



**Model Answer
End-Sem Examination, Winter 2025**

Academic Year:2025-2026

Semester:III

Name of Programme:MBA

Pattern:2024

Name of Course:Lean Six Sigma

Course Code:2410614D

Q. No.	Details
Q.1	<p style="text-align: center;">Unit 1 (6 marks)</p> <p>Apply the DMAIC methodology to improve a process with high defect rates in a manufacturing organization. (1 mark for each step)</p> <p>Ans.Application of DMAIC to Reduce High Defect Rates</p> <p>1. Define</p> <ul style="list-style-type: none">Clearly define the problem: <i>High defect rate in the manufacturing process leading to rework, scrap, and customer complaints.</i>Identify customer requirements (CTQs – Critical to Quality).Define project scope, objectives, timeline, and team responsibilities.Example goal: <i>Reduce defect rate from 8% to 2% within six months.</i> <p>2. Measure</p> <ul style="list-style-type: none">Collect data on current defect levels, defect types, and process performance.Establish baseline metrics such as defect per unit (DPU), defect rate, and process capability (Cp, Cpk).Validate the measurement system to ensure data accuracy (e.g., Gage R&R). <p>3. Analyze</p> <ul style="list-style-type: none">Identify root causes of defects using tools like Pareto charts, cause-and-effect (fishbone) diagrams, and process flow analysis.Analyze data to determine key factors causing defects (e.g., machine variation, operator error, poor raw material quality).Verify root causes using statistical analysis. <p>4. Improve</p> <ul style="list-style-type: none">Develop and implement solutions to eliminate root causes.Examples: machine calibration, operator training, process standardization,



	<p>improved raw material inspection.</p> <ul style="list-style-type: none">• Conduct pilot runs to test improvements and confirm defect reduction.• Optimize process parameters to achieve desired quality levels. <p>5. Control</p> <ul style="list-style-type: none">• Implement control plans to sustain improvements.• Use control charts and standard operating procedures (SOPs) to monitor the process.• Train employees and assign process ownership.• Continuously review performance to prevent recurrence of defects.
Q.2	<p style="text-align: center;">Unit 2 (6 marks)</p> <p>A team has a DPMO of 20,000. They inspect 2,000 units, with each unit having 4 opportunities. Calculate how many total defects occurred? (2 marks each step)</p> <p>Ans Given:</p> <ul style="list-style-type: none">• DPMO = 20,000• Units = 2,000• Opportunities per unit = 4 <p>✓ Step-by-Step Calculation:</p> <p>1. Total opportunities =</p> $2,000 \text{ units} \times 4 = 8,000 \text{ opportunities}$ <p>2. Plug into the formula:</p> $\text{Defects} = \frac{20,000 \times 8,000}{1,000,000}$ <p>3. Calculate:</p> $\text{Defects} = \frac{160,000,000}{1,000,000} = 160$ <hr/> <p>🎯 Answer:</p> <p>160 defects occurred in total.</p>
Q.3	<p>a) What is Process Capability Index? A machining process produces shafts with a specification limit of: LSL = 48 mm, USL = 52 mm. From process data, the following statistics are obtained: Process mean (μ) = 50.5 mm. Process standard deviation (σ) = 0.5 mm. Calculate (C_p), (C_{pk}) and Comment on the process capability and</p>



centering.(8 marks),(2 marks for each answer)

Ans. Process Capability Index (PCI):The Process Capability Index is a statistical measure used in Six Sigma and quality management to determine how well a process can produce output within specified limits. It compares the natural variation of a process to the customer's specification limits. **Interpretation**

- **Cp or Cpk < 1.0** → Process is **not capable**
- **Cp or Cpk = 1.0** → Process barely meets specifications
- **Cp or Cpk ≥ 1.33** → Process is capable
- **Cp or Cpk ≥ 2.0** → World-class performance (Six Sigma)

2. Calculate Cpk

$$Cpk = \min \left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right)$$

$$\frac{USL - \mu}{3\sigma} = \frac{52 - 50.5}{1.5} = 1.0$$

$$\frac{\mu - LSL}{3\sigma} = \frac{50.5 - 48}{1.5} = 1.67$$

$$Cpk = \min(1.0, 1.67) = 1.0$$

Solution

1. Calculate Cp

$$Cp = \frac{USL - LSL}{6\sigma}$$

$$Cp = \frac{52 - 48}{6 \times 0.5} = \frac{4}{3} = 1.33$$

3. Interpretation

- **Cp = 1.33** indicates the process has **potential capability**.
- **Cpk = 1.0** shows the process is **not well centered** and is closer to the USL.
- Since **Cpk < Cp**, the process mean is shifted.

OR

b) What is Process Capability Index? A machining process produces shafts with a specification limit of: **LSL = 95 mm, USL = 105 mm**. From process data, the following statistics are obtained: **Process mean (μ) = 102 mm, Process standard deviation (σ) = 1.5 mm**. Calculate (Cp), (Cpk) and Comment on the process capability. (8 marks),(2 marks for each answer)

Ans. Process Capability Index (PCI)

Process Capability Indices are statistical measures that indicate how well a process can



produce output within specified limits. They compare the natural variability of a process (based on its standard deviation) with the engineering specification limits.

Given data

- Lower Specification Limit (LSL) = 95 mm
- Upper Specification Limit (USL) = 105 mm
- Process mean, $\mu=102$ mm
- Process standard deviation, $\sigma=1.5$ mm

1. Process Capability Index, C_p

$$C_p = \frac{USL - LSL}{6\sigma}$$
$$C_p = \frac{105 - 95}{6 \times 1.5} = \frac{10}{9} = 1.11$$

↓

2. Process Capability Index considering centering, C_{pk}

$$C_{pk} = \min \left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right)$$
$$\frac{USL - \mu}{3\sigma} = \frac{105 - 102}{4.5} = 0.67$$
$$\frac{\mu - LSL}{3\sigma} = \frac{102 - 95}{4.5} = 1.56$$
$$C_{pk} = \min(0.67, 1.56) = 0.67$$

3. Comment on process capability

- $C_p=1.11$ indicates that the **process spread is slightly better than the specification width**.
- $C_{pk}=0.67 < 1$ shows that the **process is not well centered** and is shifted toward the upper specification limit.
- Although the variability is acceptable, the process **is not capable of consistently meeting specifications** due to poor centering.

Conclusion: The process needs mean adjustment (centering) to improve capability.



c) Analyse and write in detail the key steps in Process Analysis
(8 marks), (1.5 marks for each step)

Ans.1. Process Identification

The first step is to identify the process that needs analysis. This includes defining the process boundaries, objectives, and its importance to organizational goals. Key stakeholders and customers of the process are also identified.

2. Process Mapping

A detailed process map or flowchart is created to visually represent each step involved in the process. This helps in understanding the sequence of activities, decision points, inputs, outputs, and interactions between different functions.

3. Data Collection

Relevant data related to the process is collected, such as cycle time, defect rates, cost, delays, and resource utilization. Accurate data is essential to evaluate current process performance objectively.

4. Process Performance Measurement

Performance metrics are established to assess how well the process is performing. Common measures include efficiency, effectiveness, quality, throughput, and customer satisfaction. This step helps in identifying performance gaps.

5. Identification of Bottlenecks

The process is analyzed to identify bottlenecks, redundancies, delays, and non-value-added activities. Tools such as value stream mapping and Pareto analysis are often used to pinpoint problem areas.

6. Root Cause Analysis

Once problem areas are identified, root cause analysis is conducted to determine the underlying reasons for inefficiencies or defects. Techniques such as fishbone diagrams, 5-Why analysis, and cause-and-effect matrices are commonly used.

7. Development of Improvement Opportunities

Based on the analysis, potential improvement solutions are generated. These may include process simplification, automation, elimination of waste, resource reallocation, or standardization of procedures.

8. Implementation and Monitoring

The selected improvements are implemented, and the process is continuously monitored using performance indicators and control measures. Feedback is used to ensure sustained improvement and to make further refinements if necessary.

OR

d) Analyse the given below tools used in process analysis with a diagram and example

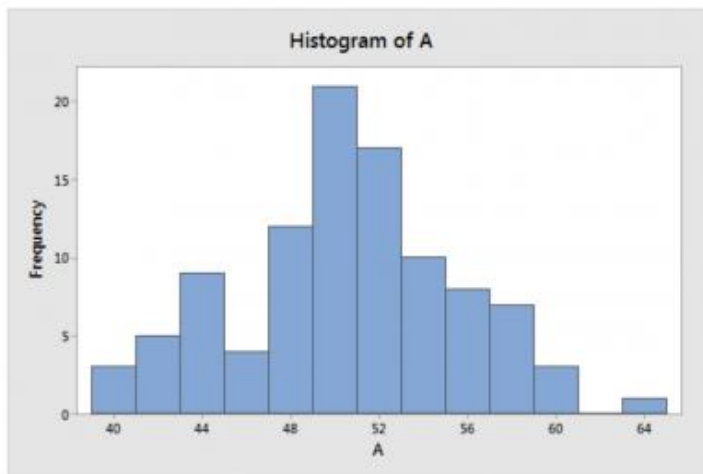
1) Value Stream Mapping 2) Histogram

(8 marks) ,(4 marks for each)

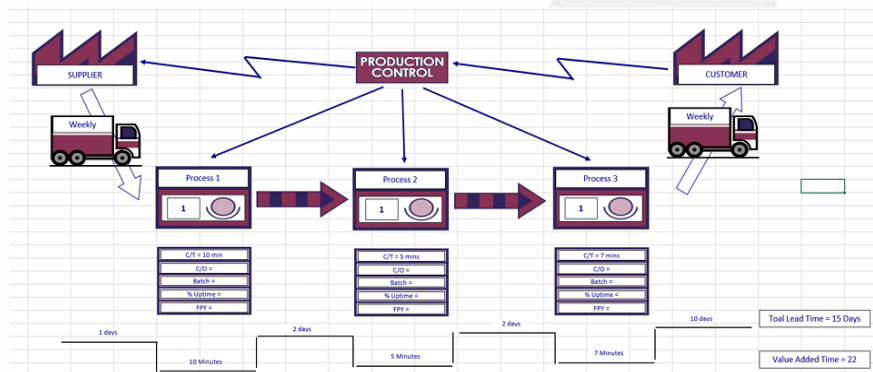
Ans. This is a bar graph showing the frequency of a set of data, usually continuous data. The histogram allows you to see

- the center of the data,
- the range of the data, and
- the distribution of the data.

It is a very useful snapshot. The downside is that you can't see the sequence or order of the data.



Value Stream Maps



2)



Value Stream Maps

Typical Process Information

C/T =	Cycle Time
C/O =	Changeover Time
Batch =	Batch Size
% Uptime =	% Uptime
FPY =	First Pass Yield

a) A teacher wants to check whether **different teaching methods** affect students' **test scores**. **Method 1:** Lecture, **Method 2:** Group discussion, **Method 3:** Online learning. The teacher tests **4 students per method** after a month. Apply ANOVA to interpret the results. Given value of alpha is 0.05

Method	Student 1	Student 2	Student 3	Student 4
Lecture	70	75	72	68
Group Discussion	78	82	85	80
Online Learning	88	85	90	92

(8 marks), (1.5 marks each step)

Ans. Step 1: State the Hypotheses

- **Null Hypothesis (H_0):** All teaching methods have the **same mean score**.
- **Alternative Hypothesis (H_1):** At least one teaching method has a **different mean score**. Significance level: $\alpha = 0.05$

• **Number of groups (k) = 3**, • **Number of observations per group (n) = 4**, • **Total observations (N) = 12**

Step 3: Calculate the Grand Mean

$$\text{Grand Mean (GM)} = \frac{\text{Total of all scores}}{N} = \frac{285 + 325 + 355}{12} = \frac{965}{12} = 80.42$$

Q.4



Step 4: Calculate Sum of Squares Between Groups (SSB)

$$SSB = \sum n_i(\bar{X}_i - GM)^2$$

- Lecture: $4 \times (71.25 - 80.42)^2 = 4 \times (-9.17)^2 = 4 \times 84.06 = 336.24$
- Group Discussion: $4 \times (81.25 - 80.42)^2 = 4 \times (0.83)^2 = 4 \times 0.69 = 2.76$
- Online Learning: $4 \times (88.75 - 80.42)^2 = 4 \times (8.33)^2 = 4 \times 69.39 = 277.56$

$$SSB = 336.24 + 2.76 + 277.56 = 616.56$$

Step 5: Calculate Sum of Squares Within Groups (SSW)

$$SSW = \sum (X_{ij} - \bar{X}_i)^2$$

- Lecture:
 $(70 - 71.25)^2 + (75 - 71.25)^2 + (72 - 71.25)^2 + (68 - 71.25)^2 = 1.56 + 14.06 + 0.56 + 10.56 = 26.74$
- Group Discussion:
 $(78 - 81.25)^2 + (82 - 81.25)^2 + (85 - 81.25)^2 + (80 - 81.25)^2 = 10.56 + 0.56 + 14.06 + 1.56 = 26.74$
- Online Learning:
 $(88 - 88.75)^2 + (85 - 88.75)^2 + (90 - 88.75)^2 + (92 - 88.75)^2 = 0.56 + 14.06 + 1.56 + 10.56 = 26.74$

$$SSW = 26.74 + 26.74 + 26.74 = 80.22$$

Step 6: Calculate Degrees of Freedom

- Between groups: $df_b = k - 1 = 3 - 1 = 2$
- Within groups: $df_w = N - k = 12 - 3 = 9$

Step 7: Calculate Mean Squares

$$MSB = \frac{SSB}{df_b} = \frac{616.56}{2} = 308.28$$

$$MSW = \frac{SSW}{df_w} = \frac{80.22}{9} = 8.91$$



Step 8: Calculate F-Statistic

$$F = \frac{MSB}{MSW} = \frac{308.28}{8.91} \approx 34.6$$

Step 9: Compare with Critical F-Value

- Degrees of freedom: $df_1 = 2, df_2 = 9$
- From F-table at $\alpha = 0.05$: $F_{critical} \approx 4.26$

$$F_{calculated} = 34.6 > F_{critical} = 4.26$$

Step 10: Conclusion

- Since $F_{calculated} > F_{critical}$, **reject H_0** .
- **Interpretation:** There is a **significant difference** in students' test scores between at least one pair of teaching methods.

Observation: From the means, Online Learning (88.75) > Group Discussion (81.25) > Lecture (71.25), suggesting Online Learning is the most effective.

OR

b) A teacher wants to check the effectiveness of three study methods on test scores. Apply ANOVA to interpret the results. Given value of alpha is 0.05

Group	SCORES
Group A	70,75,80
Group B	85,90,88
Group C	72,78,74

(8 marks), (1.5 marks each step)

Ans.

- **STEP 1: State the Hypotheses**
- **H_0 (Null):** $\mu_A = \mu_B = \mu_C$
→ All group means are equal
- **H_1 (Alternative):** At least one mean is different
- **STEP 2: Choose the Significance Level**
- Typically $\alpha = 0.05$
- **STEP 3: Calculate Means**
- **Group Means**
- A: $(70 + 75 + 80) / 3 = 75$
- B: $(85 + 90 + 88) / 3 = 87.67$



- C: $(72 + 78 + 74) / 3 = 74.67$
- **Grand Mean (GM)**
- All scores added:
 $70 + 75 + 80 + 85 + 90 + 88 + 72 + 78 + 74 = 712$
- Grand mean:
 $GM = 712 / 9 = 79.11$
- **STEP 4: Compute Variations**
- ANOVA separates total variation into:
- **A. Between-Group Variation (SSB)**
- Formula:

$$SSB = \sum n_i(\bar{X}_i - GM)^2$$

- Compute each term:
- Group A: $3(75 - 79.11)^2 = 3 \times 16.89 = 50.67$
- Group B: $3(87.67 - 79.11)^2 = 3 \times 73.25 = 219.75$
- Group C: $3(74.67 - 79.11)^2 = 3 \times 19.73 = 59.19$
- Total SSB = $50.67 + 219.75 + 59.19 = 329.61$
- **B. Within-Group Variation (SSW)**
- Formula:

$$SSW = \sum (X_{ij} - \bar{X}_i)^2$$

- Compute for each group:
- **Group A (mean = 75)**
- $(70-75)^2 = 25$
- $(75-75)^2 = 0$
- $(80-75)^2 = 25$
- **SSW_A = 50**
- **Group B (mean = 87.67)**
- $(85-87.67)^2 = 7.11$
- $(90-87.67)^2 = 5.44$
- $(88-87.67)^2 = 0.11$
- **SSW_B = 12.66**
- **Group C (mean = 74.67)**
- $(72-74.67)^2 = 7.11$
- $(78-74.67)^2 = 11.11$
- $(74-74.67)^2 = 0.44$
- **SSW_C = 18.66**
- Total SSW = $50 + 12.66 + 18.66 = 81.32$
- **STEP 5: Degrees of Freedom**
- **df_{between} = k - 1 = 3 - 1 = 2**



- $df_{\text{within}} = N - k = 9 - 3 = 6$
- **STEP 6: Compute Mean Squares**
- $MSB = SSB / df_{\text{between}}$
 $= 329.61 / 2$
 $= 164.81$
- $MSW = SSW / df_{\text{within}}$
 $= 81.32 / 6$
 $= 13.55$
- **STEP 7: Calculate the F-Statistic**

$$F = \frac{MSB}{MSW} = \frac{164.81}{13.55} = 12.17$$

- **STEP 8: Compare F to Critical Value or p-value**
- For $df_1 = 2$ and $df_2 = 6$ at $\alpha = 0.05$:
Critical F ≈ 5.14
- Since:
 $12.17 > 5.14 \rightarrow$ **Reject H_0**
- Meaning the study methods do **not** all perform the same.
- **STEP 9: Conclusion**

There is a **significant difference** between the means of at least one pair of study methods

c) A machine fills sugar packets with a claimed mean of 500 g.
The population standard deviation is known to be 8 g.
A sample of 40 packets has a mean weight of 497.5 g.
Test at **5% significance** if the machine is underfilling. (Zcritical from table is -1.645)
(8 marks), (2 marks for each step)

Step 1: Hypotheses

$$H_0: \mu = 500$$

$$H_1: \mu < 500$$

Step 2: Test statistic

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{497.5 - 500}{8 / \sqrt{40}} = \frac{-2.5}{1.2649} = -1.98$$

Step 3: Critical value ($\alpha = 0.05$, left-tailed)

$$Z_\alpha = -1.645$$

Step 4: Decision

$$Z = -1.98 < -1.645 \rightarrow \text{reject } H_0$$

Conclusion:

There is significant evidence the machine is underfilling.



OR

d) A sample of 16 batteries has mean life 40 hours and standard deviation 6 hours. Test at 1% significance whether the population mean life is **different from 44 hours**.

.(value of $t_{critical}$ from table is ± 2.947), (8 marks), (2 marks for each step)

Ans.

Solution:

Step 1: Hypotheses

$H_0: \mu = 44$

$H_1: \mu \neq 44$

Step 2: Test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{40 - 44}{6/\sqrt{16}} = \frac{-4}{1.5} = -2.67$$

Step 3: Degrees of freedom: 15

Critical t ($\alpha=0.01$, two-tailed): ± 2.947

Step 4: Decision

$|t| = 2.67 < 2.947 \rightarrow$ fail to reject H_0 .

Conclusion:

No significant evidence that the true mean differs from 44 hours.

a) **Evaluate** the effectiveness of \bar{X} -R charts versus p-charts in monitoring a manufacturing process, and justify which is better under different circumstances.

(8 marks), (4 marks for each chart)

Ans.1. Nature of Data Monitored

- \bar{X} -R Charts are used for **variable (continuous) data** such as length, weight, diameter, or temperature.
- **p-Charts** are used for **attribute (discrete) data**, specifically the **proportion of defective items** in a sample. **Effectiveness depends on the type of data available.**

2. Sensitivity to Process Variation

Q.5

- \bar{X} -R Charts are highly sensitive to small shifts in process mean and variability because they track both the average (\bar{X}) and range (R).
- **p-Charts** are less sensitive to small changes and mainly detect larger shifts in defect proportions. **\bar{X} -R charts are more effective for precise process**



control.

3. Information Provided

- **\bar{X} -R Charts** provide detailed insight into process behavior, allowing identification of assignable causes affecting variability.
- **p-Charts** only indicate whether the fraction defective is in or out of control, offering limited diagnostic information. **\bar{X} -R charts offer richer process information.**

4. Sample Size Requirements

- **\bar{X} -R Charts** require small, consistent sample sizes taken at regular intervals.
- **p-Charts** can accommodate **varying sample sizes**, making them more flexible in inspection-based processes. **p-Charts are effective where sample sizes vary.**

5. Cost and Ease of Implementation

- **\bar{X} -R Charts** require precise measurement instruments and trained personnel.
- **p-Charts** are simpler and cheaper since they rely on pass/fail inspection. **p-Charts are easier to implement in early-stage quality control.**

6. Manufacturing Context Suitability

- **\bar{X} -R Charts** are best suited for **stable, high-volume processes** where continuous measurements are feasible.
- **p-Charts** are more effective in **assembly operations or final inspection stages** where defects are counted.

OR

b) **Compare and Evaluate** the effectiveness of different types of control charts (\bar{X} -R, p-chart, c-chart) in reducing defects in a production environment.

(8 marks), (2.5 marks for each type)

Ans.1. \bar{X} -R Chart

- Used for **variable (continuous) data** such as dimensions, weight, and thickness.
- Monitors both **process mean (\bar{X})** and **variability (R)**, making it highly sensitive to small shifts.
- **Most effective in reducing defects** in precision manufacturing where tight tolerances are critical.
- Requires accurate measurement systems and consistent sample sizes.



2. p-Chart

- Used for **attribute data**, specifically the **proportion of defective items**.
- Effective in monitoring **overall defect rates** rather than process variation.
- Suitable for assembly lines and final inspection stages where items are classified as good or defective.
- Less sensitive to small changes but easier and cheaper to implement.

3. c-Chart

- Used for **counting the number of defects per unit** when inspection area or unit size is constant.
- Effective in detecting repeated or multiple defects in a single product (e.g., surface defects).
- Helps identify processes with recurring quality issues.
- Not suitable when sample sizes or inspection units vary.

Comparative Evaluation

- **\bar{X} -R charts are most effective** for early detection of variation and long-term defect prevention.
- **p-charts are effective** for monitoring defect proportions in inspection-based environments.
- **c-charts are effective** for identifying defect concentration in individual units.

c) A process manufactures bolts. Every hour, a **sample of 4 bolts** is taken and their **lengths (mm)** are measured. SPC Constants (for sample size $n = 4$) $A_2 = 0.729, D_3 = 0, D_4 = 2.282$. Apply SPC to find if the process is under control

Sample Measurements (mm)

1	50, 51, 49, 50
2	50, 50, 51, 49
3	51, 50, 50, 51
4	49, 50, 50, 49

(8 marks) (2 mark for each step)



Step 1: Organize Data and Calculate Sample Mean (\bar{X}) and Range (R)

Sample	Measurements (mm)	\bar{X}	R
1	50, 51, 49, 50	$(50+51+49+50)/4 = 50.0$	$51-49 = 2$
2	50, 50, 51, 49	$(50+50+51+49)/4 = 50.0$	$51-49 = 2$
3	51, 50, 50, 51	$(51+50+50+51)/4 = 50.5$	$51-50 = 1$
4	49, 50, 50, 49	$(49+50+50+49)/4 = 49.5$	$50-49 = 1$

Ans.

Step 2: Calculate Overall Mean and Average Range

$$\bar{\bar{X}} = \frac{\text{Sum of sample means}}{\text{number of samples}} = \frac{50.0 + 50.0 + 50.5 + 49.5}{4} = 50.0$$

$$\bar{R} = \frac{\text{Sum of sample ranges}}{4} = \frac{2 + 2 + 1 + 1}{4} = 1.5$$

Step 3: Calculate Control Limits

\bar{X} -Chart Control Limits

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} = 50.0 + 0.729 \times 1.5 = 50.0 + 1.0935 \approx 51.09$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R} = 50.0 - 1.0935 \approx 48.91$$

R-Chart Control Limits

$$UCL_R = D_4\bar{R} = 2.282 \times 1.5 \approx 3.42$$

$$LCL_R = D_3\bar{R} = 0 \times 1.5 = 0$$



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Step 4: Check if Process is Under Control

Sample	\bar{X}	R	\bar{X} within LCL-UCL?	R within LCL-UCL? 
1	50.0	2	Yes (48.91-51.09)	Yes (0-3.42)
2	50.0	2	Yes	Yes
3	50.5	1	Yes	Yes
4	49.5	1	Yes	Yes

Step 5: Conclusion

- The process is **under statistical control**.
- Variation is within expected limits.
- No immediate corrective action is required.

OR

d) A process produces shafts. Every hour, a sample of 4 shafts is taken and their diameters (mm) are measured. SPC Constants (for sample size $n = 4$) $A_2 = 0.729, D_3 = 0,$

$D_4 = 2.282$. Apply SPC to find if the process is under control

Sample Measurements (mm)

1	20, 21, 19, 20
2	20, 20, 21, 19
3	21, 20, 20, 21
4	19, 20, 20, 19

(8 marks) , (2 mark for each point)

Step 1: Organize Data and Calculate Sample Mean (\bar{X}) and Range (R)

Sample	Measurements (mm)	\bar{X}	R
1	20, 21, 19, 20	$(20+21+19+20)/4 = 20.0$	$21-19 = 2$
2	20, 20, 21, 19	$(20+20+21+19)/4 = 20.0$	$21-19 = 2$
3	21, 20, 20, 21	$(21+20+20+21)/4 = 20.5$	$21-20 = 1$
4	19, 20, 20, 19	$(19+20+20+19)/4 = 19.5$	$20-19 = 1$

Ans.



Step 2: Calculate Overall Mean and Average Range

$$\bar{\bar{X}} = \frac{20.0 + 20.0 + 20.5 + 19.5}{4} = 20.0$$

$$\bar{R} = \frac{2 + 2 + 1 + 1}{4} = 1.5$$

Step 3: Calculate Control Limits

\bar{X} -Chart Control Limits

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} = 20.0 + 0.729 \times 1.5 = 20.0 + 1.0935 \approx 21.09$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R} = 20.0 - 1.0935 \approx 18.91$$

R-Chart Control Limits

$$UCL_R = D_4\bar{R} = 2.282 \times 1.5 \approx 3.42$$

$$LCL_R = D_3\bar{R} = 0 \times 1.5 = 0$$

Step 4: Check if Process is Under Control

Sample	\bar{X}	R	\bar{X} within LCL-UCL?	R within LCL-UCL?
1	20.0	2	Yes (18.91–21.09)	Yes (0–3.42)
2	20.0	2	Yes	Yes
3	20.5	1	Yes	Yes
4	19.5	1	Yes	Yes

All sample means and ranges lie within the control limits.

Step 5: Conclusion: The process is **under statistical control**, Variation in shaft diameters is **within acceptable limits**, No immediate corrective action is required.