K. K. Wagh Institute of Engineering Education and Research, Nashik

(An Autonomous Institute from A.Y. 2022-23)

Marking Scheme

End-Sem Examination-I, Winter 2025

Academic Year: 2025-2026	Semester: II
Class: F.Y. M. Tech	Program: M. Tech
Branch Code: CIV	Pattern: 2024
Name of Course: Finite Element Method	Course Code: 2404512

Q.	Answer	Step Marks
1	Different types of elements in FEM 1D elements…….. (1) 2D elements- ……. (3) 3D elements……… (2)	 1 3 2
2	Explain in detail all the steps of Finite Element Method. Each step 1 mark	6
3	a) Using generalized co-ordinate approach find shape functions for two nodded bar element where $N_1 = \frac{x_2 - x}{l}$ and $N_2 = \frac{x - x_1}{l}$ Thus the shape function $[N]$ is $[N] = [N_1 \quad N_2] = \begin{bmatrix} \frac{x_2 - x}{l} & \frac{x - x_1}{l} \end{bmatrix}$ ***Answer***	8 1 mark each step
	OR	
	b) Determine the shape functions for the Constant Strain Triangle (CST) using polynomial functions. ***Solution***: Figure 5.7 shows a typical CST element. Let the nodal variables be u_1, u_2, u_3, v_1, v_2 *and* v_3 i.e., and $N_1 = \frac{a_1 + b_1 x + c_1 y}{2A}$ $N_2 = \frac{a_2 + b_2 x + c_2 y}{2A}$ and $N_3 = \frac{a_3 + b_3 x + c_3 y}{2A}$	8 1 mark each step
	c) A three nodded triangular element as shown in fig. 3(c) is used in plane elasticity problem. Find shape functions. (8) $[N] = [P][A]^{-1} \ldots\ldots\ldots (1)$ $[N] = [1 \quad x \quad y] \begin{bmatrix} 1 & 0 & 0 \\ 1 & 7 & 0 \\ 1 & 3 & 5 \end{bmatrix}^{-1} \ldots\ldots\ldots.. (1)$ $[N] = [1 \quad x \quad y] \frac{1}{35} \begin{bmatrix} 35 & 0 & 0 \\ -5 & 5 & 0 \\ -4 & -3 & 7 \end{bmatrix} \ldots\ldots\ldots\ldots (3)$ $N_1 = \frac{35 - 5x - 4y}{35} \ldots\ldots.. (1)$ $N_2 = \frac{5x - 3y}{35} \ldots\ldots.. (1)$ $N_3 = \frac{7y}{35} \ldots\ldots.. (1)$	8
	OR	

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d) Coordinates of nodes of CST are shown in fig. 3(d). At interior point P if x = 2.8 and value of $N_1 = 0.3$, Find coordinate of point P and values of $N_2 \; and \; N_3$.

$$[N] = [P][A]^{-1} \ldots\ldots\ldots (1)$$

$$[N] = [1 \quad x \quad y]\begin{bmatrix} 1 & 6 & 4 \\ 1 & 4 & 7 \\ 1 & 2 & 3 \end{bmatrix}^{-1} \ldots\ldots\ldots..(1)$$

$$[N] = [1 \quad x \quad y]\frac{1}{14}\begin{bmatrix} 3 & -10 & 26 \\ 4 & -1 & -3 \\ -2 & 4 & -2 \end{bmatrix} \ldots\ldots\ldots\ldots (3)$$

$$y = 5.0 \ldots\ldots\ldots (1)$$

Using $x = 2.8 \; and \; y = 5.0$

$$N_2 = \frac{-10 - x + 4y}{14} = \frac{-10 - 2.8 + (4 \times 5.0)}{14} = 0.51 \ldots\ldots..(1)$$

$$N_3 = \frac{26 - 3 \times 2.8 - (2 \times 5.0)}{14} = 0.54 \ldots\ldots..(1)$$

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a) For an axisymmetric element state relation between Strain and Displacement. (8)

Thus, there are four strain components present in this case and is given by

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{rz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial r} \\ \frac{\partial w}{\partial z} \\ \frac{u}{r} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial r} & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{1}{r} & 0 \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial r} \end{bmatrix} \begin{Bmatrix} u \\ w \end{Bmatrix} \qquad (5.6.5)$$

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b) Derive Jacobian matrix for four nodded iso-parametric quadrilateral element.

1. Shape functions
2. Displacement
3. Strains
4. Jacobian Matrix

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{Bmatrix} = \begin{bmatrix} \frac{\partial \zeta}{\partial x} & \frac{\partial \eta}{\partial x} \\ \frac{\partial \zeta}{\partial y} & \frac{\partial \eta}{\partial y} \end{bmatrix} \begin{Bmatrix} \frac{\partial N_i}{\partial \zeta} \\ \frac{\partial N_i}{\partial \eta} \end{Bmatrix}$$

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \zeta} & \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \zeta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

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2 marks each step</td></tr>
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c) Determine Jacobian Matrix for four nodded iso-parametric quadrilateral element as shown in fig. 4c (8)

1. Parent element- (1)
2. Shape function- (1)
3. Jacobian Matrix- (6)

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	$[J] = \begin{bmatrix} \frac{7-3\eta}{4} & \frac{1+3\zeta}{4} \\ \frac{2}{4} & \frac{8}{4} \end{bmatrix}$	
	OR	
	d) Determine natural coordinates (ζ, η) of the any point P whose cartesian coordinates are (3, 4) for four nodded iso-parametric quadrilateral elements as shown in fig. 4(d) 1. Parent element 2. Shape function- 3. $\zeta, \eta =$	8 1 1 6
5	a) Explain with neat sketches the various three-dimensional elements used in the analysis of shells. (8) 1. Flat Elements 2. Curved Elements 3. Solid Elements 4. Degenerated Solid Elements. The above elements are briefly explained below and their performance is commented.	 2 2 2 2
	OR	
	b) Explain Mindlin's theory of plate element. Explanation	8
	c) Continuity Category I: C^2-Continuity element i.e. second order continuity elements in which seconderivates of '*w*' are also nodal unknowns. Category II: C^1-Continuity elements i.e. first order continuity elements in which highest order of derivatives of '*w*' is one only. Category III: C^0-Continuity element i.e. the elements in which only continuity of nodal variables are to be ensured.	2 3 3
	OR	
	d) Write displacement fields in 4 nodded degenerated shell elements stepwise derivation Substituting equation 16.13 in equation 16.9 we get $\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{4} N_i \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} + \frac{\zeta h_i}{2} \begin{Bmatrix} n_{3i}\theta_{yi} - m_{3i}\theta_{zi} \\ -n_{3i}\theta_{xi} - l_{3i}\theta_{zi} \\ m_{3i}\theta_{xi} - l_{3i}\theta_{yi} \end{Bmatrix}$	8