

May-2016

Total No. of Questions—8]

[Total No. of Printed Pages—4+1

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[4957]-1076

S.E. (Computer/IT) (II Semester) EXAMINATION, 2016

ENGG. MATHEMATICS-III

(2012 PATTERN)

Time : Two Hours

Maximum Marks : 50

N.B. :— (i) Attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.

(ii) Neat diagrams must be drawn wherever necessary.

(iii) Figures to the right indicate full marks.

(iv) Use of electronic pocket calculator is allowed.

(v) Assume suitable data, if necessary.

1. (a) Solve any two :

[8]

(i) $(D^2 - 2D)y = e^x \sin x$

(ii) $\frac{dx}{3z-4y} = \frac{dy}{4x-2z} = \frac{dz}{2y-3x}$

(iii) $(D^2 + 9)y = x^2 + 2x + \cos 3x.$

(b) By considering Fourier cosine transform of $f(x) = e^{-mx}$ ($m > 0$) prove that :

[4]

$$\int_0^\infty \frac{\cos \lambda x}{\lambda^2 + m^2} d\lambda = \frac{\pi}{2m} e^{-mx}, \quad m > 0, x > 0.$$

P.T.O.

Or

2. (a) A capacitor 10^{-3} farad is in series with emf of 20 volts and an inductor of 0.4 H at $t = 0$, the charge Q and current I are zero, find Q at any time. [4]

- (b) Find the inverse z transform (any one) : [4]

(i) $F(z) = \frac{1}{(z-2)(z-3)}$, by using inversion integral method.

(ii) $F(z) = \frac{1}{(z-3)(z-4)}$, $|z| > 4$.

- (c) Solve the following difference equation to find $f(k)$ [4]

$$f(k+2) + 3f(k+1) + 2f(k) = 0$$

$$f(0) = 0, f(1) = 2, k \geq 0.$$

3. (a) The first four moments about the working mean 3.5 of a distribution are 0.0375, 0.4546, 0.0609 and 0.5074. Calculate the first four moments about the mean. Also calculate the coefficient of skewness. [4]

- (b) On an average a box containing 10 articles is likely to have 2 defectives. If we consider a consignment of 100 boxes, how many of them are expected to have three or less defectives? [4]

- (c) If the directional derivative of $\phi = axy + byz + czx$ at $(1, 1, 1)$ has maximum magnitude 4 in a direction parallel to x-axis, find the values of a, b, c . [4]

Or

4. (a) Prove the following (any one) : [4]

$$(i) \quad \nabla \left(\frac{\vec{a} \cdot \vec{r}}{r^2} \right) = \frac{\vec{a}}{r^2} - \frac{2(\vec{a} \cdot \vec{r})}{r^4} \vec{r}$$

$$(ii) \quad \nabla^2 \left[\nabla \cdot \left(\frac{\vec{r}}{r^2} \right) \right] = \frac{2}{r^4}$$

- (b) Show that the vector field $\vec{F} = f(r)\vec{r}$ is always irrotational. Also determine $f(r)$ such that the field \vec{F} is solenoidal. [4]
- (c) Calculate the coefficient of correlation from the following information :

$$n = 10, \Sigma x = 40, \Sigma x^2 = 190, \Sigma y^2 = 200, \Sigma xy = 150, \Sigma y = 40. \quad [4]$$

5. (a) Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = \sin z \hat{i} + \cos x \hat{j} + \sin y \hat{k}$, and C is the boundary of the rectangle $0 \leq x \leq \pi, 0 \leq y \leq 1$ and $z = 3$. [4]
- (b) Use divergence theorem to evaluate :

$$\iiint_s (y^4 z^4 \hat{i} + z^4 x^4 \hat{j} + x^4 y^4 \hat{k}) \cdot d\vec{s}, \text{ where } s \text{ is the upper part of the sphere } x^2 + y^2 + z^2 = 4 \text{ above the } xoy \text{ plane.} \quad [4]$$

- (c) Use Stokes' Theorem to evaluate $\int_C (5y\hat{i} + 2x\hat{j} + 7y\hat{k}) \cdot d\vec{r}$, where C is the curve of intersection $x^2 + y^2 + z^2 = 4z$ and $x = z - 2$. [5]

Or

6. (a) Find the work done of \vec{F} round the curve C , where $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$ and C is the circle $x^2 + y^2 = 1$ and $z = 0$. [4]

- (b) Using divergence theorem evaluate $\iiint_s \vec{F} \cdot \hat{n} \, ds$, where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and s is all the surface, bounding the cube, $x = 0, x = 2, y = 0, y = 2, z = 0$ and $z = 2$. [5]

- (c) Evaluate $\iint \text{curl } \vec{F} \cdot \hat{n} \, ds$ for the surface of the paraboloid $z = 9 - (x^2 + y^2)$ above the xy plane, where $\vec{F} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xz + z^2)\hat{k}$. [4]

7. (a) If $V = r^n \sin n\theta$ find u such that $f(z) = u + iv$ is analytic and determine $f(z)$ in terms of z . [4]

- (b) Evaluate $\oint_c \frac{2z^2 + 2z + 1}{(z+1)^3(z-3)} dz$ where 'c' is $|z - 3| = 2$. [5]

- (c) Find the bilinear transformation which maps the points $-i, 0, 2 + i$ of the z -plane on to the points $0, 2i, 4$ of w -plane. [4]

Or

8. (a) If $f(z)$ is an analytic function of z , and $f(z) = u + iv$ prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) | \operatorname{Re} f(z) |^2 = 2 | F'(z) |^2 \quad [4]$$

(b) Evaluate $\oint_C \frac{dz}{(z^2 + 4)^2}$ where 'C' is $|z - i| = 2$ [5]

- (c) Show that the transformation $W = \frac{z-i}{z+i}$ maps the real axis in the z -plane into the circle $|W| = 1$. [4]