quad x_0 = 0, \quad y_0 = 1, \quad h = 0.2.$$

$$K_1 = hf(x_0, y_0)$$

$$= 0.2 f(0, 1)$$

$$\boxed{K_1 = 0.2}$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$

$$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right)$$

$$= 0.2 f(0.1, 1.1)$$

$$= (0.2)(0.8333)$$

$$\boxed{K_2 = 0.1667}$$

$$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$

$$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.1667}{2}\right)$$

$$= 0.2 f(0.1, 1.0834)$$

$$= (0.2)(0.8450)$$

$$\boxed{K_3 = 0.1690}$$

$$K_4 = hf(x_0 + h, y_0 + K_3)$$

$$= 0.2 f(0 + 0.2, 1 + 0.1690)$$

$$= 0.2 f(0.2, 1.1690)$$

$$= (0.2)(0.7305)$$

$$\boxed{K_4 = 0.1461}$$

$$\begin{aligned}
 K &= \frac{1}{6} [K_1 + 2K_2 + 2K_3 + K_4] \\
 &= \frac{1}{6} [0.2 + 2(0.1667) + 2(0.1690) + 0.1461] \\
 &= \frac{1}{6} [0.2 + 0.3334 + 0.338 + 0.1461] \\
 &= \frac{1}{6} (1.0175)
 \end{aligned}$$

$$\boxed{K = 0.1696}$$

$$\begin{aligned}
 y_1 &= y_0 + K \\
 &= 1 + 0.1696
 \end{aligned}$$

$$\boxed{y_1 = 1.1696} \text{ at } x_1 = 0.2$$

$$d) \quad \frac{dy}{dx} = 1 + y^2$$

$$\begin{aligned}
 f_1 &= f(x_1, y_1) = f(0.2, 0.2027) \\
 &= 1.0411
 \end{aligned}$$

$$\begin{aligned}
 f_2 &= f(x_2, y_2) = f(0.4, 0.4228) \\
 &= 1.1788
 \end{aligned}$$

$$\begin{aligned}
 f_3 &= f(x_3, y_3) = f(0.6, 0.6841) \\
 &= 1.468
 \end{aligned}$$

Using Predictor formula -

$$y_4^P = y_0 + \frac{4h}{3} [2f_1 - f_2 + 2f_3]$$

$$= 0 + \frac{4(0.2)}{3} [2(1.0411) - 1.1788 + 2(1.468)]$$

f(B)

f'

$$y_4^p = 0.2667 [2.0822 - 1.1788 + 2.9360]$$

$$= (0.2667)(3.8394)$$

$$\boxed{y_4^p = 1.0240}$$

$$f_4 = f(x_4, y_4^p) = f(0.8, 1.0240)$$

$$\boxed{f_4 = 2.0486}$$

Using Corrector formula -

$$y_4^c = y_2 + \frac{h}{3} [f_2 + 4f_3 + f_4]$$

$$= 0.4228 + \frac{0.2}{3} [1.1788 + 4(1.468) + 2.0486]$$

$$= 0.4228 + 0.0667 [1.1788 + 5.8720 + 2.0486]$$

$$= 0.4228 + (0.0667)(9.0994)$$

$$\boxed{y_4^c = 1.0297} \text{ at } x_4 = 0.8.$$

e) Drug exposure = \int_0^9 Concentration dx .

$$\text{Total drug exposure} = \frac{h}{2} [(y_0 + y_9) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8)]$$

$$= \frac{1}{2} [(0 + 0.2) + 2(1.5 + 3.2 + 4.1 + 4.0 + 3.2 + 2.1 + 1.2 + 0.6)]$$

$$= \frac{1}{2} [0.2 + 39.8]$$

$$= 20 \text{ mg/L.}$$

Volume = \int_0^{10} Flow of water.

Using Simpson's $\frac{1}{3}$ rd Rule.

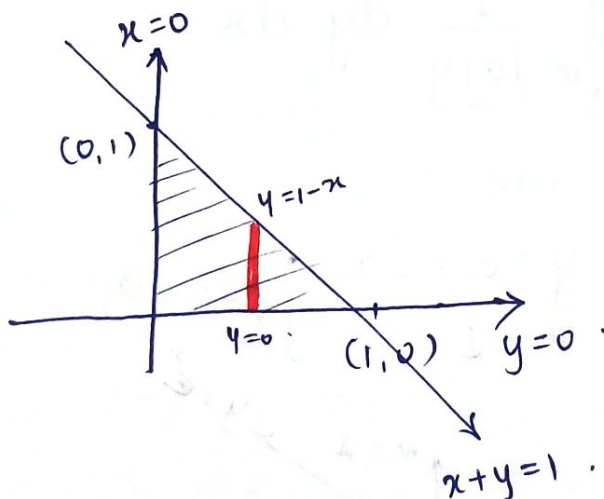
$$\text{Total Volume of water} = \frac{h}{3} [(y_0 + y_{10}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)]$$

$$= \frac{1}{3} \left\{ (0+0) + 4[6+20+30+18+4] + 2[14+25+26+10] \right\}$$

$$= \frac{1}{3} \{ 0 + 312 + 150 \}$$

$$= 154 \text{ ~~liters~~ Liters.}$$

a)



$$\begin{aligned} x+y &= 1, \\ x=0, y &= 1. \\ y=0, x &= 1. \\ (0,1) & (1,0) \end{aligned}$$

limits are,

$$y=0 \text{ to } y=1-x.$$

$$x=0, \text{ to } x=1.$$

$$I = \int_0^1 \left(\int_0^{1-x} e^{2x+3y} dy \right) dx.$$

$$= \int_0^1 \left[\frac{e^{2x+3y}}{3} \right]_0^{1-x} dx$$

$$= \frac{1}{3} \int_0^1 [e^{2x+3(1-x)} - e^{2x}] dx$$

$$= \frac{1}{3} \int_0^1 (e^{3-x} - e^{2x}) dx$$

$$= \frac{1}{3} \left[\frac{e^{3-x}}{-1} - \frac{e^{2x}}{2} \right]_0^1$$

$$= \frac{1}{3} \left[\left(\frac{e^2}{-1} - \frac{e^2}{2} \right) - \left(\frac{e^3}{-1} - \frac{e^0}{2} \right) \right]$$

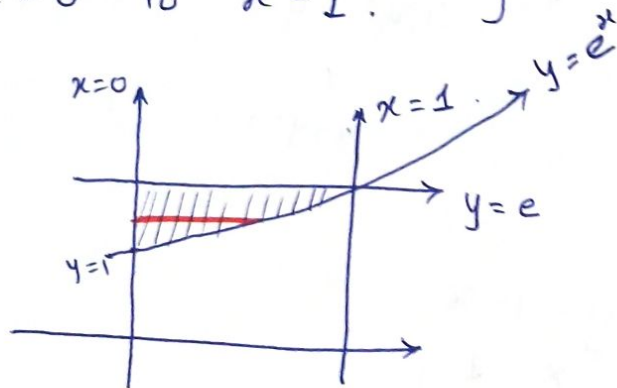
$$= \frac{1}{3} \left[-\frac{3}{2} e^2 + e^3 + \frac{1}{2} \right]$$

$$\boxed{I = -\frac{e^2}{2} + \frac{e^3}{3} + \frac{1}{6}}$$

b) Given $I = \int_0^1 \int_{e^x}^e \frac{1}{\log y} dy dx$.

Given limits are -

$$\left. \begin{array}{l} y = e^x \text{ to } y = e = 2.71827 \\ x = 0 \text{ to } x = 1. \end{array} \right\} \text{vertical strip.}$$



$$\left. \begin{array}{l} y = e^x \\ x = 0 \\ y = 1 \\ x = 1 \\ y = e \end{array} \right\} (0, 1) (1, e).$$

Considers horizontal strip.

New limits : $x = 0$ to $x = \log y$
 $y = 1$ to $y = e$.

$$\begin{aligned}
 I &= \int_1^e \left(\int_0^{\log y} \frac{1}{\log y} \right) dx dy \\
 &= \int_1^e \frac{1}{\log y} [x]_0^{\log y} dy \\
 &= \int_1^e \frac{1}{\log y} [\log y - 0] dy \\
 &= \int_1^e 1 dy \\
 &= [y]_1^e \\
 \boxed{I = e - 1}
 \end{aligned}$$

c) $y^2 = 4x$, & the line $y = x - 8$

$$y^2 = 4x$$

$$(x-8)^2 = 4x$$

$$x^2 - 16x + 64 = 4x$$

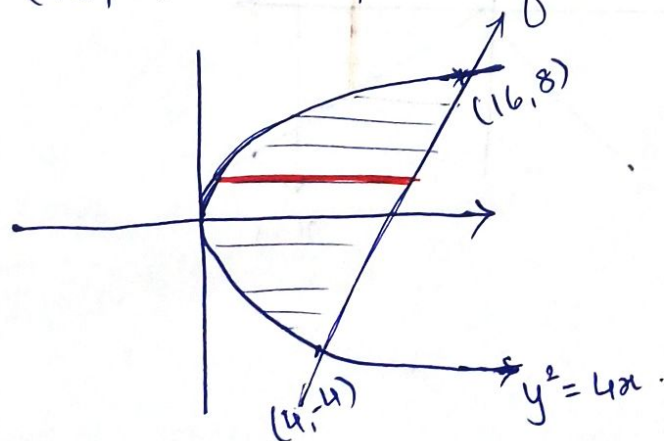
$$x^2 - 20x + 64 = 0$$

$$x = 4, x = 16$$

$$\text{if } x = 4, y = -4$$

$$\text{if } x = 16, y = 8$$

$(4, -4)$ $(16, 8)$ are points of intersection.



Consider horizontal strip.

Limits are

$$x = \frac{y^2}{4} \text{ to } x = y + 8$$

$$y = -4 \text{ to } y = 8$$

$$\text{Area} = \iint dx dy$$

$$= \int_{-4}^8 \left(\int_{\frac{y^2}{4}}^{y+8} dx \right) dy$$

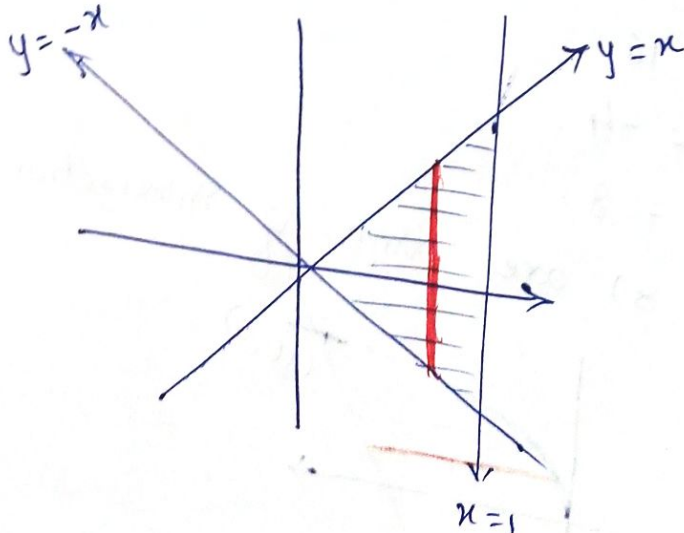
$$= \int_{-4}^8 (y + 8 - \frac{y^2}{4}) dy$$

$$= \left[\frac{y^2}{2} + 8y - \frac{y^3}{12} \right]_{-4}^8$$

$$= \left[\left(\frac{64}{2} + 64 - \frac{512}{12} \right) - \left(\frac{16}{2} - 32 + \frac{64}{12} \right) \right]$$

$$\boxed{A = 72}$$

d)



Consider vertical strip

Limits are

$$y = -x \text{ to } y = x$$

$$x = 0 \text{ to } x = 1$$

C.G. lies on x-axis

$$\therefore \bar{y} = 0.$$

$$\therefore \bar{x} = \frac{N_1}{D}$$

$$\begin{aligned} D &= \iint_R dx dy \\ &= \int_0^1 \left(\int_{-x}^x dy \right) dx \\ &= \int_0^1 (x + x) dx \\ &= \int_0^1 2x dx \\ &= \left. \frac{2x^2}{2} \right|_0^1 \end{aligned}$$

$$\boxed{D = 1}$$

$$\begin{aligned} N_1 &= \iint_R x dx dy \\ &= \int_0^1 \left(\int_{-x}^x x dy \right) dx \end{aligned}$$

$$= \int_0^1 x [y]_{-x}^x dx$$

$$= \int_0^1 x [x+x] dx$$

$$= \int_0^1 2x^2 dx$$

$$= 2 \left[\frac{x^3}{3} \right]_0^1$$

$$\boxed{N_1 = \frac{2}{3}}$$

$$\therefore \bar{x} = \frac{N_1}{D} = \frac{2}{3}$$

$$\therefore \text{C.G} = (\bar{x}, \bar{y}) \\ = \left(\frac{2}{3}, 0 \right)$$

e) $x = r \sin \theta \cdot \cos \phi$
 $y = r \sin \theta \cdot \sin \phi$
 $z = r \cos \theta$

$$dx dy dz = r^2 \sin \theta dr d\theta d\phi$$

limits are

$$r : 0 \rightarrow 1$$

$$\theta : 0 \rightarrow \pi/2$$

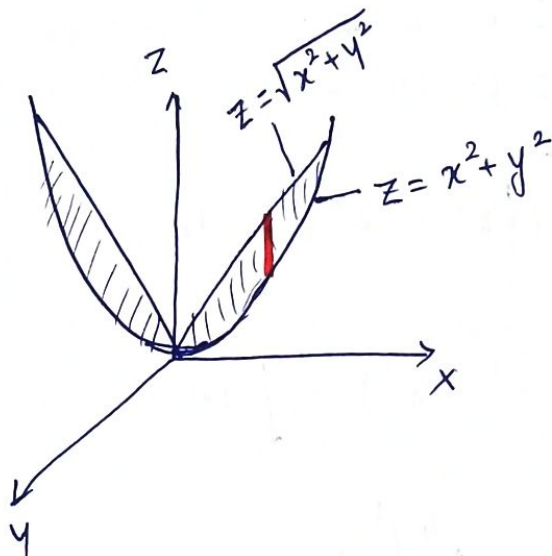
$$\phi : 0 \rightarrow \pi/2$$

$$I = \int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \frac{r^2 \sin \theta dr d\theta d\phi}{r^2}$$

$$\begin{aligned}
 &= \int_0^{\pi/2} d\phi \int_0^{\pi/2} \sin\theta d\theta \int_0^1 dr \\
 &= [\phi]_0^{\pi/2} \cdot [-\cos\theta]_0^{\pi/2} [r]_0^1 \\
 &= \left(\frac{\pi}{2} - 0\right) \left(-\cos\frac{\pi}{2} + \cos 0\right) [1 - 0] \\
 &= \left(\frac{\pi}{2}\right) (0 + 1)(1)
 \end{aligned}$$

$$I = \frac{\pi}{2}$$

f)



Consider strip parallel to z -axis

$$\therefore z = x^2 + y^2 \text{ to } z = \sqrt{x^2 + y^2}$$

$$\therefore V = \iint_R \left[\int_{x^2 + y^2}^{\sqrt{x^2 + y^2}} dz \right] dx dy$$

$$= \iint_R \left[\sqrt{x^2 + y^2} - (x^2 + y^2) \right] dx dy$$

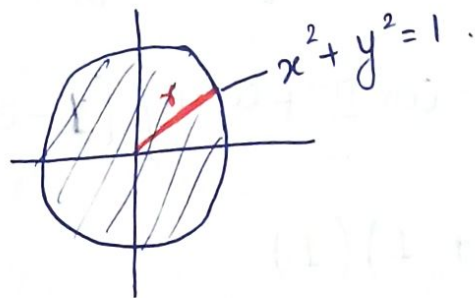
Region of Integration :

Intersection of $z = x^2 + y^2$ & $z = \sqrt{x^2 + y^2}$

$$\sqrt{x^2+y^2} = x^2+y^2$$

$$\therefore x^2+y^2 = 1.$$

Circle centre at $(0,0)$ & radius 1.



Put $x = r \cos \theta$, $y = r \sin \theta$, $dx dy = r dr d\theta$.

r : 0 to 1

θ : 0 to 2π .

$$V = \int_0^{2\pi} \int_0^1 (\sqrt{r^2} - r^2) r dr d\theta.$$

$$= \int_0^{2\pi} \left[\int_0^1 (r^2 - r^3) dr \right] d\theta.$$

$$= \int_0^{2\pi} \left[\frac{r^3}{3} - \frac{r^4}{4} \right]_0^1 d\theta$$

$$= \int_0^{2\pi} \left[\frac{1}{3} - \frac{1}{4} \right] d\theta$$

$$= \frac{1}{12} \int_0^{2\pi} d\theta.$$

$$= \frac{1}{12} \left[\theta \right]_0^{2\pi}$$

$$= \frac{1}{12} [2\pi]$$

$$V = \frac{\pi}{6}$$