Model Answer

End-Sem Examination, Winter 2025

Academic Year: 2025-2026	Semester: I
Class: PG-I	Program: MTech
Branch Code: CIV	Pattern: 2024
Name of Course: Numerical Methods	Course Code: 2404501

Q1)

Given equation:
$2x^3 - 3x + 4 = 0$
Let
$f(x) = 2x^3 - 3x + 4$
$f'(x) = 6x^2 - 3$
Newton–Raphson formula:
$x(n+1) = x(n) - f(x(n)) / f'(x(n))$
Take initial approximation:
$x_0 = -1.500$
ITERATION 1
$f(-1.500) = 2(-1.500)^3 - 3(-1.500) + 4$
$f(-1.500) = 1.750$
$f'(-1.500) = 6(-1.500)^2 - 3$
$f'(-1.500) = 10.500$
$x_1 = -1.500 - (1.750 / 10.500)$
$x_1 = -1.667$
ITERATION 2
$f(-1.667) = 0.179$
$f'(-1.667) = 13.667$
$x_2 = -1.667 - (0.179 / 13.667)$
$x_2 = -1.680$
ITERATION 3
$f(-1.680) = -0.015$
$f'(-1.680) = 13.934$
$x_3 = -1.680 - (-0.015 / 13.934)$
$x_3 = -1.648$
FINAL ANSWER
The real root of the given equation is:
$x = -1.648$

Python Code:

```
def f(x):
return 2x**3 - 3x + 4
def df(x):
return 6*x**2 - 3
```

```
x = -1.5
for i in range(5):
x = x - f(x)/df(x)
print(round(x, 3))
```

Q2)

Eulers Method

Given:	x =	0	y =	1
	dy/dx =	2x-3y		
To find:	y when x =	0.5	h =	0.1

x	y	dy/dx	new y = old y +h(dy/dx)
0.000	1.000	-3.000	**0.700**
0.100	0.700	-1.900	**0.510**
0.200	0.510	-1.130	**0.397**
0.300	0.397	-0.591	**0.338**
0.400	0.338	-0.214	**0.317**
0.500	0.317		

The solution is 0.317

Q3a)

Simpson's 1/3rd Rule

Given: $y = \int_0^1 \frac{1}{1+x^2} dx$

Integration Limits: a = 0 b = 1

n = 4 h = 0.2

The values of x and y are as follows

x	0	0.2	0.4	0.6	0.8	1
y	**1.000**	**0.962**	**0.862**	**0.735**	**0.610**	**0.500**
	y0	**y1**	**y2**	**y3**	**y4**	**y5**

By Simpson's 1/3rd Rule, we have,

$$y = \int_0^1 1/(1+x^2)dx = \frac{h}{3}[(y0 + yn) + 4(odd\ values\ of\ y)$$

$$+2(even\ values\ of\ y)]$$

y = 0.749

Q3b)

Trapezoidal Rule

Given: $y = \int_{0.5}^{1.5} \left(e^{x^3} + 2x^2 + 3\right) dx$

Integration Limits: a = 0.5 b = 1.5

n = 5 h = 0.2

The values of x and y are as follows

x	0.500	0.700	0.900	1.100	1.300	1.500
y	4.633	5.389	6.693	9.205	15.378	36.724
	y0	**y1**	**y2**	**y3**	**y4**	**y5**

By Trapezoidal Rule, we have

$$y = \int_{0.5}^{1.5} \left(e^{x^3} + 2x^2 + 3\right) dx = \frac{h}{2}[(y0 + yn) + 2(remaining\ values\ of\ y)]$$

y = 11.469

Q3c)

Given: $y = \int_{0}^{6} \frac{dx}{1 + 2x + 3x^2}$

Integration Limits: a = 0 b = 6

n = 6 h = 1

The values of x and y are as follows

x	0	1	2	3	4	5	6
y	1.000	0.167	0.059	0.029	0.018	0.012	0.008
	y0	**y1**	**y2**	**y3**	**y4**	**y5**	**y6**

By Simpson's 3/8th Rule, we have,

$$y = \int_{0}^{6} \frac{dx}{1 + 2x + 3x^2} = \frac{3h}{8}[(y0 + yn) + 3(values\ of\ y\ except\ multiple\ of\ 3) + 2(values\ of\ y\ in\ multiple\ of\ 3)]$$

y = 0.687

Q3d)

Double Integration Trapezoidal's rule					
y\x	**0**	**0.25**	**0.5**	**0.75**	**1**
0	0.000	0.063	0.250	0.563	1.000
0.25	0.063	0.188	0.438	0.813	1.313
0.5	0.250	0.438	0.750	1.188	1.750
0.75	0.563	0.813	1.188	1.688	2.313
1	1.000	1.313	1.750	2.313	3.000

$$\int_0^1 \int_0^1 x^2 + xy + y^2 \, dxdy = 0.938$$

Q4a)

Method of Least Squares

x	0	2	4	6	8
y	8	13	21	34	55

	x	y	xy	x^2
	0	8	0	0
	2	13	26	4
	4	21	84	16
	6	34	204	36
	8	55	440	64
Σ=	**20**	**131**	**754**	**120**

Normalized equations:

131 = 5c + 20m
754 = 20c + 120m

Solving simultaneously:

m = 5.75
c = 3.2

y = 5.75x + 3.2

Q4b)

Polynomials

x	1	3	5	7	9
y	15	24	33	41	48

	x	y	xy	x^2y	x^2	x^3	x^4
	1	15	15	15	1	1	1
	3	24	72	216	9	27	81
	5	33	165	825	25	125	625
	7	41	287	2009	49	343	2401
	9	48	432	3888	81	729	6561
Σ=	**25**	**161**	**971**	**6953**	**165**	**1225**	**9669**

Normalized equations:

161 = 5a + 25b +
165c
971 = 25a + 165b +1225c
6953 = 165a + 1225b +9669c

Solving simultaneously:
a = 9.932
b = 5.043
c = -0.089
y = 9.932 5.043x +-0.089x^2

Q4c)

Lagrange's Interpolation			Find f(4)	4
x	1	3	5	7
y	6	10	14	18

$$y = \frac{(4-3)(4-5)(4-7)}{(1-3)(1-5)(1-7)} \times 6$$

$$+ \frac{(4-1)(4-5)(4-7)}{(3-1)(3-5)(3-7)} \times 10$$

$$+ \frac{(4-1)(4-3)(4-7)}{(5-1)(5-3)(5-7)} \times 14$$

$$+ \frac{(4-1)(4-3)(4-5)}{(7-1)(7-3)(7-5)} \times 18$$

= -0.375 + 5.625 + 8 + -1.125

= 12

Q4d)

Spline Interpolation — Find f(40) — 40

x	0	17	32	54
y	27	69	95	132

$$y = 69 + \frac{(95 - 69)}{(32 - 17)} \times (40 - 17)$$

y = 109

Q5a)

x	0	1	2	3
y	1	2	1	10

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	**1**			
		1		
1	**2**		-2	
		-1		12
2	**1**		10	
		9		
3	**10**			

Using Newton's forward interpolation formula;

$f(x) = 2x^3-7x^2+6x+1$

Q5b)

x(ht)	100	150	200	250	300	350	400
y(dist)	10.63	13.03	15.04	16.81	18.42	19.9	21.27

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
100	**10.63**						
		2.4					
150	**13.03**		-0.39				
		2.01		0.15			
200	**15.04**		-0.24		-0.07		
		1.77		0.08		0.02	
250	**16.81**		-0.16		-0.05		0.02
		1.61		0.03		0.04	
300	**18.42**		-0.13		-0.01		
		1.48		0.02			
350	**19.9**		-0.11				
		1.37					
400	**21.27**						

Using Newton's backward interpolation formula;

y_{410} = 21.53 nautical miles

Q5c)

x	10	11	12	13	14
y	23967	28060	31788	35209	38368

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
10	**23967**				
		4093			
11	**28060**		-365		
		3728		58	
12	**31788**		-307		-13
		3421		45	
13	**35209**		-262		
		3159			
14	**38368**				

Using Stirling's formula;

$y_{12.2}$ = 32497

Q5d)

x	20	25	30	35	40	45
y	354	332	291	260	231	204

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
20	**354**					
		-22				
25	**332**		-19			
		-41		29		
30	**291**		10		-37	
		-31		-8		45
35	**260**		2		8	
		-29		0		
40	**231**		2			
		-27				
45	**204**					

Using Gauss' forward interpolation formula;

f(22) = 347.983