NOV - 2017

Total No. of Questions—8] [Total No. of Printed Pages—5

Seat No.	

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S.E. (Comp/IT.) (Second Semesters) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. :— (*i*) Attempt *four* questions : Q. **1** *or* Q. **2**; Q. **3** *or* Q. **4**, Q. **5** *or* Q. **6**, Q. **7** *or* Q. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^4 - 1)y = \cosh x \sinh x$

(*ii*) $(D^2 - 4D + 4)y = e^{2x} \sec^2 x$ (By variation of parameters)

(*iii*) $(x+1)^2 \dfrac{d^2y}{dx^2} + (x+1)\dfrac{dy}{dx} = (2x+3)(2x+4)$

(*b*) Find the Fourier sine integral of : [4]

$$f(x) = x^2, \quad 0 < x < a$$
$$= 0, \quad x > a$$

P.T.O.

Or

2. (*a*) An electric current consists of an inductance 0.1 henry, a resistance R of 20 ohms and a condenser of capacitance C of 25 microfarads. If the differential equation of electric circuit is : [4]

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=0,$$ then find the charge q and current i at any time t, given that at $t = 0$, $q = 0.05$ Coulombs,

$$i=\frac{dq}{dt}=0 \text{ when } t = 0.$$

(*b*) Find the Inverse Z–transform (any *one*) : [4]

(*i*) $F(z)=\frac{1}{(z-a)^3}$ (By using Inversion Integral Method).

(*ii*) $F(z)=\frac{z^2}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}, \quad |z| > \frac{1}{4}$

(*c*) Solve the following difference equation to find $\{f(k)\}$: [4]

$$f(k + 2) + 3f(k + 1) + 2 f(k) = 0,$$
$$f(0) = 0, f(1) = 1.$$

3. (*a*) Calculate the correlation coefficient for the following data : [4]

x	1	2	3	4	5
y	2	5	2	7	6

(*b*) A firm produces articles of which 0.1% are defective out of 600 articles. If wholesaler purchases 1000 such cases, how many can be expected to have two defectives ? [4]

(c) Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 8$ at the point $(1, -2, 1)$. [4]

Or

4. (a) Find the directional derivative of $xz^3 - x^2yz$ at the point $(2, 1, -1)$ in the direction of tangent to the curve $x = e^t\cos t,\ y = e^t\sin t,\ z = e^t$ at $t = 0$. [4]

(b) If $\bar{u}$ and $\bar{v}$ are irrotational vectors, then prove that $\bar{u} \times \bar{v}$ is solenoidal vector. [4]

(c) A random sample of 500 screws is drawn from a population which represents the size of screws. If a sample is distributed normally with a mean 3.15 cm and standard deviation 0.025 cm, find expected number of screws whose size falls between 3.12 cm and 3.2 cm. (Given for z = 1.2, area = 0.3849, $z = 2.0$, area = 0.4772). [4]

5. (a) Evaluate :

$$\int_C \bar{F}.d\bar{r} \quad \text{where } \bar{F} = z\bar{i} + x\bar{j} + y\bar{k} \quad \text{and}$$

C is the arc of the curve $x = \cos t,\ y = \sin t,\ z = t$ from $t = 0$ to $t = \pi$ [5]

(b) Evaluate $\iint_S (\nabla \times \bar{F}).\,d\bar{S}$ for $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$ where S is the surface of paraboloid $z = 9 - x^2 - y^2,\ z \geq 0$. [4]

(c) If $\bar{E} = \nabla\phi$ and $\nabla^2\phi = -4\pi\rho$, then prove that $\iint_S \bar{E}.\,d\bar{S} = -4\pi\iiint_V \rho\, dv$. [4]

Or

6. (*a*) Using Green's theorem, evaluate :

$$\int_C \left(\frac{1}{y}dx + \frac{1}{x}dy\right)$$ where C is the boundary of the region bounded by the parabola $y = \sqrt{x}$ and line $x = 1$ and $x = 4$. [5]

(*b*) Use divergence theorem to evaluate

$$\iint_S \left(y^2z^2\bar{i} + z^2x^2\bar{j} + x^2y^2\bar{k}\right).d\bar{S}$$

where S is the upper part of the sphere $x^2+y^2+z^2 = 9$ above XOY plane. [4]

(*c*) Prove that :

$$\int_C (\bar{a}\times\bar{r}).d\bar{r} = 2\bar{a}.\iint_S d\bar{S}$$

where S is any open surface with boundary C. [4]

7. (*a*) Determine the analytic function $f(z) = u + iv$ in terms of z. Whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$. [5]

(*b*) Using Cauchy's Integral Formula evaluate $\int_C \frac{\cos \pi z}{z^2-1}dz$ where C is the rectangle with vertices $2\pm i$, $-2\pm i$. [4]

(*c*) Find the bilinear transformation which maps the points $1, i, -1$ from z–plane onto the points $i, 0, -i$ of the W–plane. [4]

Or

8. (*a*) If $f(z) = u + iv$ be an analytic function find $f(z)$. If $u + v = r^2 (\cos 2\theta + \sin 2\theta)$. [5]

(*b*) Using residue theorem evaluate : [4]

$$\int_c \frac{z^3 - 5}{(z+1)^2 (z-2)} dz \quad \text{where C is } |z| = \frac{3}{2}.$$

(*c*) Find the mapping of the line $2y = x$ under the transformation $W = \frac{2z-1}{2z+1}$. [4]