

NOV 2015

Total No. of Questions—8]

[Total No. of Printed Pages—4+2

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S.E. (Comp./I.T.) (Second Sem.) EXAMINATION, 2015

ENGINEERING MATHEMATICS-III

(2012 PATTERN)

Time : Two Hours

Maximum Marks : 50

N.B. :— (i) Attempt four questions : Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6 and Q. No. 7 or Q. No. 8.

- (ii) Neat diagrams must be drawn wherever necessary.
- (iii) Figures to the right indicate full marks.
- (iv) Use of non-programmable electronic pocket calculator is allowed.

(v) Assume suitable data, if necessary.

1. (a) Solve any two : [8]

(i)  $(D^2 - 2D - 3)y = 3e^{-3x} \sin e^{-3x} + \cos(e^{-3x})$

(ii)  $(D^2 - 2D + 2)y = e^x \tan x$ . (By variation of parameters)

(iii)  $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^2$ .

(b) Find the Fourier transform of  $e^{-x^2}$  and hence show that : [4]

$$\int_{-\infty}^{\infty} \frac{e^{i\lambda x}}{1 + \lambda^2} d\lambda = \pi e^{-x^2}.$$

P.T.O.

Or

2. (a) An uncharged condenser of capacity C charged by applying an e.m.f. of value  $E \sin \frac{t}{\sqrt{LC}}$  through the leads of inductance L and of negligible resistance. The charge Q on the plate of condenser satisfied the differential equation : [4]

$$\frac{d^2 Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L} \sin \frac{t}{\sqrt{LC}}.$$

Prove that the charge at any time t is given by :

$$Q = \frac{EC}{2} \left[ \sin \frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}} \cos \frac{t}{\sqrt{LC}} \right].$$

- (b) Find the Inverse Z-transform (any one) : [4]

(i)  $F(z) = \frac{z+2}{z^2-2z+1}$  for  $|z| > 1$ .

(ii)  $F(z) = \frac{10z}{(z-1)(z-2)}$  (Use inversion integral method).

- (c) Solve the following difference equation to find  $\{f(k)\}$  : [4]

$$f(k+1) + \frac{1}{4} f(k) = \left(\frac{1}{4}\right)^k, \quad k \geq 0, \quad f(0) = 0.$$

3. (a) The first four moments of a distribution about the value 4 are -1.5, 17, -30 and 108. Obtain the first four central moments, mean, standard deviation and coefficient of skewness and kurtosis. [4]

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- (b) If the probability that a concrete cube fails is 0.001. Determine the probability that out of 1000 cubes : [4]

(i) exactly two

(ii) more than one cubes will fail.

- (c) Show that : [4]

$$\vec{F} = (y \sin z - \sin x) \vec{i} + (x \sin z + 2yz) \vec{j} + (xy \cos z + y^2) \vec{k}$$

is irrotational and hence find scalar function  $\phi$  s.t.  $\vec{F} = \nabla\phi$ .

Or

4. (a) Find the directional derivative of  $\phi = 4xz^3 - 3x^2y^2z$  at  $(2, -1, 2)$  along a line equally inclined with co-ordinate axes. [4]

- (b) For a solenoidal vector field  $\vec{F}$ , show that : [4]

$$\text{curl curl curl } \vec{F} = \nabla^4 \vec{F}.$$

- (c) The regression equations are :

$$8x - 10y + 66 = 0 \text{ and } 40x - 18y = 214.$$

The value of variance of  $x$  is 9. Find :

- (i) The mean values of  $x$  and  $y$   
(ii) The correlation coefficient between  $x$  and  $y$   
(iii) The standard deviation of  $y$ . [4]

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P.T.O.

5. (a) Find the work done in moving a particle once round the ellipse : [4]

$$\frac{x^2}{16} + \frac{y^2}{4} = 1, \quad z = 0$$

under the field of force given by :

$$\vec{F} = (2x - y + z) \vec{i} + (x + y - z^2) \vec{j} + (3x - 2y + 4z) \vec{k}.$$

- (b) Evaluate : [4]

$$\iint_S (\nabla \times \vec{F}) \cdot \vec{n} \, dS$$

$$\text{where } \vec{F} = (x^3 - y^3) \vec{i} - xyz \vec{j} + y^2 \vec{k}$$

and  $S$  is the surface  $x^2 + 4y^2 + z^2 - 2x = 4$  above the plane  $x = 0$ .

- (c) Evaluate : [5]

$$\iint_S \vec{F} \cdot d\vec{S}$$

using divergence theorem, where

$$\vec{F} = x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}$$

and  $S$  is the surface of sphere  $x^2 + y^2 + z^2 = a^2$ .

Or

6. (a) If [4]

$$\vec{F} = x^2 \vec{i} + (x - y) \vec{j} + (y + z) \vec{k}$$

displaces a particle from  $A(1, 0, 1)$  to  $B(2, 1, 2)$  along the straight line  $AB$ , find work done.

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(b) Evaluate :

[4]

$$\int_C (e^x dx + 2y dy - dz)$$

where C is the curve  $x^2 + y^2 = 4$ ,  $z = 2$ .

(c) Evaluate :

[5]

$$\iint_S \vec{F} \cdot d\vec{S}$$

using Gauss divergence theorem, where :

$$\vec{F} = 2xy\vec{i} + yz^2\vec{j} + xz\vec{k}$$

and S is the region bounded by :

$$x = 0, y = 0, z = 0, y = 3, x + 2z = 6.$$

7. (a) Show that  $u = y^3 - 3x^2y$  is harmonic function. Find its harmonic conjugate and the corresponding analytic function  $f(z)$  in terms of  $z$ . [5]

(b) Using Cauchy's integral formula, evaluate : [4]

$$\int_C \frac{2z^2 + z + 5}{(z - 3/2)^2} dz$$

where C is  $\frac{x^2}{4} + \frac{y^2}{9} = 1$ .

(c) Find the bilinear transformation which maps the points  $z = 1, i, -1$ , onto the points  $w = 0, 1, \infty$ . [4]

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P.T.O.

Or

8. (a) If  $f(z)$  is an analytic function  $v^2 = u$ , then show that  $f(z)$  is constant function. [4]

(b) Using residue theorem evaluate : [5]

$$\int_C \frac{z}{z^4 + 13z^2 + 36} dz$$

where 'C' is the circle  $|z| = \frac{5}{2}$ .

(c) Find the map of the circle  $|z - i| = 1$  under the transformation  $w = \frac{1}{z}$  into  $w$ -plane. [4]

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